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Earthquake Response Control of 3-Story Building Structures by Tuned Mass Damper

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Abstract*—The optimum design of tuned mass damper (TMD) for seismically excited building structures considering system parameter uncertainties is investigated in this paper. This involves optimization of the frequency and damping properties of TMD considering uncertain system parameters. In the design process the three story building structures are considered so that it makes improvement to the design procedures so far, where usually only single mode model is considered. The first order Taylor series expansion and total probability concept is applied to evaluate the unconditional response of three story building structures. The conditional second order information of the response quantities are obtained in random vibration framework using state space formulation. Subsequently, the maximum unconditional root mean square displacement of the three story building structures is considered as the objective function to find the optimum value TMD parameters. The numerical example is taken to elucidate the effect of parameters uncertainties on the optimization of TMD parameters and system performance.

***Index Terms*— Earthquake, Optimization, Tuned Mass Damper, Uncertain Parameters, Vibration Control.**

I. INTRODUCTION

A tuned mass damper (TMD) is a passive vibration control device consisting of a mass, damping, and a spring; it is attached to a main building structure for suppressing undesirable vibrations induced by earthquake loads. The natural frequency of the TMD is tuned in resonance with the fundamental mode of the building structure, so that the huge amount of the structural vibrating energy is transferred to the TMD and dissipated by the damping as the building structure is subjected to earthquake loads. One of the most important design issues is the parameter optimization. The parameters to be optimally designed include the mass, damping and stiffness of the TMD. The optimal design of TMD assuming deterministic system parameters has been developed by [1]-[3]. A major limitation of the deterministic approach is that the uncertainties in the performance-related decision variables cannot be included in the optimization process. But, the efficiency of dampers may be drastically reduced if the parameters are off tuned to the vibrating mode it is designed due to unavoidable presence of uncertainty in the system parameter. Thus, the probabilistic vibration control under uncertain parameters is gaining more important in recent past. Papadimitriou et al [4] proposed the reliability-based design (RBD) approach for passive control applications for systems with probabilistic parameter uncertainty. The control problems for a wide class of mechanical system with uncertainties were recently presented by Ferrara and Giacomini [5]. The present study investigated the optimum parameters of TMD system considering system parameter uncertainties and determined the performance of TMD to minimizing the structural responses. The theoretical sensitivity formulation is developed by considering the properties of primary structure and seismic load parameters as uncertain. The unconditional responses are obtained by using the Taylor's series expansion based perturbation technique. The numerical example is taken to elucidate the effect of parameters uncertainties on the optimization of TMD parameters and system performance.

II. THEORETICAL FORMULATION

Structure Motion Equation:

The equation of motion of an md of system attached with TMD can be expressed as,

$$\mathbf{M}\ddot{\mathbf{Y}} + \mathbf{C}\dot{\mathbf{Y}} + \mathbf{K}\mathbf{Y} = -\mathbf{M}\bar{\mathbf{r}}\ddot{z} \qquad (1)$$

Where, $\mathbf{Y} = [x_t, x_n, x_{n-1}, ..., x_1]^T$ is the relative displacement vector, and $\bar{\mathbf{r}} = \begin{bmatrix} 0 & \mathbf{I} \end{bmatrix}^T$, where **I** is an nx1 unit vector. **M**, **C**, and **K** represent the mass, damping and stiffness matrix of the combined system.

Introducing the state space vector, $\mathbf{Y}_s = (x_t, x_n, x_{n-1}, ...x_1, \dot{y}, \dot{x}_n, \dot{x}_{n-1}, ...\dot{x}_1)^T$, Equation (1) can be written as,

$$\dot{\mathbf{Y}}_s = \mathbf{A}_s \mathbf{Y}_s + \tilde{\mathbf{r}}\ddot{z} \ \text{ where, } \mathbf{A}_s = \begin{bmatrix} 0 & \mathrm{I} \\ \mathbf{H}_k & \mathbf{H}_c \end{bmatrix} \qquad (2)$$

Where, $\mathbf{H}_k = \mathbf{M}^{-1}\mathbf{K}$, $\mathbf{H}_c = \mathbf{M}^{-1}\mathbf{C}$

