ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 2, Issue 12, June 2013

Performance Analysis of Cross QAM over Beckmann Fading Channel

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Abstract—***In this paper the SEP (symbol error probability) performance of Cross QAM (quadrature amplitude modulation) is analyzed over Beckmann fading channel. The SEP expression is obtained by using MGF approach and it is composed of linear sum of integrals involving product of Q-function and PDF (probability density function) of Beckmann fading channel. Complex integrals are transformed into integrals involving moment generating function (MGF) and has finite limits that allows numerical integration with more accuracy. SEP performance of Cross QAM is analyzed for various special cases of Beckmann distribution including Rayleigh, Nakagami-q, Nakagami-n and single-sided Gaussian channel. Tight approximation for SEP of Cross QAM is considered in AWGN channel that only contains Q-functions. SEP Performance of Cross QAM (quadrature amplitude modulation), when compared with Rectangular QAM can obtain gain of at least 1.1 dB for SEP less than 0.3. The better performance is due to the unique constellation structure of Cross QAM that enables the modulation scheme to have relatively lesser average and peak energy and making it an energy efficient modulation scheme.***

Index Terms—**MGF (moment generating function), SEP (symbol error probability), QAM (quadrature amplitude modulation) & Gaussian distribution.**

I. INTRODUCTION

QAM is a preferred modulation scheme for digital communication over fading channels. In ASK (amplitude shift keying) the transmitted signal information is stored in the amplitude of the carrier that is very sensitive towards channel noise. Similarly in PSK (phase shift keying) for larger constellation size ‘M’ the phase variations of the carrier signal is very complex and to detect that a very expensive and complicated receiver is required. To provide tradeoff between performance and complexity a Hybrid of ASK/PSK is commonly used that is known as QAM. When transmission of even bits per symbol is the case then square QAM is preferred constellation but when odd bits per symbol is required then both Rectangular and Cross QAM can be considered. Cross QAM is usually the better choice as it is energy efficient scheme. Cross QAM has comparatively lesser average and peak energy than that of Rectangular QAM. In other words we can say that to achieve a particular SEP, Cross QAM require lesser SNR in dBs then that required by the Rectangular QAM.

Cross QAM have many applications such as adaptive modulation scheme where the transmission rate is improved according to the channel quality. When channel quality is better than the constellation size is increased by ‘k+1’ bits per symbol. If we are to consider only square QAM then the increment would be ‘k+2’ bits per symbol (we have to go from 16 to 64 to 256 QAM...). Use of square QAM makes the step size quite large for smaller constellation sizes. Cross QAM however provides the opportunity to reduce the intermediate step size from ‘k+2’ to ‘k+1’ bits per symbol (we need to go from 16 to 32 to 64 QAM…). Use of Cross QAM for single bit increment makes the change relatively smoother and enables the system to perform better over a determined data rate. Cross QAM with symbol length of 5 to 15 bits are commonly used in ADSL and VDSL, also Cross QAM has many application regarding blind equalization.

In [1],author derived the average SEP for Cross QAM in AWGN and fading channels (including Rayleigh, Nakagami-m, Nakagami-q & Nakagami-n) using MGF approach and the expression obtained contains finite integrals with integrands that are composed of exponential functions. In [2] we have mathematical models of various frequency flat and frequency selective channels that are useful for performance analysis of various modulation schemes. In [3]-[4] exact average SEP expressions for 32,128 and 512 Cross QAM had been derived for AWGN channel while [5] provides generalized SEP expression for Rayleigh fading channel. In [6] rectangular QAM had been analyzed for AWGN and Rayleigh channels. In [7] Exact expression for BER (bit error rate) of Cross QAM had been derived according to the constellation structure of Cross QAM (each bit is considered individually) and Smith’s approximation for BER of Cross QAM is also discussed and compared with the exact expression obtained. It can be observed that when SNR is high Smith’s approximation is very tight and for low SNR it has certain amount of deviation which increases with increase in constellation size.

