

Performance Analysis of Space-Time Block Codes Achieving Full Diversity with Linear Receivers Using MIMO

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Abstract--Space-time block codes have been shown to perform well with Multiple-Input Multiple Output (MIMO) systems. STBC is a MIMO transmit strategy which exploits transmit diversity and high reliability. The existence of real orthogonal design has been proved by Radon. As for a complex orthogonal design, it was proved by Tarokh that for the corresponding STBC one cannot achieve a full transmission rate and full diversity when the number of transmitting antennas is larger than two. Therefore, some Quasi-orthogonal space-time block codes have been designed. In most of the existing space-time code designs, achieving full diversity is based on maximum-likelihood (ML) decoding at the receiver that is usually computationally expensive. Orthogonal space-time block codes (OSTBC) achieve full diversity when a linear receiver, such as, zero-forcing (ZF) or minimum mean square (MMSE), is used. By choosing different parameters, codes with different symbol rates and orthogonality can be obtained. In this paper, we compare the performance of a family of space-time codes that achieve full diversity when linear receivers, such as, zero-forcing (ZF) or minimum mean square error (MMSE) is used.

Index Terms—Full Diversity, Maximum Likelihood (ML) Detector, Minimum Mean Square Error (MMSE), Multiple-Input Multiple-Output (MIMO), Space-Time Block Codes (STBC), Zero-Forcing (ZF) Equalizer.

I. INTRODUCTION

MIMO technology means multiple antennas at both the ends of a communication system, that is, at the transmitting end and receiving end. The idea behind MIMO is that the transmit antennas at one end and the receive antennas at the other end are connected and combined in such a way that the bit error rate (BER), or the data rate for each user is improved. MIMO has the capacity of producing independent parallel channels and transmitting multipath data streams and thus meets the demand for high data rate wireless transmission. This system can provide high frequency spectral efficiency and is a promising approach with tremendous potential [1]. Independent channel fading caused by multipath between different transmitting and receiving antenna pairs provides a significant capacity gain and link reliability over conventional single antenna system [2]. The multipath nature of the wireless channel results in a superposition of the signals of each path at the receiver. This can lead to either constructive or destructive interference. Strong destructive interference is referred to as a deepfade and may result in temporary failure of communication due to a severe drop in the channel

signal-to-noise ratio (SNR). To avoid this situation, transmit diversity scheme is used. By having more than one antenna at the transmitter and receiver, spatial diversity methods can be employed in order to overcome the fading problem [3], [4]. MIMO is mostly used in conjunction with OFDM, a modulation technology that is part of the IEEE 802.16 and IEEE 802.11n “High-throughput standards”. Several transmission schemes have been proposed that utilize the MIMO channel in different ways, for example, spatial multiplexing, space-time coding or beam forming. Multiple antennas allow MIMO systems to perform precoding (multilayer beam forming), diversity coding (space-time coding) & spatial multiplexing. Advantages of MIMO systems include:

1. Beam forming: Beam forming consists of transmitting the same signal with different gain and phase over all transmit antennas such that the receiver signal is maximized. A transmitter receiver pair can perform beam forming and direct their main beams at each other, thereby increasing the receiver’s received power and consequently the SNR.

2. Spatial Diversity: Diversity consists of transmitting a single space-time coded stream through all antennas. A signal can be coded through the transmit antennas, creating redundancy, which reduces the outage probability.

3. Spatial Multiplexing: Spatial multiplexing increases network capacity by splitting a high data rate signal into multiple lower data rate streams and transmitting them through the different antennas. A set of streams can be transmitted in parallel, each using a different transmit antenna element. The receiver can then perform the appropriate signal processing to separate the signals. Orthogonal space-time codes, first presented by Alamouti, are codes that maximize the diversity gain of a MIMO channel and allow a simple individual decoding of each symbol of the space-time block, leading to a reduced complexity implementation of the linear decoder. These attributes are achieved by sacrificing the rate of the code, such that no OSTBC with complex constellation can achieve full rate for more than two transmitting antennas [5]. Therefore some Quasi-orthogonal space-time block codes have been designed. In this structure, transmission matrix can be divided into groups to achieve full transmission rates while sacrificing the full diversity. When the number of transmit antennas is fixed, the decoding complexity of quasi-orthogonal STBC increases not as much as orthogonal STBC [6]. In other words, the

detection complexity of quasi-orthogonal STBC can be significantly reduced.

II. MIMO SYSTEM MODEL

MIMO systems are composed of three main elements, namely the transmitter (T_x), the channel (H) and the receiver (R_x). In this paper, N_t is denoted as the number of antenna elements at the transmitter and N_r is denoted as the number of elements at the receiver. Fig. 1 depicts the block diagram of such a MIMO system.

