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Air Gap Control of an Electromagnetically Levitated System

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Abstract* — Air-gap control of an electromagnetically levitated system is presented in this paper. Initially the system is linearised about an operating point and a PID controller is designed for the system. But with parameter perturbations the controller performance deteriorates. So later a sliding mode controller was developed for the nonlinear system, which will gives satisfactory performance even under perturbed conditions.

Keywords- **Electromagnetically Levitated System; PID Controller; Sliding Mode Controller.**

I. INTRODUCTION

Magnetically levitated systems are receiving increasing attention as it eliminates contact friction. It has wide range of applications such as high speed maglev trains, frictionless bearings, levitation of metal slabs during manufacturing etc. In recent years, a lot of works have been reported in the literature for controlling magnetic levitation systems. The feedback linearization technique has been used to design control laws for magnetic levitation systems [1]. Control laws based on, neural network techniques [2] have also been used to control magnetic levitation systems. One of the first applications of sliding mode control to magnetic levitation systems was carried out by Cho et al. [3] , Chen et al. [4] where they designed an adaptive sliding mode controller for a magnetic levitation system. N. Al-Muthari [5] designed static and dynamic sliding mode controllers for the magnetic levitation system. This paper presents air gap control of an electro magnetically levitated system. In this paper after making appropriate assumptions, the non-linear equations are linearised about an operating point and a linear model has been derived.PID controller is developed for this linearised model. But if the system parameters vary the system no longer remains linear, and the controller performance deteriorates. So in order to overcome that problem a sliding mode controller was developed for the nonlinear system, which will give a satisfactory performance even if the parameter varies.

II. ELECTRO MAGNETICALLY LEVITATED SYSTEM

The structure of electro magnetically levitated system is shown in Figure 1. The magnetic force produced by the current flowing through the coil of electromagnet is used to resist gravity to suspend iron ball without any contact. CdS Position sensors are used to measure the gap between the magnet and the ball .When the light intensity increases resistance of CdS sensor decreases. With this sensor, change of air gap is converted into a variation of voltage signal which is used to regulate the current to electromagnet, and to maintain equilibrium position of ball. The details of magnet are given in Table I. When magnet cum controller unit working satisfactorily the ball gets desired vertical lift.

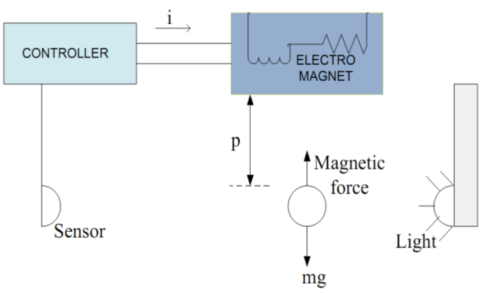

Fig 1: Electromagnetically Levitated System

III. SYSTEM MODELING

Assuming all the flux generated by the electromagnet passes through the ferromagnetic guide-way, the instantaneous coil inductance may be expressed as:

$$L(P) = \frac{\mu_o N^2 A}{P(t)} \tag{1}$$

N = no. of turns of the coil,

At any instant of time, the force of attraction between the electromagnet and the ferromagnetic rail is given by

$$F(i,p) = -\frac{d}{dz}\left[\frac{1}{2}L(p)i(t)^2\right] \tag{2}$$

Substituting the inductance value from (2) into (3) we get:

$$F(i,p) = \frac{\mu_o N^2 A}{2}\left[\frac{i(t)}{p(t)}\right]^2 \tag{3}$$

At the equilibrium position (i_0, p_0), the normalized force equation is :

$$m\ddot{p}(t) = -F(i,p) + mg \tag{4}$$

The dynamics of the electromagnet are given by the following equations:

$$m\ddot{p}(t) = -F(i,p) + mg \tag{5}$$

$$m\ddot{p}(t) = -\frac{\mu_o N^2 A}{2}\left[\frac{i(t)}{p(t)}\right]^2 + mg \tag{6}$$

The system dynamic equations are thus nonlinear and hence difficult to analyze. So the equations are linearised

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about a suitable operating point (i_0,p_0) and the linearised model may be found as described below. If the mass of the electromagnet is displaced by an amount $\Delta p(t)$ from the stable point, then let the corresponding change in current be $\Delta i(t)$.The small perturbation linear equations (neglecting higher order terms) of the system are:

$$m\Delta\ddot{p}(t) = -\frac{\mu_o N^2 A}{4}\left[\frac{i_0 + \Delta i(t)}{p_0 + \Delta p(t)}\right]^2 + mg \qquad (7)$$

On simplification,

$$m\Delta\ddot{p}(t) = -\frac{2C}{i_o}\Delta i(t) + \frac{2C}{p_o}\Delta\ddot{p}(t) \qquad (8)$$