Based on nature of propagation environment there are different types of channel models that describes the behavior of multipath fading envelope. When signal is transmitted through multi path environment it experiences scattering and reflection from rough surfaces, such channel model is presented in [8]. Beckmann distribution is presented to mathematically model behavior of such fading channels. However, [10]-[11] analyzes SEP performance of rectangular QAM over Nakagami-m fading channel without and with MRC diversity respectively.[12] derives generalized SEP expression for cross QAM with MRC diversity over different fading channels (including Rayleigh, Nakagami-m, Nakagami-q & Nakagami-n).

In this paper, we consider the SEP performance of Cross QAM in channel with Beckmann distribution. In next section we describe channel model by addressing the probability density function (PDF) and moment generating function (MGF). In section (III) we consider SEP expression for Cross

QAM in AWGN channel. In section (IV) SEP expression relating to Beckmann distribution is derived using MGF approach. Section (V) gives numerical results of SEP performance of Cross QAM in Beckmann channel for various special cases. Section (VI) describes simulation setup. Section (VII) provides conclusion.

II. CHANNEL MODEL

A. *Probability Density Function (PDF)*

The Beckmann distribution has four parameters as its envelope is composed of two independent Gaussian random variables with their own mean and variance. Fading channels like Rayleigh, Nakagami-q (Hoyt), Nakagami-n (Rice) and one sided Gaussian are the special cases of such distribution.

If we consider 'S' and 'T' as two independent Gaussian RVs with parameters such as (μs,σs) and (μt,σt) respectively, then the fading envelope would be,

$$ß = \sqrt{S^2 + T} \quad (1)$$

The Probability density function (PDF) of the Beckmann distribution as described in [2] is given as,

$$Pß(ß) = \frac{ß}{2\pi\sigma s\sigma t}\int_0^{2\pi}\exp\left[-\frac{(ß\cos\theta - \mu s)^2}{2\sigma s^2} - \frac{(ß\sin\theta - \mu}{2\sigma t^2}\right] d\theta,\ ß \quad \geq (2)$$

Other expressions relating to PDF of Beckmann distribution includes products of modified Bessel function but this form in (2) is very useful expression and we will use it to derive SEP expression for M-ary Cross QAM in Beckmann fading channel. **Fig.1** shows Probability density function (PDF) of fading envelope 'ß' of Beckmann distribution for various special cases.

B. *Moment Generating Function (MGF)*

Relating to the PDF in (2) the Moment Generating Function (MGF) in terms of instantaneous SNR as in [2] is given as,

$$M\gamma(S) = \frac{1}{\sqrt{\left(1-2\sigma s^2\frac{Es}{No}S\right)\left(1-2\sigma t^2\frac{Es}{No}S\right)}} \times$$

$$\exp\left[\frac{\mu s^2\frac{Es}{No}S}{1-2\sigma s^2\frac{Es}{No}S} + \frac{\mu t^2\frac{Es}{No}S}{1-2\sigma t^2\frac{Es}{No}S}\right] \quad (3)$$

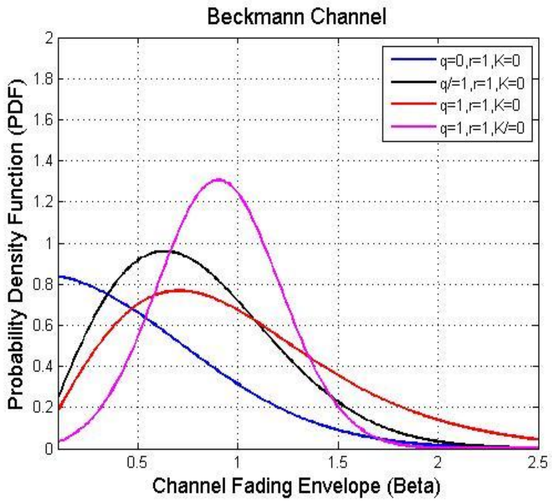

Fig.1 Probability density function (PDF) of Beckmann fading envelope.