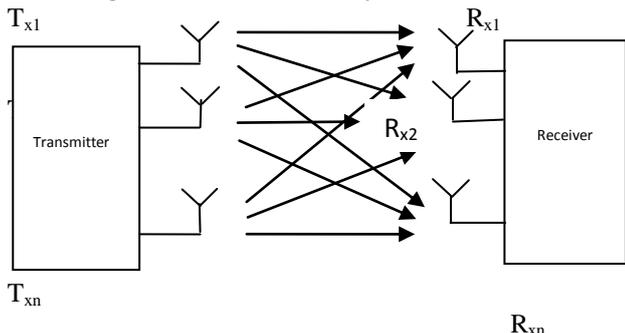


Fig. 1: Block diagram of MIMO system

Let $h_{i,j}$ be a complex number corresponding to the channel gain between transmit antenna i & receive antenna j respectively. If at a certain time instant the complex signals $\{s_1, s_2, s_3, \dots, s_n\}$ are transmitted via N_t transmit antennas, then N_r the received antenna can be expressed as:

$$y_i = \sum_{j=1}^{N_t} h_{i,j} s_j + n_i \quad (1)$$

Where n_i is a noise term combining all the received signals in a vector y , equation (1) can be expressed in the form:

$$y = Hs + n \quad (2)$$

The channel with N_t inputs and N_r outputs is denoted as $N_r \times N_t$ channel matrix:

$$H = \begin{pmatrix} h_{1,1} & \dots & h_{1,N_t} \\ \vdots & \ddots & \vdots \\ h_{N_r,1} & \dots & h_{N_r,N_t} \end{pmatrix}$$

Where each entry $h_{i,j}$ denotes the attenuation and phase shift (transfer function) between the j^{th} transmitter and the i^{th} receiver. It is assumed throughout this paper that the MIMO channels behaves in a quasi-static manner, that is, the channel varies randomly between burst to burst, but fixed within a transmission.

For simplicity, channel capacity for MIMO system can be approximated to

$$C = m \cdot \log(1 + SNR) \quad (4)$$

Where m is the number of antennas in the transmitter and receiver sides. From equation (4), it is observed that the spectral efficiency for a MIMO system is increased linearly with the increase of the number of antennas. Multiple parallel channels are created to deliver more data traffic streams simultaneously.

III. SPACE-TIME BLOCK CODE

Spatial diversity can be achieved by transmitting several replicas of the same information through each antenna. The different replicas sent for exploiting diversity are generated by a space-time encoder which encodes a single stream through space using all the transmit antennas and through time by sending each symbol at different times. By doing so, the probability of losing the information decreases exponentially [6]. This form of coding is called space-time coding (STC). Due to their decoding simplicity, the most dominant form of STCs is space-time block codes (STBC).

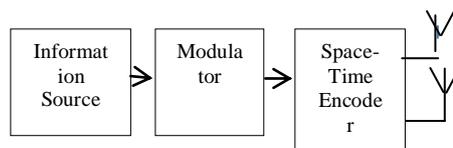


Fig. 2: Block Diagram of Space-Time Coding

A. Alamouti's STBC:

Alamouti's scheme was the first STBC that provides full diversity at full data rate for two transmit antennas. This scheme has full rate (i.e. a rate of 1) since it transmits two symbols every two time intervals. The information bits are first modulated using a digital modulation scheme, and then the encoder takes the block of two modulated symbols s_1 and s_2 in each encoding operation. Here we adopt multilevel modulation. First, we modulate m ($m = \log_2 M$) bits as a group, then the channel encoder will get two modulated signals s_1, s_2 as a group each time when encoding, and map the two signals into the transmit antennas according to the following encoding matrix:

$$S = \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix} \quad (5)$$

The outputs of the encoder are transmitted by the two transmit antennas during two continuous periods. During the first period, signals s_1, s_2 are transmitted at the same time by antenna 1 and antenna 2 respectively while during the second period, signal $-s_1^*$ is transmitted by antenna 2 and s_2^* by antenna 1, where s_1^* is the complex conjugate number of s_1 . We can express transmit sequences from antennas 1 and 2 as follows:

$$s^1 = (s \quad -s_2^*) \quad (6)$$

$$s^2 = (s_2 \quad s_1^*) \quad (7)$$

$$s_1 \cdot s_2 = s_1 s_2^* - s_2^* s_1 = 0 \quad (8)$$

Now we can find that the inner product of s^1 and s^2 is zero, that is, the two transmit sequences are orthogonal.

B. Orthogonal Space-Time Block Codes

For complex orthogonal space-time block codes, due to the orthogonality of their codes, their maximum likelihood (ML) decoding is linear and hence they achieve full diversity with linear receivers [7]. OSTBCs can be expanded to any number of transmit antennas. The real orthogonal designs exist only for $N = 2, 4$, and 8 .

