

Structural Behavior Analysis of Two-Ways (Waffle) Composite Slabs

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Abstract— For few decades and since the well-known composite slabs consisting of reinforced concrete poured on corrugated steel sheeting was achieved, the use of composite structures has grown widely. However and since this type of slabs acts as a one-way slab due to the steel sheeting ribs configurations, some problems have been created such as having two parallel heavily loaded beams in each panel while the other two perpendicular beams are almost unloaded; which is uneconomic. The steel sheeting stiffness is satisfactorily high in one direction and relatively low in the other which may require propping due to relatively high loaded perpendicular to strong direction steel beams during the construction stage.

For the above reason and others, the idea of using two-way (or waffle) composite slabs was brought up. Carrying out such research required firstly choosing the most appropriate finite element modeling technique in representing the two-way or waffle composite slab and comparing its behavior to that of one-way. Three modeling techniques named *real shape*, *equivalent* and *grillage* models were tested and represented by two and three-dimensional finite element techniques to detect the best of them. Since no previous experimental, numerical or empirical results of waffle composite slab analysis were yet available to evaluate the modeling results, it was necessary to study also the behavior of the well-known one-way composite slab using the pre-mentioned modeling techniques.

After the best finite element model was selected, an extensive parametric study in which many influencing parameters such as slab aspect ratio, slab boundary conditions, steel sheet depth, corrugation cell aspect ratio and its orientation were tested, was carried out in order to investigate the overall behavior of two-way composite waffle slab under different conditions. Many encouraging results and recommendations were obtained opening the door for this new configuration of composite slabs.

Index Terms— Composite, Slabs, Waffle.

I. INTRODUCTION

The two-way behavior is always preferable to that of one-way in all slab types due to its better structural behavior and economy in the cross sectional dimensions. Many researchers studied experimentally and numerically the one-way behavior of composite slabs reinforced with steel sheet corrugated in one direction [1]-[2]-[3]-[4]-[5]. However no researchers studied slabs with steel sheeting corrugated in the two main orthogonal directions. In this paper a new suggested composite slab structural system named “Waffle Composite Slab” is presented. In addition to the expected and pre-mentioned better structural behavior of waffle composite slab, the way by which the corrugated steel sheet is formed is expected to prevent the horizontal slippage

between the concrete and the steel sheet which is considered a very serious mode of failure in one-way composite slabs.

The main aim of this research is to analyze the structural behavior of the new suggested composite slab type under different conditions using the well known structural analysis program; SAP2000 [6]. Indeed, this analysis requires a very careful choice of the used modeling technique to accurately simulate the waffle composite slabs. Therefore firstly, a modeling techniques evaluation of waffle composite slabs was achieved numerically using the finite element technique and compared to the previously available experimental results of one-way composite slab types to achieve the best modeling among three models named *real shape*, *equivalent* and *grillage* models assuming full interaction between steel sheeting and overlaying concrete.

Many influencing parameters such as the waffle rectangularity, boundary conditions, depth and corrugation cell aspect ratio and corrugation type (trapezoidal and re-entrant) which were expected to affect the structural behavior of the two-ways waffle composite slabs were investigated under uniformly distributed loads.

II. MODELING EVALUATION

Three different finite element modelling techniques were used to model composite slabs in order to detect the best of them. Since there is no previous results to compare with, it was necessary to start with modeling the traditional one-way composite slab and compare the results with those of previous models [1]-[2]-[3].

In two ways waffle composite slabs, the corrugations may have equal or different dimensions in two directions; Fig. 1.

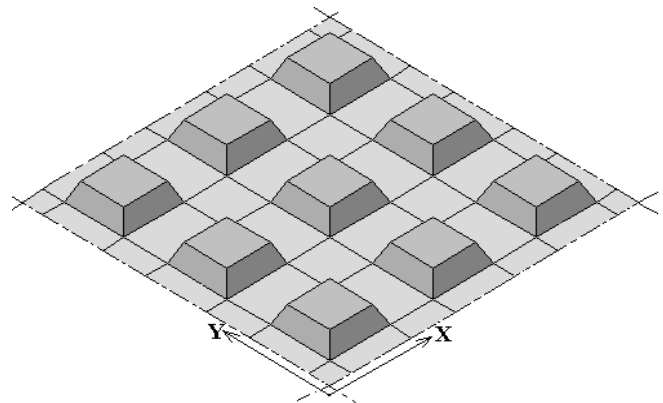


Fig. 1 : Suggested shape of waffle steel sheet [7].

