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Performance Analysis of Various Channel Estimation Techniques for Higher Order Modulation in a MIMO System

Jaspreet Kaur, Manwinder Singh

Abstract*—Digital communication using MIMO has recently emerged as one of the most significant technical breakthroughs in modern communications and also known as volume to volume wireless link. The effect of fading and interference causes an issue for signal recovery in wireless communication [1]. In this paper we have investigated the bit error rate performance characteristics of MIMO system using channel estimation techniques. Channel estimation techniques consists of linear and non linear detectors or equalizers which aid in the elimination of Inter Symbol Interference (ISI) thus improving overall performance to analyze the BER of the designed system. Firstly, the BER analysis is done for modulation schemes using linear detectors (LD) such as ZF, MMSE and non-linear detectors (NLD) such as ZF-SIC, MMSE-SIC and finally comparison of linear and non-linear detectors is done for various modulation schemes. Here 16-PSK, 32-PSK, 64-PSK modulation schemes are considered over Rayleigh channel for two transmitters and two receivers.

***Keywords*—MIMO (Multiple Input Multiple Output), BER (Bit Error Rate), SNR (Signal To Noise Ratio), PSK (Phase Shift Keying Modulation), ZF (Zero Forcing), MMSE (Minimum Mean Square Error), SIC (Successive Interference Cancellation).**

I. INTRODUCTION

Multiple-Input Multiple-Output systems (MIMO) were regarded as one of the most promising technologies in field of wireless communication. Generally considered as one of the several forms of smart antenna technology, it offers considerable increase in data throughput and link range without additional bandwidth or transmit power [2]. In conventional wireless communications, a single antenna is used at the source and destination. When EM field is met with obstructions such as hills, the scattering occurs which results in fading; it can cause a reduction in data speed and an increase in the number of errors. The use of two or more antennas at the source and the destination eliminates the trouble caused by multipath wave propagation [3]. The basic idea of MIMO is to improve quality (BER) and data rate (bits/sec) by using multiple Transmit and receive antennas [4]. Channel Estimation is part of multiple user receivers. It is well known that the wireless channel causes an arbitrary time dispersion, attenuation, and phase shift in the received signal. The use of orthogonal frequency-division multiplexing and a cyclic prefix mitigates the effect of time dispersion. However, it is still necessary to remove the amplitude and phase shift caused by the channel. The function of channel estimation is to form an estimate of the amplitude and phase shift caused by the wireless channel from the available pilot information [5]. So, the function of channel estimation is to estimate channel distortion by pilot symbols. The equalization removes the effect of the wireless channel and allows subsequent symbol demodulation. To remove Intersymbol Interference in channels various types of detectors are used. However, these detectors require knowledge on the channel impulse response (CIR), [6] which can be provided by a separate channel estimator. Usually the channel estimation is based on the known sequence of bits, which is unique for a certain transmitter and which is repeated in every transmission burst. Thus, the channel estimator is able to estimate CIR for each burst separately by exploiting the known transmitted bits and the corresponding received samples. Detectors comprising of linear detectors (LD) and non-linear detectors (NLD). Linear detectors (LD) consists of zero forcing (ZF) & Minimum mean square error (MMSE) and Non-linear detectors (NLD) consists of Zero forcing-successive Interference cancellation (ZF-SIC) & Minimum mean square error-successive Interference cancellation (MMSE-SIC) [7].

II. DETECTORS

A. Zero Forcing Equalizer (ZF)

Zero Forcing Equalizer is a linear equalization algorithm used in communication systems, which inverts the frequency response of the channel [8]. The Zero-Forcing Equalizer applies the inverse of the channel to the received signal, to restore the signal before the channel. The name Zero Forcing corresponds to bringing down the ISI to zero in a noise free case.

Let us consider a 2x2 MIMO channel, the channel is modelled as [9],

In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}\ x_1 + h_{1,2}\ x_2 + n_1 = [\ h_{1,1} \quad h_{1,2}\ ] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \qquad (1)$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1}\ x_1 + h_{2,2}\ x_2 + n_2 = [h_{2,1} \quad h_{2,2}\ ] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \qquad (2)$$

Where

y_1, y_2 are the received symbol on the first and second antenna respectively,

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$h_{1,1}$ is the channel from 1st transmit antenna to 1st receive antenna,

$h_{1,2}$ is the channel from 2nd transmit antenna to 1st receive antenna,

$h_{2,1}$ is the channel from 1st transmit antenna to 2nd receive antenna,

$h_{2,2}$ is the channel from 2nd transmit antenna to 2nd receive antenna,

x_1, x_2 are the transmitted symbols

n_1, n_2 are the noise on 1st and 2nd receive antennas.

