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Mitigation Curves for Determination of Relief Holes to Mitigate Stress Concentration Factor in Thin Plates Loaded Axially for Different Discontinuities

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Abstract*— In many structural elements, holes and notches of different shapes and orientations are made with as per the requirements. The presence of holes and notches in structural elements creates stress concentration, which eventually reduces the mechanical strength of the structure. Efforts are being made to reduce this stress concentration effect. Mitigation of maximum stress can be obtained by area reduction method, which can be achieved by introducing relief holes around main hole or notch. However the size and place of these relief holes and relief notches depends on the geometry of the structural element and are required to be analyzed or experimented individually. An analytical form for determination of the location and size of relief holes will be very useful to mitigate the maximum stress. In the present work a generalized analytical form for mitigation curve is proposed which can be used for similar type of discontinuity. FEA has been used to analyze and determine the optimum size and position of relief holes. The analysis is used to formulate the expression for mitigation curve. The use of these mitigation curves are carried out to generate the optimum results which are then validated through analysis and experiments carried out earlier.

Keywords: **Mitigation Curves, Mitigation of Stress Concentration Factor, Finite Element Analysis, Stress Concentration Factor.**

Nomenclature

A Width of rectangular plate, mm
b Depth of V- notch
b_1 Depth of first set of relief V-notch
b_2 Depth of second set of relief V-notch
c Distance of centre of main hole and centre of plate , in Eccentric holes
δ Peripheral distance between two consecutive Holes/notches
$\delta 1$ Peripheral distance between V notch and first set of Semicircular notch
$\delta 2$ Peripheral distance between V notch and second set of Semicircular notch
Kt SCF, Stress concentration factor = $\sigma_{max} / \sigma_{nom}$
L Length of rectangular plate, mm
L1 Distance between centre of main hole/notch and first set Of auxiliary holes/notches
L2 Distance between centre of main hole/ notch and second Set of auxiliary holes/ notches
L3 Distance between centre of main hole/ notch and third Set of auxiliary holes/ notches
r Radius of first set of semi circular holes
r_1 Radius of second set of semi circular holes
r_2 Radius of third set of semi circular holes
R Radius of main hole , mm
R1 Radius of first set of auxiliary holes, mm
R2 Radius of second set of auxiliary holes, mm
R3 Radius of third set of auxiliary holes, mm
2α Angle of V- notch
σ Stress applied in the plate lengthwise, N/mm^2
σ_{max} Maximum stress at discontinuity , N/mm^2
σ_{nom} Nominal stress at discontinuity = Applied load / Area at discontinuity
x_i ,y_j Co ordinates on periphery of main and auxiliary holes/ Notches

I.INTRODUCTION

Plates and shells with different discontinuities of various sizes and shapes are frequently used as load-bearing members of thin-walled structures in various areas of engineering. In analyzing the strength of those structural members, engineers should take into account the presence of stress concentrators and other factors contributing substantially to the heterogeneity and perturbation of the strain and stress fields under static loading. Almost all structures consist of assembly of simple elements, which are connected to each other by joints. Joints or connections that are usually made in steel structures are mechanical fastening using bolts or rivets. In the mechanical fastening, holes are made to place the bolts or rivets; these make the structure weak and susceptible to failure. Therefore, it is necessary to investigate the state of stress around the holes for the safety and proper design of such structures. From the point of view of the above facts, it is of great importance to understand the behavior of the steel structures with holes/ notches. For the solution of the problem several methodologies can be followed, however, all of these methods can be classified in the following three general categories: experimental, analytical and numerical method. Though experimental methods give the most reliable results, it is very costly, as it requires special equipments, testing facilities etc. Analytical solution of every problem is almost impossible because of complex boundary conditions and shapes. For this reason the numerical methods had become the ultimate choice by the researchers in the last few decades. Invention and rapid improvement of the computing machines, i.e. sophisticated high performance computers, also played an important role for the increasing popularity of the numerical methods. Stress analysis of a steel structure with discontinuities requires the