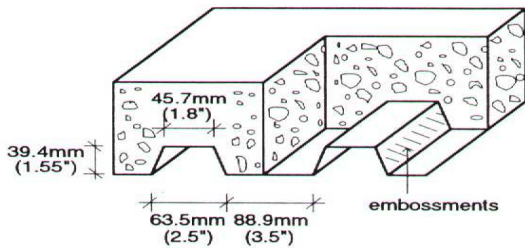


Fig. 2 : Cross sectional dimensions of composite slab [8].

Since there is no steel sheet having this shape of corrugation in the market yet, the standard corrugations dimensions of the one-way corrugated steel sheets available in market were considered. Then the dimensions of corrugations were changed gradually in one of the two directions to study their effect on the slab behaviour; as will be detailed later.

A. Model Dimensions And Applied Loading Configurations

For the sake of comparison, the dimensions and the loading configurations of previously investigated models [8] were maintained as shown in Fig.'s 2 and 3. The slab had dimensions of (3.66 × 4.88 m) (L2 × L1). For one-way slab, The steel sheeting was oriented in direction of the short span. The steel sheet used had rolled embossments on the webs to provide longitudinal shear resistance at the concrete–sheet interface. Four symmetrical concentrated loads were applied to ultimate capacity. This was depending on the assumption that a slab element with concentrated loads at the quarter points behaves in a similar manner to a slab element with uniform loading. The ultimate load on each loading point was 6 tons. In waffle composite slab, the sheet dimensions were the same in the two directions of the slab. Depending on full bond between steel sheet and concrete and linear elastic un-cracked behavior of concrete assumptions, the maximum load was obtained once the stress in any steel element reaches the yield strength or the compressive strain in concrete reaches the maximum compressive strain or if the maximum deflection of the slab reaches the allowable deflection value [9].

B. Three-Dimensional Real Shape Model

Both one-way and waffle composite slabs were modelled using the three-dimensional finite element real technique.

The concrete body was divided into four layers; as shown in Fig. 4. Layers 1 & 2 were in compression zone having the concrete compressive stress.

C. Two-Dimensional Equivalent Model

References [4] and [5] introduced this model which simulates the beam-plate bending model as shown in Fig. 5 .

For one-way composite slab, the concrete layer was modelled as a series of two adjacent plate bending elements which had the same properties and different thickness to simulate the depth variation of concrete layer due to the steel sheet corrugation geometry. The steel-deck was modelled as beam elements in direction of the ribs “strong direction”.

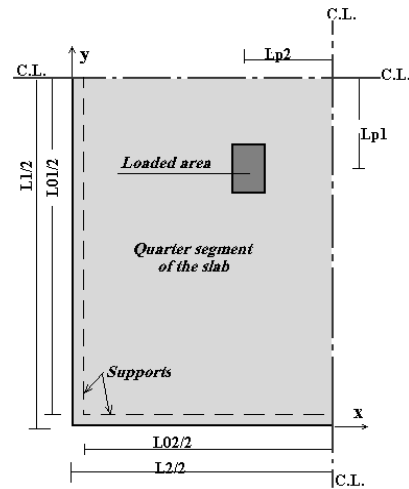


Fig. 3 : Slab dimensions, loading positions and supported edges [8].

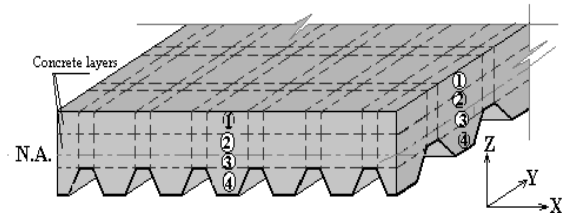


Fig. 4 : 3-D Real shape model of waffle composite slab.