The equation can be represented in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (3)$$

Equivalently, $y = Hx + n$ (4)

To solve for x, we need to find a matrix W which satisfies $WH = I$. The Zero Forcing (ZF) detector for meeting this constraint is given by,

$$W_{ZF} = (\mathbf{H}^{\dagger}\mathbf{H}\ [\mathbf{H})]^{\dagger}(-1)\ \mathbf{H}^{\mathbf{H}} \quad (5)$$

Where W = Equalization matrix H = Channel matrix

This matrix is Known as Pseudo inverse for a general m $\times$ n matrix where

$$H^H H = \begin{pmatrix} h^*_{1,1} & h^*_{2,1} \\ h^*_{1,2} & h^*_{2,2} \end{pmatrix} \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \quad (6)$$

The off diagonal elements in the matrix $\mathbf{H}^{\mathbf{H}}$ H are not zero, because the off diagonal elements are non zero in values. Zero forcing equalizer tries to null out the interfering terms when performing the equalization, i.e. when solving for x_1 the interference from x_2 is tried to be nulled and vice versa. While doing so, there can be an amplification of noise. Hence the Zero forcing equalizer is not the best possible equalizer. However, it is simple and easy to implement. [10]

B. Minimum Mean Square Error Equalizer (MMSE)

A minimum mean square error (MMSE) estimator describes the approach which minimizes the mean square error (MSE), which is a common measure of estimator quality [8]. The main feature of MMSE equalizer is that it does not usually eliminate ISI completely but, minimizes the total power of the noise and ISI components in the output. Let us now try to understand the math for extracting the two symbols which interfered with each other. In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}\, x_1 + h_{1,2}\, x_2 + n_1 = [\, h_{1,1} \quad h_{1,2}\,] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad (7)$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1}\, x_1 + h_{2,2}\, x_2 + n_2 = [\, h_{2,1} \quad h_{2,2}\,] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad (8)$$

The equation can be represented in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (9)$$

Equivalently, $y = Hx + n$ (10)

MMSE approach tries to find a coefficient W which minimizes the E{[W_{y-x}][$[W_1(y - x)]]^{\dagger}\mathbf{H})$ } criterion

To solve for x, we need to find a matrix W which satisfies $WH = I$. The Zero Forcing (ZF) detector for meeting this constraint is given by,

$$W_{MMSE} = (\mathbf{H}^{\dagger}\mathbf{H}\ [\mathbf{H} + \mathbf{N}_1 0)]^{\dagger}(-1)\ \mathbf{H}^{\dagger}\mathbf{H}$$

(11)

This matrix is Known as Pseudo inverse for a general m $\times$ n matrix where

$$H^H H = \begin{pmatrix} h^*_{1,1} & h^*_{2,1} \\ h^*_{1,2} & h^*_{2,2} \end{pmatrix} \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \quad (12)$$

When comparing the eq.(11) to the eq.(5) in Zero Forcing equalizer, apart from NoI the term both the equations are comparable. In fact, when the noise term is zero, the MMSE equalizer reduces to Zero Forcing equalizer [11].

C. Zero Forcing With Successive Interference Cancellation (ZF-SIC)

Successive interference cancellation (SIC) is a physical layer capability that allows a receiver to decode packets that arrive simultaneously. Since SIC [12] is known to be sensitive to error propagation a careful adjustment of the data rate at each spatial layer is mandatory and decoding of already detected spatial layers is required prior to an interference cancelation step.

Using the Zero Forcing (ZF) equalization approach described above, the receiver can obtain an estimate of the two transmitted symbols x_1, x_2 i.e.

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = (\ \mathbf{H}^{\dagger}\mathbf{H}\ [\mathbf{H})]^{\dagger}(-1)\ \mathbf{H}^{\dagger}\mathbf{H} \quad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

(13)

Take one of the estimated symbols (for example $\hat{x}_2$) and subtract its effect from the received y_1 and y_2, i.e.

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,2}\,\hat{x}_2 \\ y_2 - h_{2,2}\,\hat{x}_2 \end{bmatrix} = \begin{bmatrix} h_{1,1}\, x_1 + n_1 \\ h_{2,1} x_1 + n_2 \end{bmatrix} \quad (14)$$

Expressing in matrix notation,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} x_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (15)$$

$r = h x_1 + n$ (16)

The above equation is same as equation obtained for receive diversity case. The equalized symbol is,

$$\hat{x}_1 = \frac{h^H r}{h^H h} \quad (17)$$

This forms the simple explanation for Zero Forcing Equalizer with Successive Interference Cancellation (ZF-SIC) approach.