Taking Laplace transforms on both sides of equation (8) and after rearranging, the transfer function of the plant is written as:

$$Gp(s) = \frac{\left(\frac{\left(\frac{2C}{i_o}\right)}{m}\right)}{\left(s^2 - \frac{\left(\frac{2C}{P_o}\right)}{m}\right)} \qquad (9)$$

Table I: Parameters of the levitated system

Parameters	Values
Resistance of coil (R)	6.6 ohm
Inductance of the coil (L)	0.470H
Magnetic force constant (C)	1.24×10^{-4}
Mass of levitated substance (m)	11.87g

IV. PID CONTROLLER

Generally a PID controller with input e and output u_{pid} is given as:

$$u_{pid}(t) = K_p\left[e(t) + \frac{1}{T_i}\int_0^\tau e(\tau(\tau) + T_d\frac{d}{dt}e(t)\right] \qquad (10)$$

This can be rewritten as:

$$u_{pid}(t) = K_p e(t) + K_i\int_0^\tau e(\tau(\tau) + K_d\frac{d}{dt}e(t) \qquad (11)$$

Where $K_i = \frac{K_p}{T_i}$ is the integral gain and K_d=K_pT_d,K_p is the derivative gain ,K_p is proportional gain ,T_i is integral time constant and T_d is the derivative time constant. For the magnetically levitated system which was linearized around a particular operating point, PID controller was developed. But the PID controller performance deteriorates with parameter perturbances. In order to overcome this problem we go for a sliding mode controller.

V. SLIDING MODE CONTROL OF MAGNETIC LEVITATION SYSTEM

Sliding mode control is a variable structure control utilizing a high-speed switching control law to drive a system state trajectory onto a specified and user chosen surface, so called sliding surface, and to maintain the system state trajectory on the sliding surface at subsequent times[5]. The discontinuous nature of the control action in Sliding Mode controller is claimed to result in outstanding robustness features for both system stabilization and output tracking problems. The very good performance also includes insensitivity to parameter variations and rejection of disturbances. A sliding mode controller can withstand large variations in system parameters.

VI. STATE SPACE MODEL OF THE MAGNETIC LEVITATION SYSTEM

Dynamics of the electromagnet is given by, (9)

$$m\ddot{p}(t) = -F(i,p) + mg \qquad (12)$$

$$m\ddot{p}(t) = -\frac{\mu_o N^2 A}{4}\left[\frac{i(t)}{p(t)}\right]^2 + mg \qquad (13)$$

$$m\frac{dv}{dt} = mg_c - C\left[\frac{i}{p}\right]^2 \qquad (14)$$

Let the states and the control input be chosen such that $x_1 = p$, $x_2 = v$, $x_3 = i$, $u = e$

Thus, the state-space model of the magnetic levitation system can be written as:

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2 \qquad (15)$$

$$\frac{dx_3}{dt} = -\frac{R}{L}x_3 + \frac{2C}{L}\left(\frac{x_2 x_3}{x_1^2}\right) + \frac{1}{L}u$$

Let x_{1d}, x_{2d}, and x_{3d} be the desired values of x_1, x_2, and x_3, respectively. The equilibrium point for the system is xe = $(x_{1e}\ 0\ x_{3e})^T$, where x_{3e} satisfies $x_{3e} = \sqrt{g_c m / C x_{1e}}$. Therefore, we can conclude that x_{2d} is equal to zero. The objective of the control schemes is to drive the states x_1, x_2, and x_3 to their desired constant values x_{1d}, x_{2d}, and x_{3d}, respectively. Now, consider the following nonlinear change of coordinates:

$$p_1 = x_1 - x_{1d}$$

$$p_2 = x_2 \qquad (16)$$

$$p_3 = g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2$$

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If p_1, p_2, p_3 are driven to zero as $t \to \infty$ then x_1 will converge to x_{1d}, x_2 will converge to zero, and x_3 will converge to $x_{3e} = \sqrt{g_c m / C x_{1e}}$ as $t \to \infty$

The dynamic model of the magnetic levitation system in the new coordinates system can be written as:

$$\dot{p}_1 = p_2$$
$$\dot{p}_2 = p_3$$
$$\dot{p}_3 = f(p) + g(p)u \qquad (17)$$

where,

$$f(p) = 2(g_c - p_3)\left(\left(1 - \frac{2C}{L(p_1 + x_{1d})}\right)\frac{p_2}{(p_1 + x_{1d})} + \frac{R}{L}\right),$$

$$(18) \quad g(p) = \frac{-2}{L(p_1 + x_{1d})}\sqrt{\frac{C}{m}(g_c - p_3)}$$

The functions *f(p)* and *g(p)* correspond in the original coordinates to the following functions, respectively:

$$f_1(x) = \frac{2C}{m}\left(\left(1 - \frac{2C}{Lx_1}\right)\frac{x_2 x_3^2}{x_1^3} + \frac{R}{L}\frac{x_3^2}{x_1^2}\right),$$

$$(19) \quad g_1(x) = -\frac{2Cx_3}{Lmx_1^2}$$

Where $f_1(x) = f(p)$, $g_1(x) = g(p)$

Let the output of the system be

$$y = p_1 = x_1 - x_{1d}. \qquad (20)$$

VII. DESIGN OF SLIDINGMODE CONTROLLER

The first step in designing an SMC scheme for the system is to select the switching surface. Let the switching surface be :

$$s = \ddot{y} + \lambda_1 \dot{y} + \lambda_2 y$$
$$= p_3 + \lambda_1 p_2 + \lambda_2 p_1$$
$$(21) \quad = \ddot{p}_1 + \lambda_1 \dot{p}_1 + \lambda_2 p_1$$