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help of partial differential equations. There are various numerical methods available which use partial differential equations. Among them most popular methods are: Finite Element Method (FEM) and Finite Difference Method (FDM). The finite element method is a numerical technique for obtaining approximate solution to a wide variety of engineering problems. The finite difference model of a problem gives a point wise approximation to the governing equations. This model is improved as more points are used. With finite difference techniques we can treat some fairly difficult problems; for example, when we encounter irregular geometries or an unusual specification of boundary conditions, we find that finite difference techniques become hard to use. According to Heywood [1], stress concentration can be reduced by introducing smaller auxiliary holes on either side of the original hole, which smoothen the flow of the tensile principal stress trajectories past the original hole. Peterson [2] has compiled the work of many researchers and developed charts and excellent methodology in graphical form for evaluation of SCF in different objects with different types of discontinuities for many types of loading condition. Rajaiah [3] proposed hole shape optimization in a finite plate .He applied photo elasticity techniques for analysis. Auxiliary hole with optimum size and shape around the central hole has been proposed. SCF is minimized up to 30%. Giare et al. [4] presented a method for the reduction of stress concentration around the hole in an isotropic plate under in-plane loading by using composite material. Hole in a finite plate was reinforced by composite material. Sanyal, Yadav [5], [6] proposed multiple relief holes around the main hole in infinite thin plates for reduction of stress concentration. FEM was applied for analysis. They proposed the optimum centre distance between the main hole and relief holes for mitigation of stresses; they have also extended the work by introducing multiple relief holes. Mittal, Jain [7],[8] proposed optimization of square plate with circular hole by FEM and reported around 30% reduction in SCF. A further modification of work was carried out by optimization of hole shape & optimization of auxiliary hole shape by giving trapezoidal shape to the auxiliary holes. Optimal hole shape for minimum stress concentration in two dimensional finite plates using parameterized geometry models are also given by Zhixue Wu [9]. Work done by the various researchers with different approximation are excellent for reduction of stress concentration, but the literature available for the analysis of all cases of stress concentration around different discontinuities in isotropic plates subjected to in-plane loading conditions is limited. Hence, the need arises for the analysis of more cases of stress mitigation in isotropic plates with different discontinuities subjected to in-plane loadings. It has already been established in literature that the presence of auxiliary holes/ notches will reduce the SCF value around main discontinuity. In this paper, an analysis of rectangular isotropic plate with different discontinuity has been carried out to study the effect of size of discontinuity on SCF. The present work deals with the mitigation of SCF in isotropic rectangular plates with central circular hole, eccentric circular hole , opposite V notch under in-plane static loading and gets the analytical solution. The reduction in SCF is achieved by introducing two sets of auxiliary holes/ notches on both side of main hole.

II- METHOD

It is assumed that a mitigation curve passes through the ordinates of the main discontinuity and relief holes. The nature of mitigation curve is proposed to vary with the number of relief holes and is determined from their optimum size and location. To determine the optimum size and location of relief holes for maximum mitigation of stress values detail finite element parametric analysis has been carried out. The results thus obtained in the form of optimum size and location of relief holes is then utilized for determining the nature of mitigation curves. These expressions can then be used for determination of relief holes size and locations for any other similar nature of structural elements with different parameters – i.e. for different hole size and for different plate dimensions. The method is established through its application to following different cases.

III- PROBLEM DESCRIPTION

The basic geometry of the problem for case1 and case2 is shown in Fig1. Rectangular plate of 400mm*100mm and of 1mm thickness with opposite V-notch is considered for analysis. The isotropic material of the plate is high strength alloy steel; Poisson's ratio ν=0.3, Young's modulus E=39 GPa. Uniform tensile load (σ) is applied to the plate's lengthwise direction. An 8 nodded structural plane 82 element type with element size of 2mm was selected for free meshing on the bases of convergence test as shown inTable1. Each node has two degrees of freedom, making a total 16 degrees of freedom per element. Due to the symmetry of the problem, one quarter of the plate for each case is discretized and analyzed. For semicircular notch stress concentration factor is given by,