In which $\tilde{\mathbf{r}} = \left[0, \mathbf{I}\right]^T$ with **I** and 0 is the (n+1)x(n+1) unit and null matrices, respectively

The well known Kanai-Tajimi stochastic process model [6] which is able to characterize the input frequency content for a wide range of practical situations is used in present study to represent stochastic earthquake load. The process of excitation at the base can be described as:

$$\begin{aligned} &\ddot{x}_f(t) + 2\xi_f\omega_f\dot{x}_f + \omega_f^2 x_f = -\omega(t) \text{ and} \\ &\ddot{z}(t) = \ddot{x}_f(t) + \omega(t) = 2\xi_f\omega_f\dot{x}_f + \omega_f^2 x_f \end{aligned} \qquad (3)$$

Where, $\omega(t)$ is a stationary Gaussian zero mean white noise process, representing the excitation at the bed rock, ω_f is the base filter frequency and ξ_f is the filter or ground damping. Defining the global state space vector is defined as:

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$$Z=(x_t, x_n, x_{n-1},....x_1, x_f \dot{x}_t, \dot{x}_n, \dot{x}_{n-1},.....\dot{x}_1, \dot{x}_f)^T \quad ,$$

Equation (2) and (3) leads to an algebraic matrix equation of order six i.e. the so called Lyapunov equation [7]:

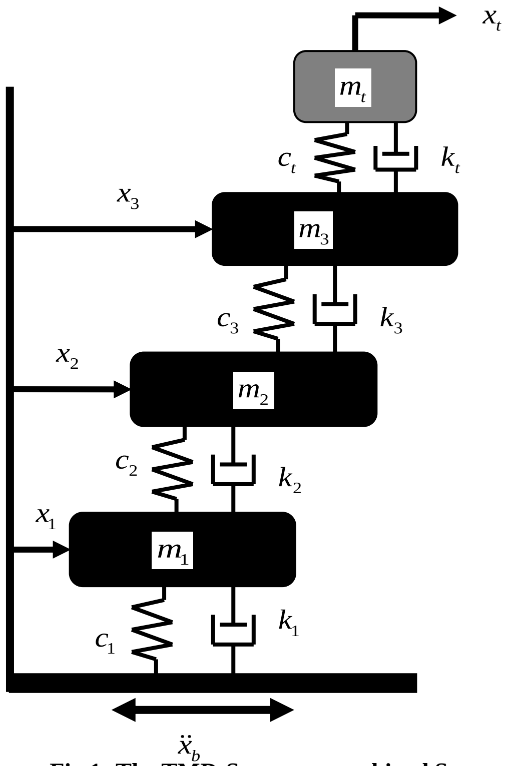

Fig.1: The TMD-Structure combined System

$$\mathbf{AR}+\mathbf{RA}^T+\mathbf{B}=0 \qquad (4)$$

Where, the details of the state space matrix **A** and **B** are provided in Appendix. The space state covariance matrix $\mathbf{R}$ is obtained as the solution of the Lyapunov equation. The state space covariance matrix is represented by the sub-matrices $R_{zz}, R_{\dot{z}z}, R_{z\dot{z}}$ and $R_{\dot{z}\dot{z}}$. The root mean square (RMS) displacement of liquid and the primary system can be then obtained as:

$$\sigma_{xt}=\sqrt{R_{ZZ}(1,1)} \quad \text{and} \quad \sigma_x=\sqrt{R_{ZZ}(2,2)} \qquad (5)$$

Conventional conditional optimization of TMD parameters:

The optimum damper parameters are usually obtained by minimizing the vibration effect of primary structure under dynamic load. The problem of optimization of TMD system of protection requires to determine the tuning ratio (g) and head loss coefficient (x) of the damper system.The optimization problem of the TMD system of protection requires determination of the tuning ratio and damping ratio of damper system. Thus, the design vector (DV) can be thus defined as: $\mathbf{b}=\left(\gamma \;\; \xi_t\right)^T$. The conventional optimization problem so defined for system subject to stochastic load can be transformed into a standard nonlinear programming problem [8] and the DV is obtained for a known mass ratio μ and the frequency and damping ratio of the primary structure. One of the much used approaches is to minimize the mean square response of the building structure. The optimization approach involving mechanical systems subject to random load can be transformed into a standard nonlinear optimization problem as below [8]:

$$\text{Find } \mathbf{b}=[\,\gamma \;\; \xi_t]^T \text{ to minimize: } f=\sigma_x \qquad (6)$$