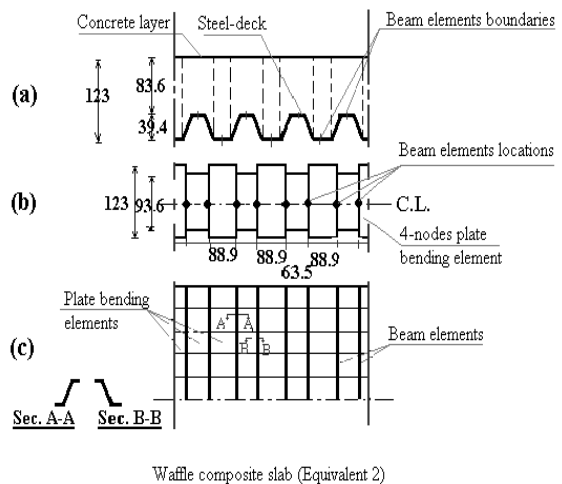
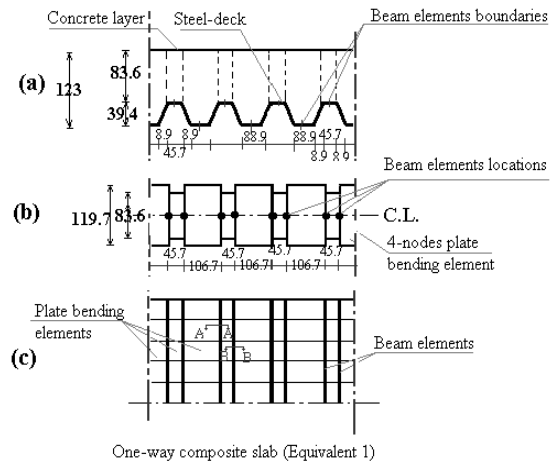


Fig. 5 : Two-dimensional equivalent model [4]-[5].

For the waffle composite slab, the steel sheet was represented by two-node beam elements in the two orthogonal directions. The concrete was represented by a series of two adjacent plate bending elements. For one-way composite slab, it had two different thicknesses of 83.6 mm and 119.7 mm, with widths of 45.7 mm and 106.7 mm respectively. In waffle composite slab, the elements representation was slightly modified to maintain the previously used procedure; Fig. 5.

D. Two-Dimensional Grillage Model

Waffle composite slab was modelled using grillage model which had grillage beams in both directions. They were chosen to coincide with the rib centre lines, to simplify data preparation. Each cycle of the waffle composite slab was idealized as an equivalent frame element having the same transformed area and moment of inertia of the real cross section, (in concrete dimensions). The remaining part of concrete over the supporting steel I-beam, and the steel I-beam itself was represented as beams having the same area and moment of inertia of the real section, (in concrete dimensions); Fig. 6.

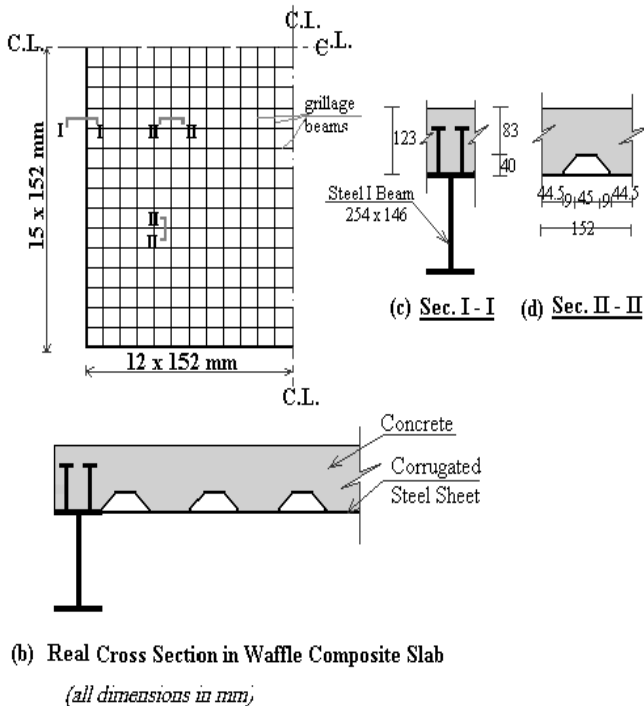
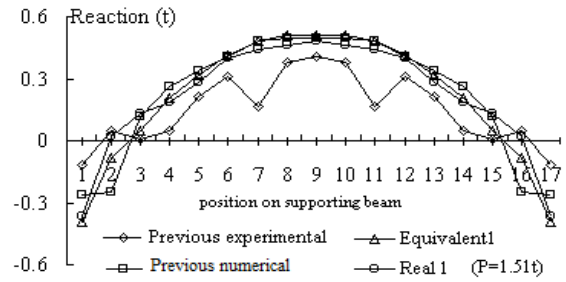


Fig. 6 : Two-dimensions grillage model .

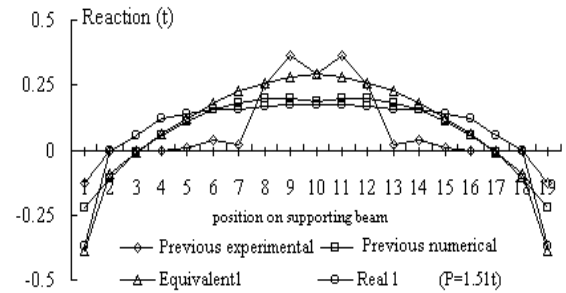
E. Results And Discussion

Comparison between the three models is based mainly on the distribution of forces at the two orthogonal longitudinal supports.