$K_{tu} = 3.065-3.472(2h/D) +1.009(2h/D)^2 +0.405(2h/D)^3$

Where, h is depth of notch and D is width of rectangular plate,

$2h/D=0.398$

$K_{tu} = 1.868533,$

For V-notch , SCF is given by

If $2h/D = 0.398,\ 90^\circ \le \alpha \le 150^\circ,\ 1.6 \le K_{tu} \le 3.5$

$K_t = C_1+C_2 \sqrt{K_{tu}} + C_3 K_{tu}$

Where,

$C1 = 5.294 - 0.1225\alpha + 0.000523\alpha^2$

$C2 = -5.0002 + 0.1171\alpha - 0.000434\alpha^2$

$C3 = 1.423 - 0.01197\alpha - 0.000004\alpha^2$

Table 1 Convergence test for element size 2b/A=0.398,b=0.0199, 2α=120^0

ELEMENT SIZE	Max Stress	Nom Stress	SCF (TH.)	SCF
0.001	10832	1666.67	1.83	6.49
0.003	7157	1666.67	1.83	4.29

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0.005	5789	1666.67	1.83	3.47
0.006	5531	1666.67	1.83	3.31
0.008	4937	1666.67	1.83	2.96
0.01	4391	1666.67	1.83	2.63
0.015	3917	1666.67	1.83	2.35
0.02	**3056**	**1666.67**	**1.83**	**1.83**
0.03	3078	1666.67	1.83	1.84

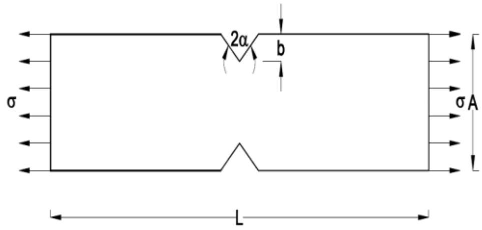

Fig1. Rectangular Plate With Opposite V-Notch

Case 1: Mitigation Curve for Two Relief-V Notches Around One Main V -Notch

Mitigation of Stress Concentration Factor

Plate has been optimized for mitigation of SCF by providing auxiliary notches is considered. For mitigation of SCF, two shapes of auxiliary notches are considered and plate has been analyzed for both types of auxiliary notches.
In this method auxiliary V -notch has been proposed. The model of the plate with two sets of relief V-notches has been shown in Fig4.Size and distance of relief V- notches from main V-notch, number of sets of auxiliary V- notches has also been optimized. The results are shown in Table-3. This method shows significant mitigation in SCF.

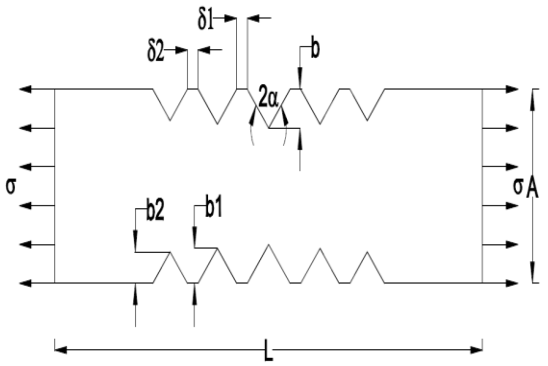

Fig2. Rectangular Plate with Main opposite V-Notch and Relief V Notches

Table-2: Optimization of Parameters for Mitigation in SCF by Providing Auxiliary V- Notches.

b/A	b	b_1 (of b)	b_2 (of b_1)	δ_1 (of b)	δ_2 (of b)	σ_{max} main	σ_{max} with two
0.1	10	0.5	0.5	0.25	0.25	9.4	6.068
0.1	10	0.5	0.5	0.5	0.5	9.4	5.923
0.1	10	0.5	0.5	1	1	9.4	5.95
0.1	**10**	**0.75**	**0.75**	**0.25**	**0.25**	**9.4**	**4.693**
0.1	10	0.75	0.75	0.5	0.5	9.4	4.811
0.1	10	0.75	0.7	1	1	9.4	5.402
0.1	10	0.75	0.75	1.5	1.5	9.4	5.299
0.1	10	0.75	0.75	3	3	9.4	5.835
0.1	10	0.9	0.9	0.25	0.25	9.4	5.316
0.1	10	0.9	0.9	0.5	0.5	9.4	5.435
0.1	10	0.9	0.9	1	1	9.4	5.656

From this mitigation method the mitigation curve follows the following quadratic equation.