Response Sensitivity Evaluation considering parameter uncertainty:

In engineering analysis and design, various response quantities such as displacement, stress, vibration frequencies and mode shapes of structure are obtained against a given set of system parameters. However, the system parameters may be uncertain because of structural complexity, error in construction techniques and inaccuracy in measurement etc. It can be noted that the matrix **A** and **B** are functions of various parameters characterizing the primary structures and stochastic load. The response statistic evaluation as discussed in previous section intuitively assumes that the system parameters are completely known. Thus, evaluation of stochastic response using (5) and subsequent solution of optimization problem to obtain optimum TMD parameters are conditional. The uncertainties of these parameters may lead to large and unexpected excursion of responses that may lead to drastic reduction in accuracy and precision of the system safety evaluation. In design of optimum TMD parameters, apart from the stochastic nature of the earthquake load, the uncertainty with regard to these parameters are expected to have influences**.** To include such parameter uncertainties effect, the total probability concept is one of the most important theorems in evaluating the unconditional response of structure [10]-[11] involving of stochastic dynamic sensitivity analysis [12]-[13]. The uncertain system parameters as mentioned above are denoted by a vector **u**. To obtain the sensitivity of responses, the first order derivative of basic Lyapunov equation can be obtained by differentiating (4) with respect to i-th parameter u_i:

$$\mathbf{AR},_{u_i}+\mathbf{R},_{u_i}\mathbf{A}^{\mathbf{T}}+\mathbf{B}_1=0, \text{where } \mathbf{B}_1=\mathbf{A},_{u_i}\mathbf{R}+\mathbf{RA}^{\mathbf{T}},_{u_i}+\frac{\partial}{\partial u_i}(\mathbf{B})$$

(7)

The sensitivity of response (RMS displacement as considered here) can be obtained directly by differentiating the corresponding expression of (5) with respect to i-th random variable u_i as following:

$$\frac{\partial}{\partial u_i}\left(\sigma_x\right) \; i.e. \; \sigma_{x,u_i}=\frac{1}{2}\frac{\mathbf{R},_{u_i}(2,2)}{\sqrt{\mathbf{R}(2,2)}} \qquad (8)$$

In which, $\mathbf{R},_{u_i}(2,2)$ is obtained by solving (7). Now, taking the second derivative with respect to j[th] parameter, one can further obtain the following:

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$$\mathbf{A}\mathbf{R}_{,u_iu_j}+\mathbf{R}_{,u_iu_j}\mathbf{A}^T+\mathbf{B}_2=0,$$
$$\text{where } \mathbf{B}_2=2[\mathbf{A}_{,u_i}\mathbf{R}_{,u_i}+\mathbf{R}_{,u_i}\mathbf{A}^T_{,u_i}]+[\mathbf{A}_{,u_iu_j}\mathbf{R}+\mathbf{R}\mathbf{A}^T_{,u_iu_j}] \qquad (9)$$

The second order sensitivity of the rmsd can be obtained by differentiating (8) with respect to j-th random variable u_j as following:

$$\sigma_{x,u_iu_j}=\frac{1}{2\sqrt{\mathbf{R}(2,2)}}\left\{\mathbf{R}_{,u_iu_j}(2,2)-\frac{1}{2}\frac{[\mathbf{R}_{,u_i}(2,2)]^2}{\mathbf{R}(2,2)}\right\} \qquad (10)$$

The steps involve to obtain the response sensitivity are straightforward. For known system parameter matrix **A** and **B**, the basic covariance matrix R is first obtained by solving the Lyapunov equation as described by (4). The first order sensitivities of the covariance matrix $\mathbf{R}_{,u_i}$ can be obtained by solving (7) and the associated terms of this matrix is used in (8) to obtain the sensitivity of RMS displacement. It can be noted that the equation need to be solved for each random variable involve in the problem. The second order sensitivities of the covariance matrix are obtained by solving (9). Finally, using this result, the second order sensitivities of responses are obtained from (10).