Fig. 7 shows the distribution of forces in the long and the short directions for the previous experimental tested [1], numerical [3], suggested equivalent [4] – [5] and Real models.



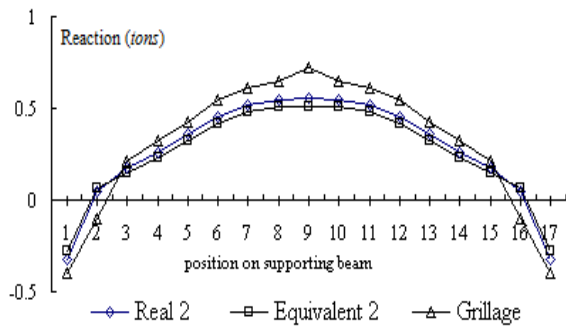
(a) Support transverse to the sheet corrugations.



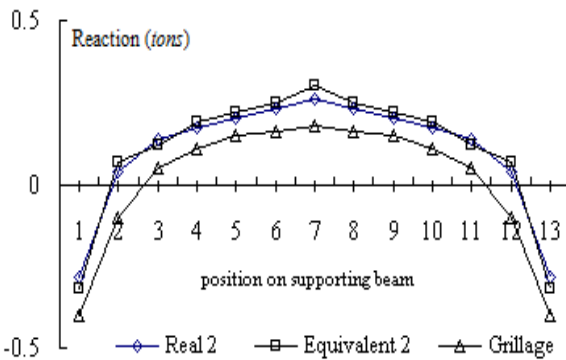
(b) Support parallel to the sheet corrugations.

Fig. 7 : Distribution of reaction forces .(one-way slab)

Fig. 8 shows the distribution of the reaction forces in the Real, Equivalent, and grillage models.



(a) Long direction



(b) Short direction

Fig. 8 : Distribution of reaction forces (waffle slab).

Table 1 shows the total transferred load in the two directions of the compared slabs. Comparison of results indicated that the three dimensional finite element real shape

model gave the best and the nearest results to those obtained experimentally.

Table 1 : Percentage of load transferred in each direction.
[at applied load (P) = 2.15 tons = ultimate load (P_u)/3]

Investigated Model	Main (%)	Secondary (%)
Previous experimental	71.6	28.4
Previous numerical	73.4	26.6
Real 1	73	27
Equivalent1	69.6	30.4
Real 2	63	37
Equivalent 2	61.1	38.9
Grillage	70.1	29.9

III. WAFFLE COMPOSITE SLAB GAINED SUPERIORITY

The most critical mode of failure in one-way composite slab is the slippage between the steel sheet and the concrete. This slippage is expected to be prevented in two ways waffle slabs due to steel sheet ribs configurations [10]. One hundred and thirty Real models were tested for the sake of comparing the traditional one-way composite slab and waffle composite slab in both construction and composite stages. Each was examined through different slab aspect ratios (r) of (1, 1.2, 1.4, 1.6, 1.8 & 2). Slabs were examined under an increasing uniformly distributed load until failure. The considered failure limits were the concrete compressive stress or the steel sheet tensile stress or the allowable slab deflection.

A. Basic Assumptions Satisfying the Waffle Composite Slab Analysis

- 1- Full bond (*interaction*) between steel and concrete.
- 2- Linear cracked elastic behavior of concrete.
- 3- The effect of supplementary (*secondary*) reinforcements against shrinkage effect and temperature is neglected .

B. Construction Stage

Deflection must be controlled in composite slabs construction phase. In one-way slab, loads were transferred mainly in direction of ribs. For example in a one-way composite slab of aspect ratio of 1.4, about 79%; in average; of the total load went through the short direction and the remaining 21% went through the long direction. However, in a waffle composite slab of the same aspect ratio, about 66% of the total load went through the short direction and the remaining 34% went through the long direction. Of course, the better load distribution of the waffle composite slab enhances its structural behaviour; as shown in Fig. 9 while in one way composite slab, the stresses are so high in one direction and much lower in the other direction, however in waffle composite slab, the stresses in the two directions were closer to each other.

C. Composite Stage

Seventy-two waffle and one-way models with different aspect ratios were analysed. All slabs were assumed to be simply supported at four edges. The dead weight of the slabs was taken into account.