$y = ax^2 + by + c$

From Table2, maximum mitigation has been reported where , $b/A=0.1, b=10, b_1=0.75b, b_2=0.75b_2, \delta_1= 0.25b, \delta_2=0.25b$

At this position of relief notches and size of relief notches, the mitigation curve passes through following co ordinates, $(x_1, y_1)= (0 ,10)$, $(x_2, y_2)=(20 ,7.5)$, $(x_3, y_3)=(35.625 ,5.625)$

By putting these values and solving the equation for these three sets, we can determine,

$a=2.5*10^{-5} \approx 0$, $b = -0.12$, $c = b$ (depth of V-notch)

the generalized equation for mitigation curve is obtained as ,

$y = -0.12x+b$

Case2: Mitigation Curve for Two Relief Semicircular Notches Around One V- Notch

In this method auxiliary notches of semi circular shape along with the main V -notch in plate has been provided and analyzed. The modified model of the plate with two sets of relief semicircular notches has been shown in Fig5. The size and distance of semicircular notches, number of sets of auxiliary semicircular notches has also been optimized by providing two sets of auxiliary notch. The mitigation results are shown in Table-5. By this method, significant mitigation in SCF has been reported.

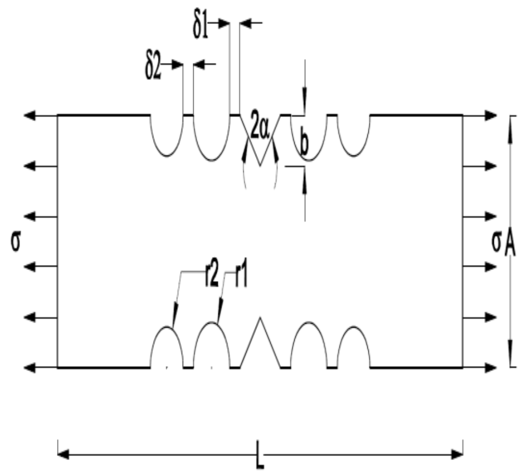

Fig3. Rectangular Plate with Main opposite V-Notch and Auxiliary Semicircular Notches

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Table 3. Mitigation of SCF by Providing Relief Semi-Circular Notch

b/A	b	r	r1	r2	δ/b	δ1	δ2	SCF WITH MAIN NOTCH	Max Stress with two sets of relief notches
0.05	5	5	4.5	4.05	0.1	0.5	0.5	4.4964	2.638
0.05	5	5	4.5	4.05	0.5	2.5	2.5	4.4964	2.914
0.05	5	5	4.5	4.05	1	5	5	4.4964	4.306
0.05	5	5	4.5	4.05	2	10	10	4.4964	4.945
0.05	5	5	4.5	4.05	5	25	25	4.4964	6.138
0.1	10	10	9	8.1	0.1	0.5	0.5	5.4256	3.843
0.1	10	10	9	8.1	0.5	2.5	2.5	5.4256	3.74
0.1	10	10	9	8.1	1	5	5	5.4256	5.934
0.1	10	10	9	8.1	2	10	10	5.4256	6.507
0.1	10	10	9	8.1	5	25	25	5.4256	7.846

From this mitigation method the mitigation curve follows the following quadratic equation.

$$y = ax^2 + by + c$$

$a=2.5*10^{-5} \approx 0$, $b = -0.12$, $c = b$ (depth of V-notch)

$$y = -0.12x+b$$

Case 3: Mitigation Curve for two relief coaxial circular holes around main eccentric circular hole

The geometry of the problem is shown in Fig6. Rectangular plate of 400mm*100mm and of 1mm thickness with eccentric circular hole is considered for analysis. The isotropic material of the plate is high strength alloy steel; Poisson's ratio ν=0.3, Young's Modulus E=39 GPa. Uniform tensile load (σ) is applied to the plate's lengthwise direction.