Unconditional Optimization with parameter uncertainty

The stochastic response of structure subjected to earthquake load is a function of the system parameters **u**. The random design parameter u_i can be viewed as the superposition of the deterministic mean component ($\bar{u}_i$) with a zero mean deviatoric component (Δu_i). Assuming uncertain variables **u** are uncorrelated, the quadratic approximation based on the Taylor series expansion of RMS displacement about its mean value provides the expected value i.e. the unconditional RMS displacement as:

$$\sigma_x=\sigma_X(\bar{u}_i)+\frac{1}{2}\sum_{i=1}^{nv}\sigma_{x,u_iu_i}\sigma^2_{u_i} \qquad (11)$$

Where, σ_{u_i} is the standard deviation of i-th random parameter, *nv* is the total number of random variables involve in the problem. It can be noted that the damper parameter optimization problem as described by (6) is conditional due to the fact that the structural and excitation model specified by **u** are assumed to be known a priori. However, knowing the conditional second order information of response quantities, the unconditional response can be obtained from (11) and this can be used as performance index to obtain the optimum TMD parameters. Thus, the TMD parameters optimization problem as defined by (6) is redefined as following:

Find $\mathrm{b}=(\gamma,\ \xi_t)^T$

$$\text{to minimize: } f=\sigma_x=\sigma_X(\bar{u}_i)+\frac{1}{2}\sum_{i=1}^{nv}\sigma_{x,u_iu_i}\sigma^2_{u_i} \qquad (12)$$

The standard gradient based techniques of optimization can be used to solve the problem. In present study, the MATLAB routine is used. However, for more complex configuration, genetic algorithm based technique are robust choice for solving the associated optimization problem as the approach is independent from the choice of an initial point and does not require any information regarding the gradient of the objective function.

III. NUMERICAL STUDY

A three degree of freedom primary system with an attached TMD subjected to stochastic earthquake excitation is undertaken to study the performance of the proposed TMD parameters optimization procedure considering random system parameters. The primary system has the following mass and stiffness values: $m_{s1}=5.0\times10^5$ kg; $m_{s2}=5.0\times10^5$ kg; $m_{s3}=4.0\times10^5$ kg; $k_1=k_2=k_3=1.0\times10^7$N/m. Unless mentioned otherwise, following nominal values are assumed for various parameters: structural damping, $\xi_s=3\%$, mass ratio, μ=3%, . The PSD of the white noise process at bed rock, S_0 is related to the standard deviation $\ddot{\sigma}_z$ of ground acceleration [9] by: $S_0=\dfrac{2\xi_f\sigma^2_{\ddot{z}_b}}{\pi(1+4\xi_f^2)\omega_f}$. For numerical study, the peak ground acceleration is taken as, *PGA*=0.15g, where 'g' is the acceleration due to gravity. It is assumed that $PGA=3\ddot{\sigma}_{z_b}$. The mean value of the filter frequency (ω_f) and damping (ξ_f) are taken as 7.5π rad/sec and 0.4, respectively. The uncertainties are considered in the primary system mass, stiffness, damping and load parameters ω_f, ξ_f and S_0.

The variation of the optimum TMD parameters and the RMS displacement of the building structure with mass ratio and different level of cov of parameters are shown in Figs. 2 to 4. The similar results are shown in Figs. 5 to 7 for varying damping ratio of building structure. It can be observed that there is a definite change in optimum results considering system parameter uncertainties and reduced the damper efficiency.

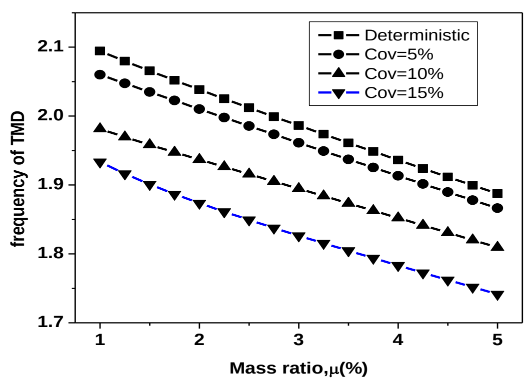

Fig.2: The optimum tuning ratio with varying mass ratio for different cov. of parameters.