STBCs based on real designs have transmission rate of 1; a number codes based on generalized real designs are constructed explicitly for $N \leq 8$.

The code for a $N = 4$ transmit antenna system is given by

$$S = \begin{bmatrix} X_1 & -X_2 & -X_3 & -X_4 \\ X_2 & X_1 & X_4 & -X_3 \\ X_3 & -X_4 & X_2 & X_1 \\ X_4 & X_3 & -X_2 & X_1 \end{bmatrix} \quad (9)$$

We find that each column of S differs from the first by a permutation and a reflection. Thus when transmitting this code over a slow fading channel, its structure will be transferred to the effective channel matrix, as in the case of the Alamouti's STBC. There are $Q = 4$ symbols being sent over $L = 4$ symbol periods, thus the rate of the space-time encoder is 1.

IV. EQUALIZATION TECHNIQUES

1. Zero-Forcing Equalizer: The ZF equalizer applies the inverse of the channel frequency response to the received signal, to restore the signal after the channel [8]. The name zero forcing corresponds to bringing down the intersymbol interference (ISI) to zero in a noise free case. For a channel with frequency response $F(f)$, the zero-forcing equalizer $C(f)$ is constructed by

$$C(f) = \frac{1}{F(f)} \quad (10)$$

Thus combination of channel and equalizer gives a flat frequency response and linear phase $F(f)C(f) = 1$.

2. Minimum Mean Square Error (MMSE): Although the ZF equalizer removes ISI, may not give the best error performance for the communication system because it does not take into account noises in the system. An equalizer that takes noise into account is the MMSE equalizer. It is based on the mean square error (MSE) criterion. A MMSE estimator describes the approach which minimizes the mean square error, which is a common measure of estimator quality. The MMSE estimator is then defined as the estimator achieving minimal MSE. Basically, MMSE is the expected value of the squared difference between desired data signal and the estimated data signal [9]. Its output can be expressed in the form

$$(\hat{f}) = E|X(f) - X^A(f)|^2 \quad (11)$$

Where X are data samples.

3. Maximum Likelihood Decoding Equalizer: Maximum likelihood estimation is a totally analytic maximization procedure (MAP). The techniques discussed previously were not optimal in terms of minimizing the average symbol error probability. Since the effect of a symbol is spread to other symbols, it is intuitive that the optimal receiver should observe not only the segment of received signal concerning the desired symbol, but the whole received signal instead [10]. Using the whole received signal, we can employ the MAP principle to develop the

optimal symbol-by-symbol detector, which decides one transmitted symbol at a time, to minimize the average symbol error probability.

We know that the system equation is expressed as

$$r = Hs + v \quad (12)$$

Where r is the received signal, Hs is the noise free signal constellation and v is the receiver noise. The optimum detector compares the received signal vector r to every possible noise free constellation point $H(s)$ [11].

V. RESULTS

Figure 3 and 4 shows the simulation results comparing various space-time block codes. It is seen from figure 3 that the bit error rate performance of orthogonal code is better as compared to Alamouti's code. Similarly, figure 4 shows that the orthogonal space-time block codes (OSTBC) provides a better BER than the quasi orthogonal codes (QOSTBC). Figure 5 shows BER plot of STBC codes using different modulation schemes. It is observed from the graph that the BER performance deteriorates as the modulation scheme is increased.

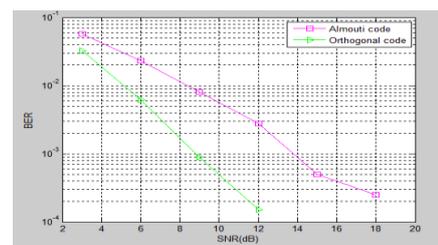


Fig. 3: BER Performance of Alamouti's and Orthogonal Codes

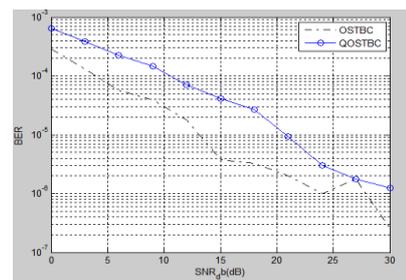


Fig. 4: BER Performance of OSTBC and QOSTBC Codes

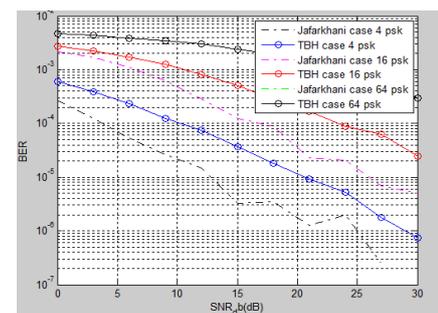


Fig. 5: BER Performance of STBC Codes Using Different Modulation Schemes

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