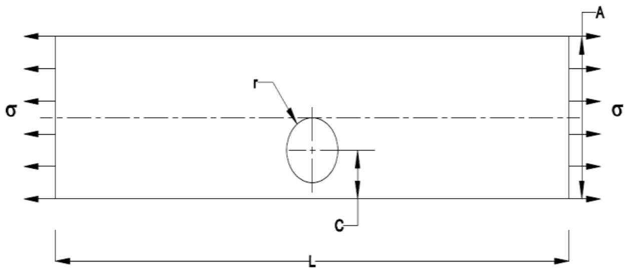

Fig4. Rectangular Plate with Eccentric Circular Hole

An 8 nodded structural plane 82 element type was selected for free meshing on the bases of convergence test. The results of convergence test are shown in Table-3. The element size of 1.75 has been selected for analysis from this test as it gives results close to the theoretical results. Each node has two degrees of freedom, making a total 16 degrees of freedom per element. As in this case the plate is not symmetrical about its axis due to eccentric circular hole, the whole plate for each case is analyzed. The results for SCF obtained from ANSYS are compared with results for SCF calculated from the equations derived by Roark as given below.

Roark's equation for eccentric circular hole in plate:

$$\sigma_{nom} = (P\sqrt{(1-(r/c)^2)} \quad 1-(c/D))/(Dt \quad 1-(r/c) \quad 1-(c/D)[2-\sqrt{(1-[(r/c)]^2)})$$

$$K_t = 3.00 - 3.13\left(\frac{r}{c}\right) + 3.66\left(\frac{r}{c}\right)^2 - 1.53\left(\frac{r}{c}\right)^3$$

Table -4: Convergence Test for Appropriate Element Size

c/A	r/c	Element Size	Nominal Stress(Theoretical)	SCF (Theoretical)	Maximum Stress (ANSYS)	SCF (ANSYS)	% AGE DIFFERENCE
0.4	0.25	1.0	1.318915147	2.42234	3.082	2.3367	3.5354
0.4	0.25	1.25	1.318915147	2.42234	3.0	2.3333	3.6761
0.4	0.25	1.5	1.318915147	2.42234	3.076	2.3322	3.7212
0.4	**0.25**	**1.75**	**1.318915147**	**2.42234**	**3.218**	**2.43988**	**0.724**
0.4	0.25	2.0	1.318915147	2.42234	3.116	2.3625	2.47
0.4	0.25	2.25	1.318915147	2.42234	3.135	2.3769	1.8758
0.4	0.25	2.5	1.318915147	2.42234	3.153	2.3906	1.31
0.4	0.25	5.0	1.318915147	2.42234	3.17	2.5334	4.58

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Four models are prepared for analysis, by considering different c/A ratio. For all c/A ratios the plate is analyzed and the results are compared with the theoretical results as shown in Table6. The results are very close to the theoretical results by considering the optimum element size and optimum element type for analysis.

Table-5: Comparison of SCF (Theoretical) and SCF (ANSYS) For Different Models

c/A	r/c	r (In mm)	Nominal Stress (Theoretical)	SCF (Theoretical)	Maximum Stress (ANSYS)	SCF (ANSYS)
0.3	0.67	20	2.52	2.3867	5.973	2.3702
0.35	0.428	15	1.67	2.21	3.81	2.2814
0.4	0.25	10	1.31	2.42234	3.218	2.4564
0.45	0.111	5	1.12	2.6952	3.114	2.7803