Where λ_1 and λ_2 are positive scalars

Using equation (16) the switching surface can be written as a function of x_1, x_2, and x_3 such that

$$s = g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2 + \lambda_1 x_2 + \lambda_2(x_1 - x_{1d}) \qquad (22)$$

The choice of the switching surface guarantees that

$y = p_1 = x_1 - x_{1d}$ converges to 0 as $t \to \infty$

Sliding surface is selected such that it satisfies the reachability condition $s\dot{s} \angle 0$

A. Control law Design

The control law is given by,

$$u(t) = u_l(t) + u_n(t) \qquad (23)$$

Where $u_l(t)$ is the linear component of control law and $u_n(t)$ is the non linear component of control law

Linear component of control law is given by:

$$u_l(t) = -(SB)^{-1} SAx(t) \qquad (24)$$

Non-Linear Component of control law is given by:

$$(25) \quad u_n(t) = -W \, sgn(s)$$

Where *W* is a positive constant. To reduce the magnitude of the steady-state error, the value of *W* should be increased. However, increasing the value of W will lead to a larger control magnitude and more chattering.

So the control law is given by,

$$u = \frac{1}{g_1}\left[-f_1 - \lambda_1\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right) - \lambda_2 x_2 \right. \qquad (26)$$
$$\left. - Wsign\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2 + \lambda_1 x_2 + \lambda_2(x_1 - x_{1d})\right)\right]$$

VIII. SIMULATION RESULTS

The proposed systems were simulated using MATLAB Simulink models and following are the results obtained. Gain values for PID controller are selected as, K_p 771, K_i= 110, K_d= 489. P_d=.01 is the desired position in meters.

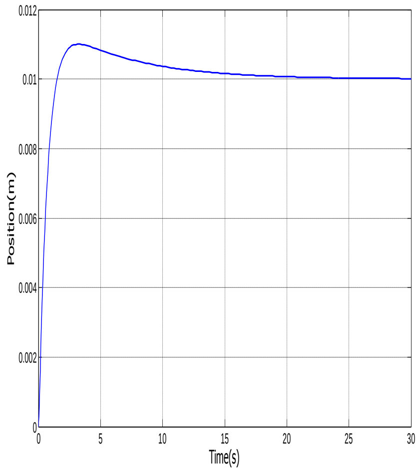

Fig 2: Position (m) Vs Time (sec) response using PID controller

Parameter values used in sliding mode controller are, λ_1 =60, λ_2 =930, *W=350.*

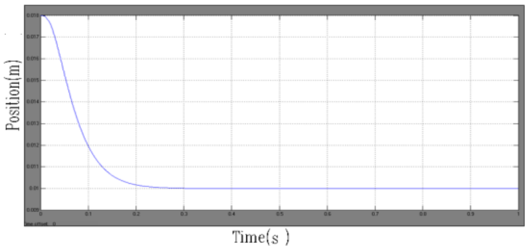

Fig 3: Position (m) Vs Time (sec) response using Sliding Mode controller

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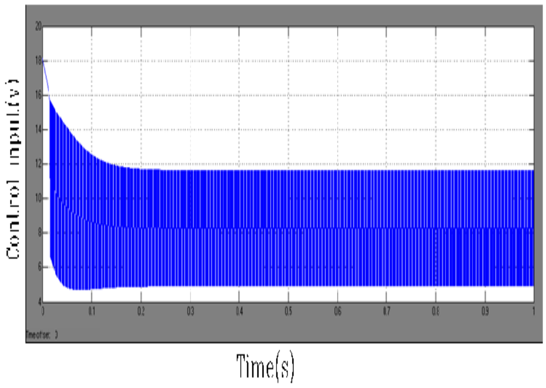

Fig 4: Control Input (v) Vs Time (sec) Response using Sliding Mode Controller

IX. CONCLUSION

A PID controller was designed for the linearized Magnetically Levitated System. But with variations in system parameters, system remains no longer linear and system performance deteriorates. In order to improve the performance of the system a sliding mode controller was designed for the system, and now the system can withstand variations in system parameters. Problem of shattering will be present for an ordinary sliding mode controller due to discontinuous control action. So a dynamic sliding mode controller can be designed for the system, which completely eliminates the problem of shattering, by its continuous control action. The overall system was modeled using simulink and satisfactory performance was obtained.

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