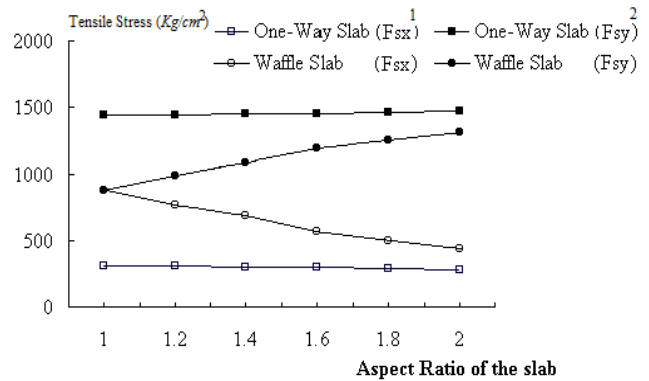


Fig. 9 : Compared steel tensile stress (L.L.³ = 450 Kg/m²) .

D. Deflection

Small slab aspect ratios gave better behavior of waffle composite slab. Fig. 10 declares that for slab aspect ratios of 1.0, 1.2, 1.4, 1.6, 1.8 & 2.0 the maximum deflection of waffle composite slab was lower than that of one-way composite slab by 19%, 18%, 16%, 14%, 9% and 6% respectively. As the slab aspect ratio increases, the behavior of waffle composite slab become closer to that of the one-way slab.

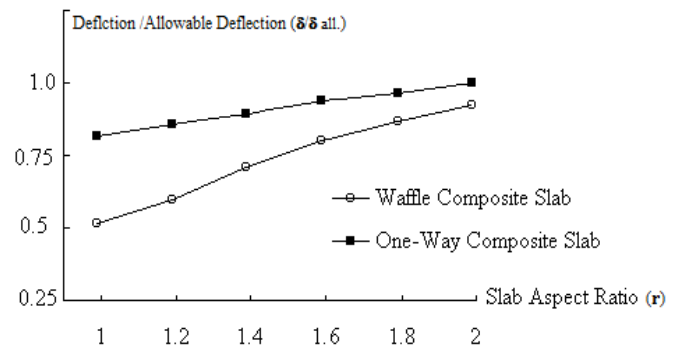


Fig. 10 : Maximum deflection (at L.L.³ = 450 Kg/m²) .

E. Load Transferred in Each Direction

In general, the load is distributed in a better manner in two-ways slabs than one-way slabs. Fig. 11 show that the load transferred in the short direction in both types of slabs increased as aspect ratio increases. For slab aspect ratio (r) of 1.0, 1.2, 1.4, 1.6, 1.8 & 2.0, the waffle slab load transferred through its short direction was reduced by 18%, 13%, 11%, 8.5%, 6.5% and 5% respectively than that recorded for one-way slab. As the slab aspect ratio increases, the behavior of both types of slabs becomes closer.

F. Maximum Loads

Fig. 12 shows that waffle composite slab could carry higher maximum load more than one-way composite slab by about 12 to 20 % depending on the slab aspect ratio. This relatively higher slab loading capacity leads to reducing the total slab thickness which; of course; means economy.

1 : F_{sx} is Steel sheet tensile stress in long direction.

2 : F_{sy} is Steel sheet tensile stress in short direction.

3 : L.L. = Live Load

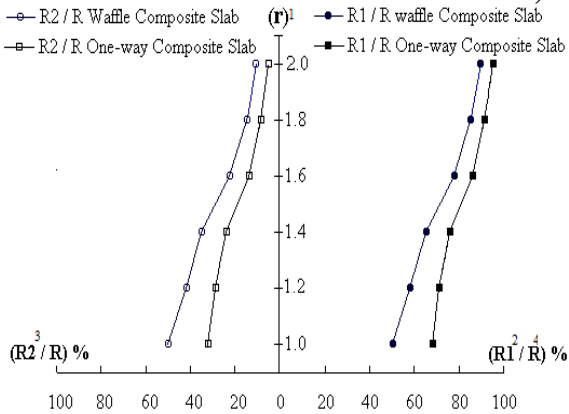


Fig. 11 : Percentage of load distributed on each side (at L.L. = 450 Kg/m²).

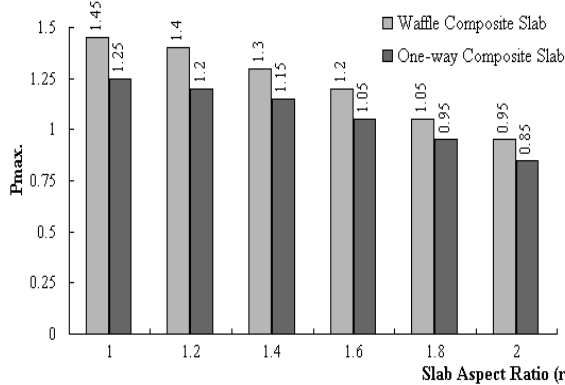


Fig. 12: Slab aspect ratio influence on slab loading capacity.