We define three parameters in terms of the μs, μt, σs^2 and σt^2 as,

$$q^2 \triangleq \frac{\sigma s^2}{\sigma t^2},\ r^2 \triangleq \frac{\mu s^2}{\mu t^2},\ K = \frac{\mu s^2 + \mu t^2}{\sigma s^2 + \sigma t^2}$$

The MGF in terms of average received SNR '$\bar{\gamma}$' is given as,

$$M\gamma(S) = \frac{(1+q^2)(1+K)}{\sqrt{[(1+q^2)(1+K) - 2q^2\bar{\gamma}S][(1+q^2)(1+K) - 2\bar{\gamma}S]}} \times$$

$$\exp\left[\frac{K\left(\frac{r^2}{1+r^2}\right)(1+q^2)\bar{\gamma}S}{(1+q^2)(1+K) - 2q^2\bar{\gamma}S} + \frac{K\left(\frac{1}{1+r^2}\right)(1+q^2)\bar{\gamma}S}{(1+q^2)(1+K) - 2\bar{\gamma}S}\right] \quad (4)$$

(4) gives the MGF of Beckmann distribution in terms of the three parameters that are defined previously.

C. SEP OF CROSS QAM IN AWGN CHANNEL

Cross QAM as described previously is an energy efficient version of QAM that is the hybrid of amplitude and phase shift modulation schemes. Tight approximation of SEP of Cross QAM in AWGN channel is well described in [1, Eq (18)] that is given by,

$$Ps(\gamma) = \frac{1}{M}\left[\begin{array}{l} M\left(4M - 6\sqrt{\frac{M}{2}}\right)Q\left(\sqrt{2A\gamma}\right) + 4Q\left(2\sqrt{A} \right. \\ \quad - \left(4M - 12\sqrt{\frac{M}{2}} + 12\right)Q^2\left(\sqrt{2A\gamma}\right) \end{array}\right. \quad (5)$$

Where,

$$A = \frac{48}{31M - 32}$$

In (5) 'M' is constellation size while 'γ' is the instantaneous SNR.

D. SEP OF CROSS QAM IN BECKMANN CHANNEL

SEP expression for Cross QAM in Beckmann fading channel can be derived by averaging the performance over fading PDF. So the average SEP of Cross QAM over Beckmann fading channel is given by,

$$Pe = \int_0^\infty Ps(\gamma)Pß(\gamma) \quad (6)$$

Where instantaneous SNR 'γ' relates to fading envelope 'ß' and symbol energy 'Es' by the following relation,

$$\gamma = \frac{ß^2}{N}$$

Ps(γ) is the SEP expression of Cross QAM in AWGN channel given by (5), also $Pß($ is PDF of Beckmann distribution in terms of instantaneous SNR 'γ'. Evaluating (6) we have one dimensional integrals that has integrands that are product of Q function and PDF of Beckmann Fading.

We define,

$$L1(b) = \int_0^\infty Q(b\sqrt{\gamma})Pß(\gamma) \quad (7)$$

$$L2(b) = \int_0^\infty Q^2(b\sqrt{\gamma})Pß(\gamma) \quad (8)$$

Where 'b' is a constant that depends on the specific type modulation/detection process used. If we consider classical definition of Q-function that is defined as,

$$Q(z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{x^2}{2}\right) x \qquad (9)$$

Classical definition of Q-function makes it very difficult to evaluate expression in (7) and (8), because of the presence of in the lower limit. [10] presents a definite integral form of Q-function that is very useful and is given by (10). The required expression is obtained by first extending expression in (9) to two dimensions (x and y) and then changing coordinates from rectangular to polar form.

$$Q(z) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} exp\left(-\frac{z^2}{2 \sin\theta^2}\right) \qquad (10)$$