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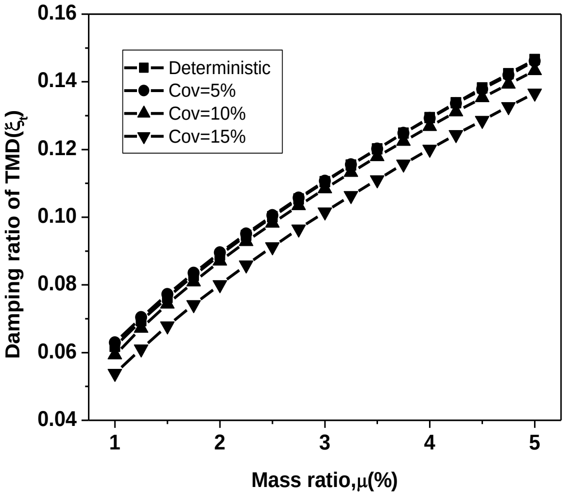

Fig.3: The optimum damping ratio of TMD with varying mass ratio for different cov. of parameters.

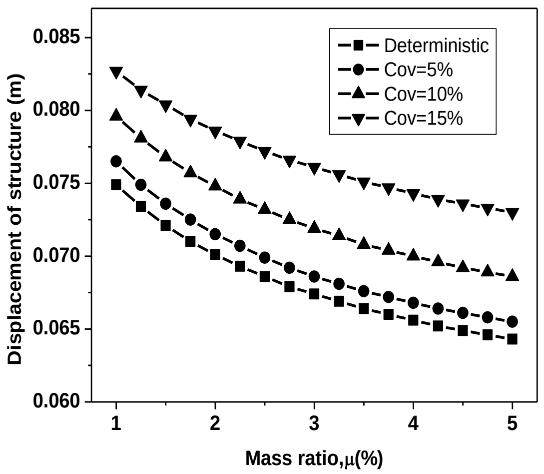

Fig.4: The R.M.S displacement of structure with varying mass ratio for different cov. of parameters.

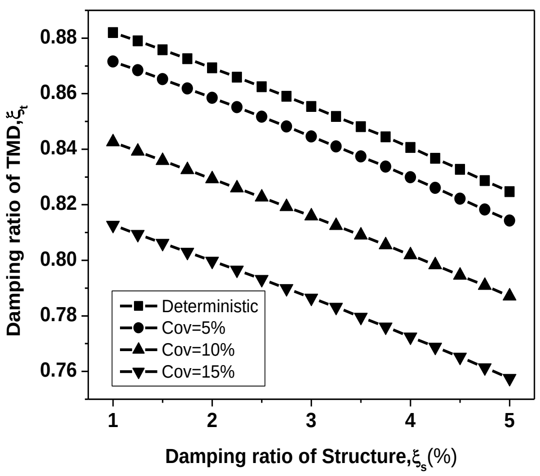

Fig.5: The optimum tuning ratio with varying damping ratio of structure for different cov. of parameters.

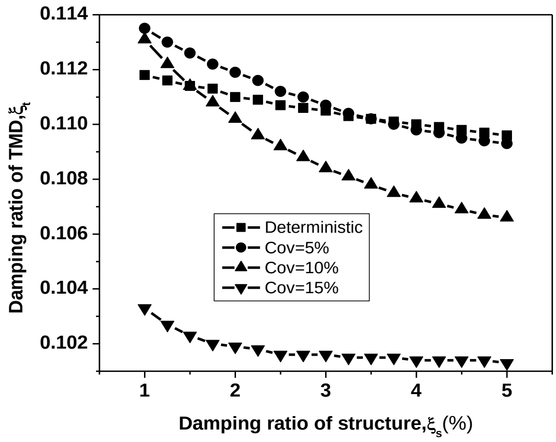

Fig.6: The damping ratio of TMD with varying damping ratio of structure for different cov. of parameters.