G. Compressive Stress in Concrete

The maximum compressive stresses were recorded and plotted for both waffle and one-way composite slabs due to live load application; as shown in Fig. 13 for r = 1.2 as an example.

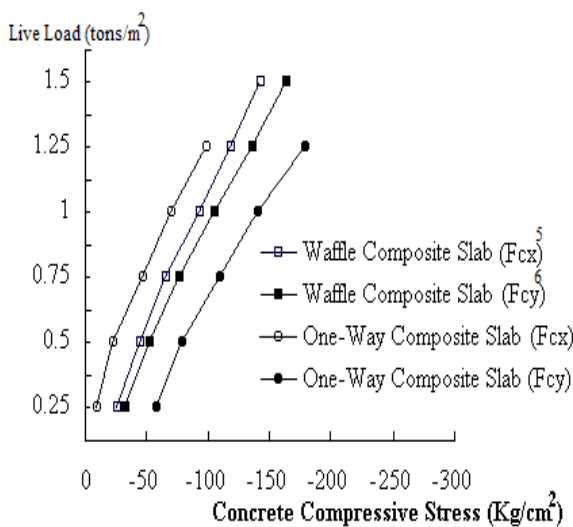


Fig. 13 : Concrete compressive stress for aspect ratio of 1.2.

- 1 : r = Composite slab aspect ratio (rectangularity) = L₁/L₂ .
- 2 : R₁ = Force at support in long direction.
- 3 : R₂ = Force at support in short direction.
- 4 : R = R₁ + R₂.
- 5 : F_{cx} : Concrete stress in long direction.
- 6 : F_{cy} : Concrete stress in short direction.

It can be noticed that failure happened due to either yielding in steel sheet or excessive slab deflection in both types of slabs. Also, Fig. 13 shows that the concrete compressive stress in the short direction of the waffle composite slab was about 19% lower than that of its one-way analogue.

H. Tensile Stress in Corrugated Steel Sheet

The tensile steel stresses in of waffle composite slab were found to be lower than the stresses in one-way composite slab, under the same load levels.

Fig. 14 shows the stresses in steel sheet in waffle slab compared to one-way slab for slab aspect ratio of 1.2 as an example. The tensile stress in the short direction in waffle slab was 17% lower than that of the one-way corrugated one.

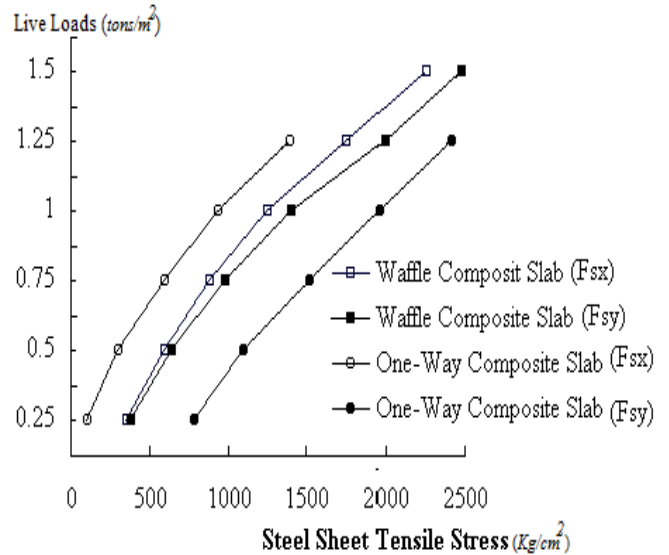


Fig. 14 : Tensile stress in corrugated steel sheet in waffle and one-way composite slabs .

I. Effect of Generating and Increasing the Number of Ribs in the Long Direction

Five-three dimensional finite element real shape slab models with dimensions (3.66 × 4.88 m) were used to study the effect of variation of composite slab behavior due to changing it gradually from one-way to waffle composite slab. Table 2 shows the percentage of load transferred in each slab direction due to this variation.