Using (10) in (7) resulting expression is given by,

$$L1\ (b) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left[\int_0^{\infty} exp\left(-\frac{b^2\gamma}{2 \sin\theta^2}\right) P_{ß}(\gamma)\, d\gamma\right] d \qquad (11)$$

In (11) the inner product integral is in form of Laplace transform of PDF of Beckmann distribution, where MGF is defined as,

$$M\gamma(S) = \int_0^{\infty} e^{S\gamma} P_{ß}(\gamma) \qquad (12)$$

Therefore (11) can be transformed in terms of MGF as,

$$L1\ (b) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} M_\gamma\left(-\frac{b^2}{2 \sin\theta^2}\right) \qquad (13)$$

Similarly (8) can be written as,

$$L2\ (b) = \frac{1}{\pi} \int_0^{\frac{\pi}{4}} M_\gamma\left(-\frac{b^2}{2 \sin\theta^2}\right) \qquad (14)$$

Integrals in (13) and (14) don't have closed form solution, so they have to be evaluated numerically.

Using (6), (13) and (14), we have average SEP expression for M-ary Cross QAM in Beckmann fading channel that is given as,

$$Pe = \frac{1}{M}\left[\begin{array}{l} M\left(4M - 6\sqrt{\frac{M}{2}}\right) L_1(\sqrt{2A}) + 4L_1(2\sqrt{A} \\ \quad - \left(4M - 12\sqrt{\frac{M}{2}} + 12\right) L_2(\sqrt{2A}) \end{array}\right. \qquad (15)$$

Where 'M' is the constellation size as described previously, 'L1'and'L2' are given by (13) and (14).

We have some special cases regarding the Beckmann distribution such as,

Rayleigh ($\mu s^2 = \mu t^2 = 0, \sigma s^2 \quad \sigma t^2$) (q=1, r=1, K=0)

Nakagami-q (Hoyt)($\mu s^2 = \mu t^2 = 0, \sigma s^2 \quad \sigma t^2$) (q 1, r=1, K=0)

Nakagami-n (Rice) ($\mu s^2 = \mu t^2 \quad 0, \sigma s^2 \quad \sigma t^2$) (q=1, r=1, K 0)

Single-Sided Gaussian ($\mu s^2 = \mu t^2 \quad 0, \sigma s^2 \quad 0, \sigma t^2 \quad 0$)(q=0,r=1, K=0)

III. NUMERICAL RESULTS

Fig.2 shows SEP performance of Cross 32-QAM versus average fading SNR of received signal, we can observe that with improvement in SNR the system performance improves. The performance of single-sided Gaussian is worst as compared to the other cases; parameters 'q' and 'K' are similar to those considered in Nakagami-q and Nakagami-n channels. We can observe that as the 'q' parameter varies from '0' to '1' while keeping other two parameters as constant performance improves and 'q=1' corresponds to Rayleigh channel. **Fig.3-4**shows SEP performance for constellation size M=128 and 512 respectively. Also it can be seen that SNR required to achieve particular SEP in 32 Cross QAM is lesser than that required for 128 and 512 Cross QAM due to increase in constellation size.

IV. SIMULATION SETUP

In this section we will discuss the simulation setup made for observing the performance of M-ary Cross QAM modulation scheme over Beckmann fading channel. Initially binary bit stream is generated that is mapped to 'M' possible symbols, each symbol comprising of 'k' bits. Symbols are then modulated using Cross QAM modulation scheme and the modulated signal is transmitted through the fading channel. The channel response is generated with the help of two Gaussian random variables whose mean and variances are taken according to the special cases described in section (IV).

The received signal is obtained by applying the channel effect along with the addition of white noise. The received signal is then first equalized by applying inverse channel response and output of the equalizer is demodulated. The received symbols are then compared with transmitted symbols to compute symbol error probability. This process is repeated for various values SNR (dB). It can be seen from **Fig. 1-3** that the simulation results very closely follow analytical ones, hence validating the accuracy of the average SEP expression derived in (15).