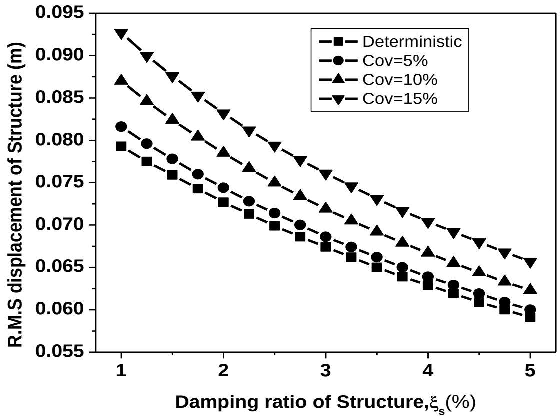

Fig.7: The R.M.S displacement of structure with varying damping ratio of structure for different cov. of parameters.

The variation of the optimum TMD parameters and the RMS displacement of the building structure with peak ground acceleration and different level of cov of parameters are shown in Figs. 8 to 10. We observed that tuning ratio and damping ratio of TMD are remaining same for different peak ground acceleration and RM.S displacement increases with increasing the peak ground accelerations.

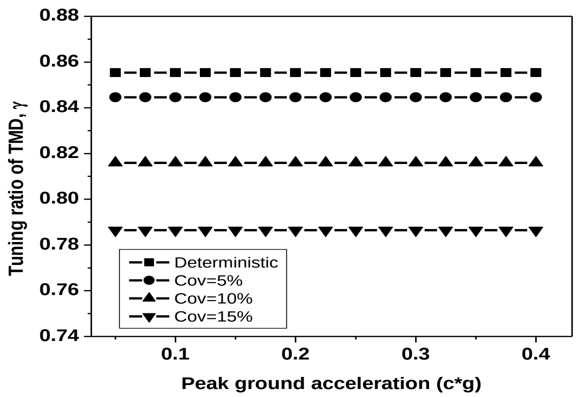

Fig.8: The optimum tuning ratio of TMD with varying peak ground acceleration for different cov. of parameters.

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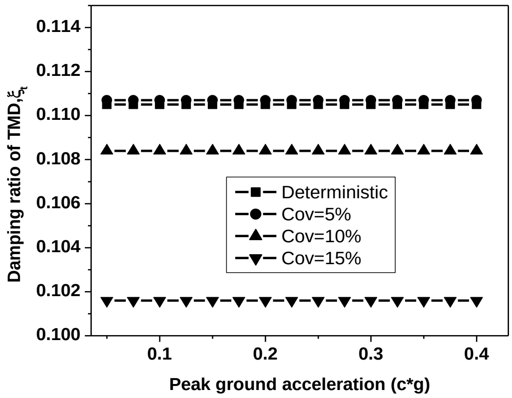

Fig.9: The optimum damping ratio of TMD with varying peak ground acceleration for different cov. of parameters.

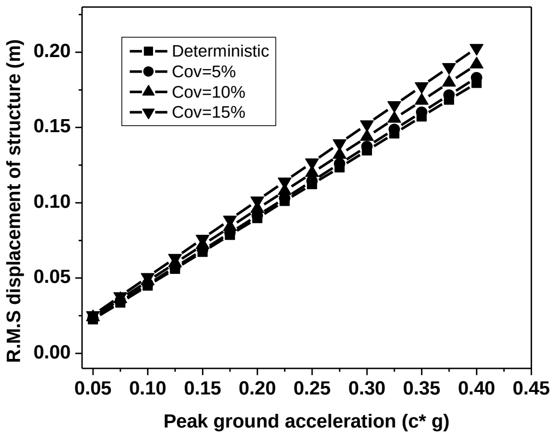

Fig.10: The R.M.S displacement of structure with varying peak ground acceleration for different cov. Of parameters.

IV. CONCLUSION

The TMD parameters optimization to minimize the maximum RMS displacement of the primary building structure subjected to stochastic earthquake load considering random system parameters is presented here. The observations of optimization results are in conformity with the well known facts in the application of TMD assuming deterministic system parameters. As expected, the RMS displacement of the primary system is quite significantly reduced with increasing mass ratio and damping ratio of the structure. However, when the system parameters uncertainties are considered, there is a definite change in the optimal tuning ratio and head loss coefficient of the TMD yielding a reduced efficiency of the system. Though the uncertainty in the excitation process dominates the response of structure, it is generally seen that the uncertainties in the system parameters have noticeable influence and can not be neglected. It is evident that if the uncertainty which affects the parameters of the system is not considered, the damper performance is overestimated leading to an unsafe design. The effect of uncertainty i.e. the visible differences of optimum results can be noted when the RMS displacement is comparatively higher. The observation is made with earthquake load of specific model. However, it needs to study further for non-stationary earthquake or other random load like wind, wave etc