III. SIMULATION RESULTS

Simulation results are provided to substantiate the analysis. The Simulation results for the performance of Alamouti STBC for different modulation techniques 16-PSK, 32-PSK and 64-PSK for Rayleigh channel are obtained using MATLAB. The BER values as function of

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SNR are determined for each modulation scheme for the purpose of comparing their relative performances as shown in Table 1.

Table 1: BER for Different Modulation Techniques at SNR = 20 Db

Modulation Schemes	Linear Detectors (LD)		Non-Linear Detectors (NLD)	
	ZF	MMSE	ZF-SIC	MMSE-SIC
16-PSK	0.1561	0.0022	0.0015	0.0006
32-PSK	0.2427	0.0043	0.0028	0.0012
64-PSK	0.3477	0.0076	0.0050	0.0021

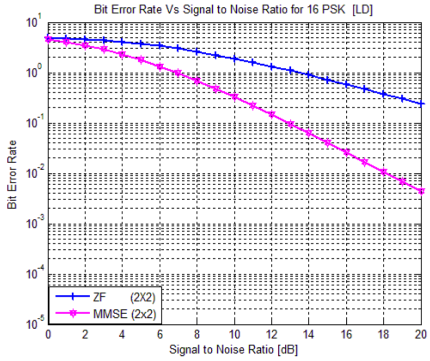

Fig 1: BER Vs SNR for 16-PSK [LD]

Figure 1 shows bit error rate vs signal to noise ratio of linear detector (LD) for 16-PSK. This figure depicts, at 20 dB, BER of MMSE is 0.0022 and ZF is 0.1561. So, MMSE achieves low bit error rate and better performance than ZF

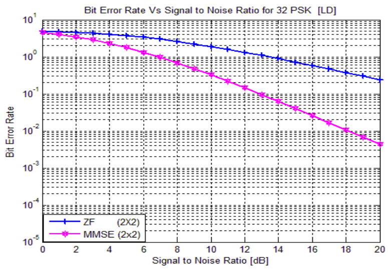

Fig 2: BER Vs SNR for 32-PSK [LD]

Figure 2 shows bit error rate vs signal to noise ratio of linear detector (LD) for 32-PSK. This figure depicts, at 20 dB, BER of MMSE is 0.0043 and ZF is 0.2427. So, MMSE achieves minimum bit error rate than ZF.

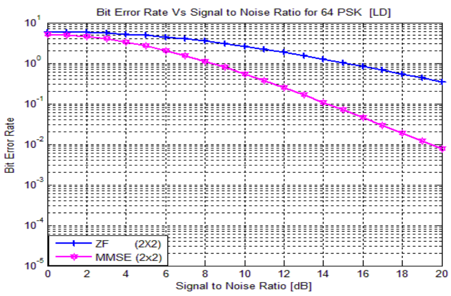

Fig 3: BER Vs SNR for 64-PSK [LD]

Figure 3 shows bit error rate vs signal to noise ratio of linear detector (LD) for 64-PSK. This figure depicts, at 20 dB, BER of MMSE is 0.0076 and ZF is 0.3477. Again, MMSE achieves low bit error rate and better performance than ZF.

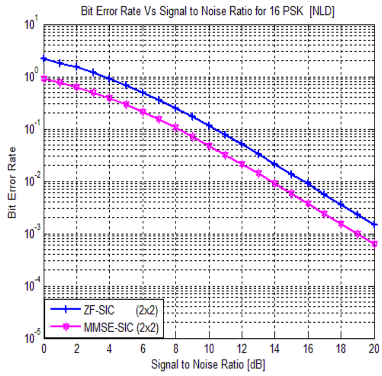

Fig 4: BER Vs SNR for 16-PSK [NLD]

Figure 4 shows bit error rate vs signal to noise ratio of Non-linear detector (NLD) for 16-PSK. This figure depicts, at 20 dB, BER of MMSE-SIC is 0.0006 and ZF-SIC is 0.0015. So, MMSE-SIC achieves minimum bit error rate and better performance than ZF-SIC.