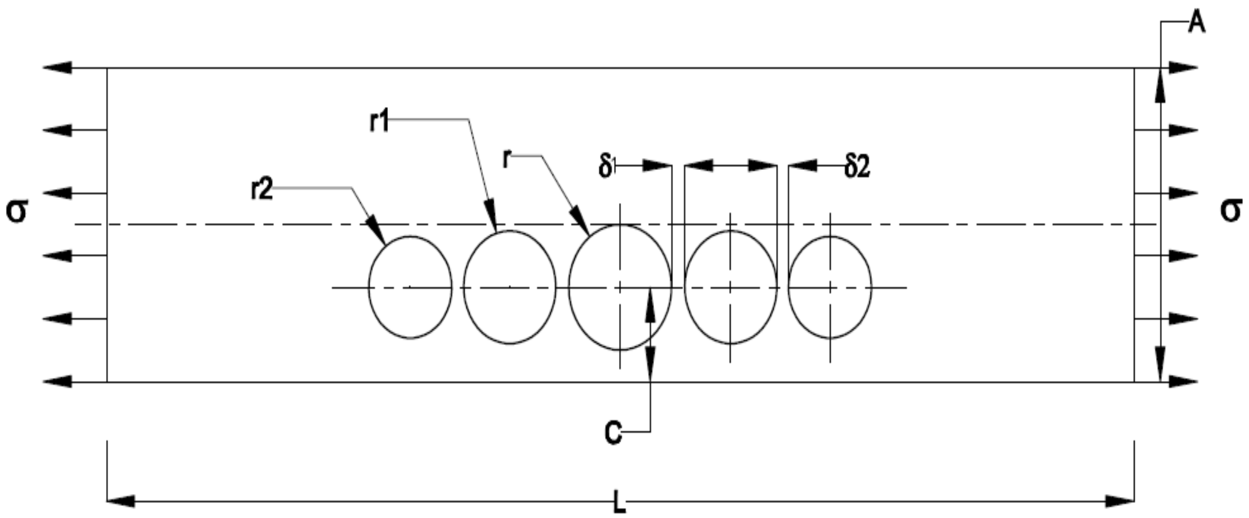

Fig5. Rectangular Plate with Eccentric Circular Hole and Relief Coaxial Holes

Table-6: Optimization of Size and Position of Relief Holes with Main Eccentric Hole

c/A	r/c	r	r1	r2	δ_1, δ_2	L1	L2 (mm)	σ_{nom}	σ_{max}	SCF (without relief holes)	SCF (with one set of relief holes)	SCF (with two sets of relief holes)	%age reduction
0.4	0.25	10	9.0	_	0.25r	21.5	_	1.31	2.607	2.4564	1.99	_	17.89
0.4	0.25	10	**9.0**	**8.1**	**0.25r**	**21.5**	**40.85**	**1.31**	**2.477**	**2.4564**	_	**1.89**	**23.06**
0.4	0.25	10	9.0	8.1	0.5r	24	45.6	1.31	2.505	2.4564	_	1.91	22.25
0.4	0.25	10	9.0	8.1	r	29	55.1	1.31	2.583	2.4564	_	1.97	19.80
0.4	0.25	10	7.5	5.625	0.25r	20	35	1.31	2.702	2.4564	_	2.06	16.14
0.4	0.25	10	7.5	5.625	0.5r	22.5	39.375	1.31	2.643	2.4564	_	2.02	17.77
0.4	0.25	10	7.5	5.625	r	27.5	48.125	1.31	2.677	2.4564	_	2.04	16.95
0.4	0.25	10	5.0	2.5	0.25r	17.5	26.25	1.31	3.011	2.4564	_	2.30	6.37
0.4	0.25	10	5.0	2.5	0.5r	20	30	1.31	2.942	2.4564	_	2.25	8.40
0.4	0.25	10	5.0	2.5	r	25	37.5	1.31	2.916	2.4564	_	2.23	9.22

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From the above results it has been analyzed that the maximum mitigation in SCF is achieved where,
r_1=0.9r, r_2=0.9r_1, δ_1 = 0.25 r and δ_2 = 0.25 r_1
By putting the optimized values in quadratic equation, we get y=-0.046511x+r
This generalized equation for mitigation curve is straight line.

IV. RESULTS AND DISCUSSIONS

A. Rectangular Plate with opposite V-notches

Rectangular plate with this type of singularities is analyzed by proposing two types of material removal. By analyzing the plate with auxiliary semi circular notch and relief v-notches it has been observed that in both the cases the optimized peripheral distance and radius/depth of auxiliary notches are same , where maximum mitigation in SCF has been reported. From these results we can conclude that both types of auxiliary notches fall on same interaction zone of main notch. Both auxiliary notches mitigate the maximum stress around the main singularity. It has been observed that the percentage of mitigation in SCF is more in case of auxiliary V notch as compared to the auxiliary semi-circular notch. The mitigation of SCF has been done by providing semi circular notches at optimum distance and of optimum size. The number of sets of auxiliary holes is increased and its effect on mitigation of SCF has been analyzed. It has been reported that maximum mitigation is by providing two sets of relief notches.