V. CONCLUSION

Previously average SEP expression for M-ary Cross QAM in AWGN and fading channels such as Rayleigh, Nakagami-m, Nakagami-n(Rice), Nakagami-q(Hoyt) for single and multiple channel receiver have been reported. In this paper, average SEP expression for M-ary Cross QAM in Beckmann fading channel have been derived using MGF (moment generating function) approach and analyzed for various special cases.

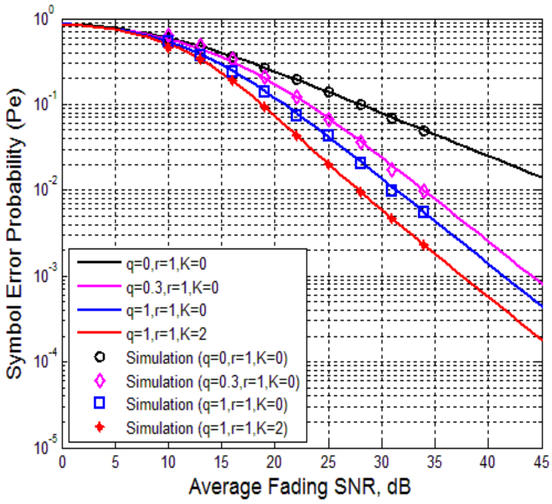

Fig.2 Average SEP performance of Cross 32-QAM in Beckmann channel for different fading parameters (q, r, K).

The future scope of this work is to improve system performance over Beckmann distribution by application of multiple diversity schemes including selection combining (SC), maximal ratio combining (MRC) and generalized selection combining (GSC).Diversity schemes combine multiple fading branches in particular fashion to improve effective SNR of overall system.

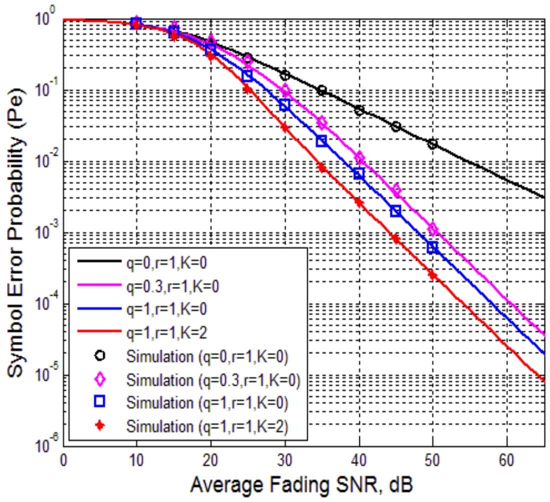

Fig.3 Average SEP performance of Cross 128-QAM in Beckmann channel for different fading parameters (q, r, K).

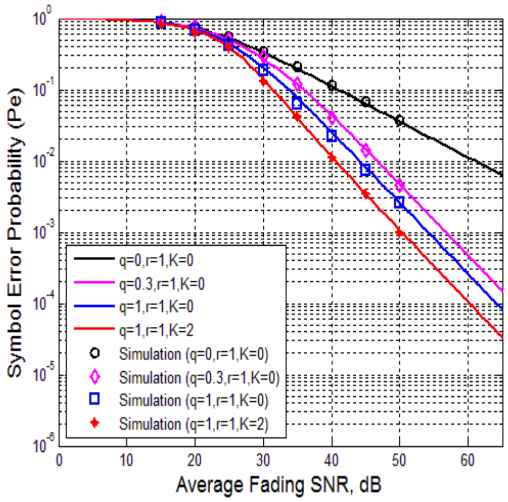

Fig.4 Average SEP performance of Cross 512-QAM in Beckmann channel for different fading parameters (q, r, K).

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