Table 2 : Decreasing number of ribs effect in long direction. [at applied L.L. (P) ≈ 25% of maximum L.L. (P_{max})]

Slab	Load transferred [%]		No of ribs in the long direction
	short	long	
Slab 2a	71.9	28.1	0.0 (one-way slab)
Slab 2b	68.3	31.7	4
Slab 2c	66.9	33.1	8
Slab 2d	63.5	36.5	16
Slab 2e	62.6	37.4	24 (two-way slab)

IV. PARAMETRIC STUDY

A. Testing Program Characteristics

One hundred waffle composite slabs were tested in order to investigate the effect of some significant parameters on their behavior. These factors were slab aspect ratio, slab boundary conditions, steel sheet profiles, varying the steel sheet corrugation cells aspect ratio, and increasing the steel sheet depth with a constant total depth of the slab. The previously mentioned three-dimensional finite element real shape model was considered. The used dimensions of the slabs varied from (7.08 × 7.08 m) to (7.08 × 3.48 m). A typical considered corrugated steel sheet deck section known as CF 46 [11] was used. Except in the comparison between the different steel sheet shapes, a trapezoidal steel sheet section called PEVA 45 [11] and a re-entrant steel sheet section were used. The details of the three different steel sheet shapes used in the study are described in Table 3 and Fig. 15. The overall slab thickness was 180 mm in all models according to the ECCS³ for one-way composite slab. The own weight of the tested slabs was taken into account. The live load was increased gradually till one of the failure limits is reached.

Table 3 : Used corrugated steel-deck sections properties.

Steel-Deck Section	Thickness (mm)		Weight (kN/m ²)	Moment of Inertia (cm ⁴ /m)	Net Section Area (cm ² /m)
	Nominal	Design			
CF 46 ¹	1.2	1.16	0.112	53.0	14.59
PEVA 45 ¹	0.94	0.9	0.102	51.5	13.23
Re-entrant Profile ²	0.9	0.86	0.127	67.0	16.55

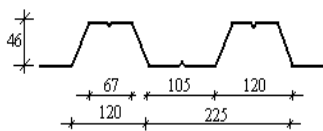
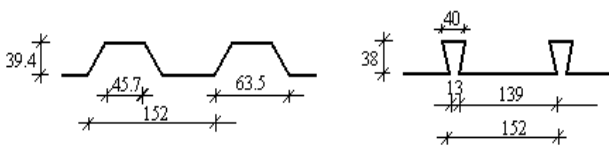


Fig. 15 : Cross section details of steel sheet cycles . All dimensions in (mms).

B. Analysis and Discussion

B-1 Slab aspect ratio

Thirty slab models were analyzed to declare the effect of slab rectangularity. The same corrugation dimensions were typical in both slab directions (cells aspect ratio of 1.0). The maximum load is shown in table 4.

- 1 : [11] .
- 2 : [12] .

It was noticed that slab deflection decreased as slab aspect ratio increased while the percentage of load transferred in the long direction decreased, and the corresponding load value transferred in the short direction increased; Fig. 16.

B-2 Slab Boundary Conditions

Slab boundary conditions was found to be a very important factor, the fixed edges are more recommended as the loads gets higher. The effect of slab boundary conditions on the results is shown in Fig. 17.

Table 4 : Slabs¹ rectangularity influence on maximum L.L. capacity (P_{max}) and failure modes.

Slabs	Aspect Ratio (r)	Ly		P _{max} (t/m ²)	Mode of failure
		Lx (m)	Ly (m)		
A1 - A5	1.0	7.08	7.08	1.0	Deflection (Serviceability)
A6 - A10	1.2	7.08	5.73	1.2	Deflection (Serviceability)
A11 - A15	1.4	7.08	5.06	1.35	Yielding in Steel Sheet
A16 - A20	1.6	7.08	4.38	1.45	Yielding in Steel Sheet
A21 - A25	1.8	7.08	3.93	1.5	Yielding in Steel Sheet
A26 - A30	2.0	7.08	3.48	1.54	Yielding in Steel Sheet

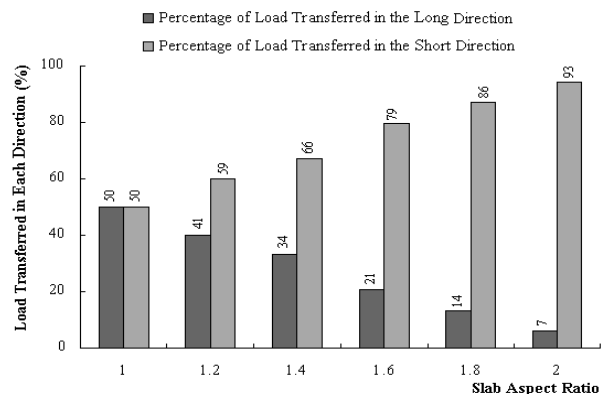


Fig. 16 : Effect of slab aspect ratio on percentage of load transferred in each direction.