B. Rectangular Plate with eccentric circular hole

Mitigation in SCF around eccentric holes has been done by providing coaxial auxiliary holes around main circular hole. It has been reported that mitigation in SCF is more by providing two sets of auxiliary holes as compared to one set of auxiliary hole.

Table 7: Mitigation of SCF by Providing Coaxial Auxiliary Holes for Different C/A

c/A	r/c	r (In mm)	r1 (In mm)	r2 (In mm)	L1 (In mm)	L2 (In mm)	Nom. Stress	Max. Stress	SCF (without auxiliary holes)	SCF (with one set of auxiliary holes)	SCF (with two sets of auxiliary holes)	%age red. In SCF
0.45	0.111	5	4.5	_	10.75	_	1.12	2.48	2.7803	2.2143	_	20.3575
			4.5	4.05	10.75	20.425	1.12	2.366	2.7803	_	2.1125	24.0189
0.4	0.25	10	9.0	_	21.5	_	1.31	2.607	2.4564	1.9900	_	17.89
			9.0	8.1	21.5	40.85	1.31	2.477	2.4564		1.89	23.06
0.35	0.428	15	13.5	_	32.25	_	1.67	3.036	2.2814	1.8179	_	20.3165
0.35	**0.428**	**15**	**13.5**	**12.15**	**32.25**	**58.23**	**1.67**	**2.724**	**2.2814**	_	**1.6311**	**28.5044**
0.3	0.67	20	18	_	43	_	2.52	5.178	2.3702	2.0547	_	13.31
0.3	0.67	20	18	16.2	43	81.7	2.52	3.157	2.3702	_	1.2528	47.15

The equation derived for stress flow lines for getting maximum mitigation in SCF is same as derived for central circular holes. From these results we can say that same mitigation techniques can be applied for eccentric circular holes as derived for central circular holes.

V.CONCLUSION

The above results shows that the radius of auxiliary holes , distance of auxiliary hole from main hole and the number of auxiliary hole sets effects the mitigation of SCF. From the derived equation we can calculate the optimum size of auxiliary hole for given distance from the main hole. Similarly, if size of auxiliary hole is given then we can fix its optimum distance from the centre of main hole. Mitigation in SCF for the case of V-notches also reported , by applying these mitigation curves , we can find the optimum position of auxiliary notch. This result helps the designer to mitigate the SCF around any discontinuity.

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REFERENCES

[1] Eywood R.B, “Designing by Photo Elasticity”, Chapman & Hall Ltd. London, 1952.

[2] R.E.Peterson, “Stress concentration design factors”, John Wiley and sons, New York, 1966.

[3] Durelli .A.J and Rajaiah.K Optimum hole shapes in finite plates under uni- axial load. Applied Mechanics 46(3), 691-695(1979).

[4] G.S.Giare and R. Shabahang, The Reduction of Stress Concentration around the hole in an isotropic plate using composite material”, Engineering Fracture Mechanics Vol.32 No.5, pp 757-766, (1989).

[5] Sanyal, Priti Yadav “ Relief Holes for Stress mitigation in Infinite Thin Plate with Single Circular Hole Loaded Axially , ASME International Mechanical Engineering Congress & Exposition , 2005.

[6] Dr. S. Sanyal, Ms. Priti Yadav “Multiple Relief Holes for Stress mitigation in Infinite Thin Plate with Single Circular Hole Loaded Axial Loading”,2nd International Conference SCCE, 5-8 July 2006.

[7] N. D. Mittal and N. K. Jain, “The optimization of a square simply supported isotropic plate with central circular hole for reduction of stress concentration subjected to transverse static loading”, Proceedings of ICTACEM -2007.

[8] N. D. Mittal and N. K. Jain, Finite Element Analysis for Stress Concentration and deflection in Isotropic and Orthotropic Rectangular Plates with Central Circular Hole Under Transverse Static Loading , Material Science and Engineering A,498 (2008)115-124.

[9] Zhixue Wu, "Optimal hole shape for minimum stress concentration using parameterized geometry models", Springer, 2008.