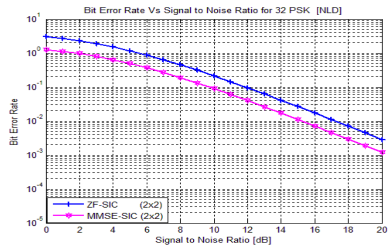

Fig 5: BER Vs SNR for 32-PSK [NLD]

Figure 5 shows bit error rate vs signal to noise ratio of Non-linear detector (NLD) for 32-PSK. This figure depicts, at 20 dB, BER of MMSE-SIC is 0.0012 and ZF-SIC is 0.0028. So, MMSE-SIC achieves low bit error rate and better performance than ZF-SIC.

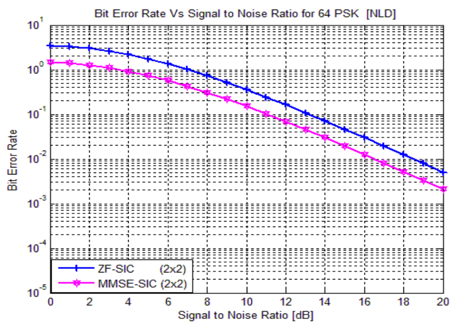

Fig 6: BER Vs SNR for 64-PSK [NLD]

Figure 6 shows bit error rate vs signal to noise ratio of Non-linear detector (NLD) for 16-PSK. This figure depicts, at 20 dB, BER of MMSE-SIC is 0.0021 and ZF-SIC is 0.0050. So, MMSE-SIC achieves low bit error rate and better performance than ZF-SIC.

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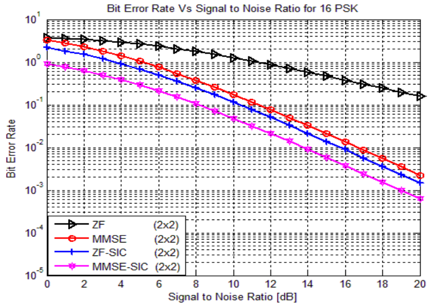

Fig 7: BER Vs SNR for 16-PSK [LD-NLD]

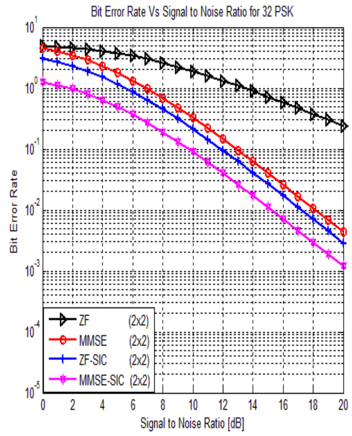

Fig 8: BER Vs SNR for 32 PSK [LD-NLD]

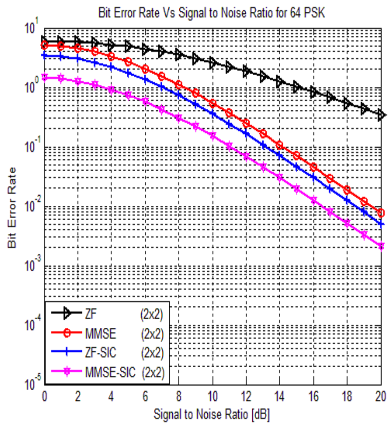

Fig 9: BER Vs SNR for 64 PSK [LD-NLD]

Figures (7 to 9) shows the comparison of linear and non-linear detector for 16-PSK, 32-PSK and 64-PSK. These figures depicts that MMSE-SIC achieves low bit error rate as compared to MMSE. Similarly ZF-SIC also achieves low bit error rate as compared to ZF. So, non-linear detectors provide better performance over linear detectors.

IV. CONCLUSION

Firstly, the BER analysis is done for various modulation schemes using linear detectors such as MMSE, ZF and non-linear detectors such as MMSE-SIC, ZF-SIC respectively. It has been concluded from results that MMSE achieves low BER and better performance over ZF equalizer in linear detectors (LD) and MMSE-SIC achieves low BER and better performance over ZF-SIC in non-linear detectors (NLD). Then, these linear and non-linear detectors are compared for various modulation schemes such as 16-PSK, 32-PSK, and 64-PSK over Rayleigh for two transmitters and receivers (2x2). It has been concluded from the results that Non-linear detectors with SIC (successive-interference cancellation) achieves low bit error rate over linear detectors. Successive interference cancellation (SIC) has capability that allows a receiver to decode packets that arrive simultaneously. SIC aids in the elimination of Inter Symbol Interference (ISI) thus improving overall performance to analyze the BER of the designed system.

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