APPENDIX

The details of the state space matrix **A** and **B** in (4) are as below: $[\mathbf{A}]=\begin{bmatrix} 0 & \mathbf{I} \\ \bar{\mathbf{H}}_k & \bar{\mathbf{H}}_c \end{bmatrix}$

$$\bar{\mathbf{H}}_k = \begin{bmatrix} & & & & \vdots & \omega_f^2 \\ & -\mathbf{M}^{-1}\mathbf{K} & & & \vdots & \omega_f^2 \\ & & & & \vdots & . \\ & & & & \vdots & \omega_f^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \ldots\ldots & 0 & \vdots & -\omega_f^2 \end{bmatrix}$$

$$\bar{\mathbf{H}}_c = \begin{bmatrix} & & & & \vdots & 2\xi_f\omega_f \\ & -\mathbf{M}^{-1}\mathbf{C} & & & \vdots & 2\xi_f\omega_f \\ & & & & \vdots & . \\ & & & & \vdots & 2\xi_f\omega_f \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \vdots & -2\xi_f\omega_f \end{bmatrix}$$ and

$$\mathbf{B} = \begin{bmatrix} 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 2\pi S_o \end{bmatrix}$$

REFERENCES

[1] G. B. Warburton, and E. O. Ayorinde, "Optimum Absorber Parameters for Simple system," Earthq. Engng. Struct. Dyn. vol. 8, pp. 197–217, 1980.

[2] A. G. Thomson, "Optimizing the Tuned Viscous Dynamic Vibration Absorber with Primary System Damping; a Frequency Locus Method," J. Sound Vibr. vol. 73, pp. 469-472 Soong,, 1980.

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 2, Issue 9, March 2013

[3] R. Rana, and T. T. Soong, "Parametric Study and Simplified Design of Tuned Mass Dampers," Engng Struct., vol. 20, pp. 193-204, 1998.

[4] C. Papadimitriou, L. S. Katafygiotis, and S.K. Au, "Effects of Structural Uncertainties on TMD Design: A Reliability-Based Approach", J. Struct. Contr., vol.4, no.1, pp. 65–88, 1997.

[5] A. Ferrara, and L Giacomini, "Control of a Class of Mechanical Systems with Uncertainties via a Constructive adaptive/second order VSC Approach," J. Dyn. Syst. Meas. Contr., vol. 122, no. 1, pp. 33-39, 2000.

[6] H. Tajimi, "A Statistical Method of Determining the Maximum Response of a Building During Earthquake," Proc. of 2nd World Conf. on Earthq. Engnng, Tokyo, Japan, 1960.

[7] L. D. Lutes, and S. Sarkani, "Random Vibration," Butterworth-Heinemann, Oxford, 2001.

[8] N. C. Nigam, "Structural Optimization in Random Vibration Environment,'' AIAA J., 10(4), pp. 551-553, 1972.

[9] S. H. Crandall, and. W. D., Mark, "Random vibration in Mechanical Systems," Academic Press, New York, 1963.

[10] Y. G. Zhao, T. Ono, and and H. Idota, "Response Uncertainty and Time-Variant Reliability Analysis for Hysteretic MDF Structures," Earthq. Engng. Struct. Dyn., vol. 28, pp. 1187-1213, 1999.

[11] A. Chaudhuri, and S. Chakra borty "Reliability of Linear Structures with Parameter Uncertainty under No stationary Earthquake," Struct. Saf., vol. 28, pp. 231-246, 2006.

[12] H. A. Jensen, "Design and Sensitivity Analysis of Dynamical Systems Subjected to Stochastic Loading," Comput. and Struc., vol. 83, pp. 1062–1075, 2005.

[13] A. Chaudhuri, and S. Chakraborty, "Sensitivity Evaluation in Seismic Reliability Analysis of] Structures," Comput. Methods Appl. Mech. Engng., vol.93, pp.59-68, 2004.

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