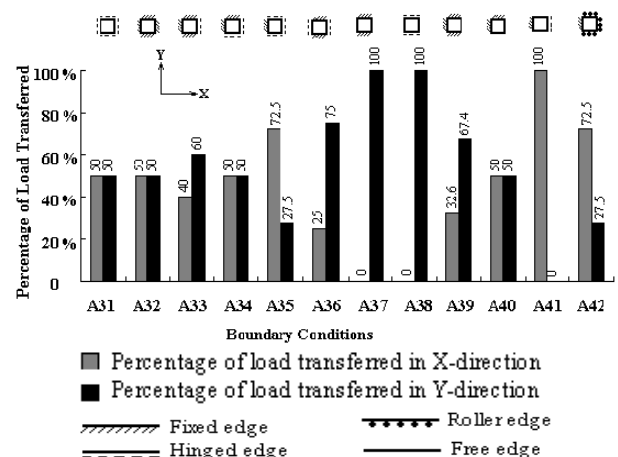


Fig. 17 : Percentage of load transferred in each direction .
1 : All slabs are simply supported at four edges using CF 46 steel sheets [11].

B-3 Steel sheet profile

A comparison was held between a waffle trapezoidal and a re-entrant steel sheet shape with almost the same corrugation cycle dimensions; Fig.'s 1 and 18. Almost the same deflection, stresses and percentage of load transferred in each direction were recorded in the two types.

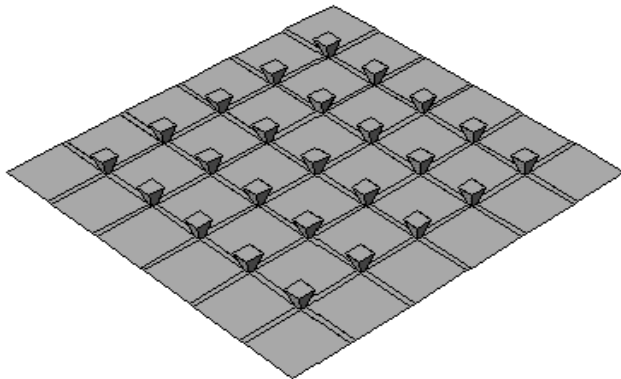


Fig. 18 : Suggested shape for re-entrant waffle sheet [7].

B-4 Steel sheet corrugation cells aspect ratio

The dimensions of the traditional steel sheet cycles of the one-way composite slab were used in the two directions of the tested slabs; Fig. 19. The cells were kept constant in one direction, and varied in the other. Eighteen models divided into two were tested. *the first group* had square slabs (7 × 7 m), where the corrugation cells were oriented parallel to Y-direction. *The second group* had rectangular slabs of aspect ratio $r = 1.4$, (7.08 × 5.05m).

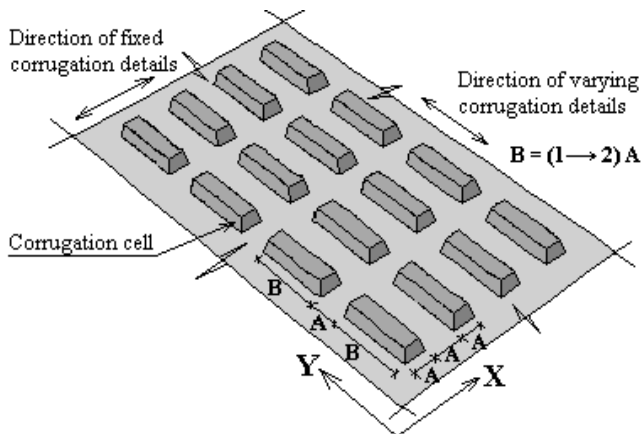


Fig. 19 : Steel sheet configurations [7].

The rectangular slab had two cases. *Case A* where the cell long direction was parallel to slab long direction and *Case B* where the long direction of the cell was perpendicular to the long direction; as shown in Fig. 20.

The increase of the corrugation cell aspect ratio decreased the percentage of load transferred in the long direction in both square and rectangular slab of (Case B), while it was increased in rectangular slabs of (Case A) as shown in Fig. 21. Deflections increased in both square and rectangular slabs of (Case A) and decreased in slabs of (Case B). Both steel and concrete stresses decreased in the long direction in square and (Case B) slabs, and increased in the long

direction in (Case A) slabs with the increase of the cell aspect ratio.

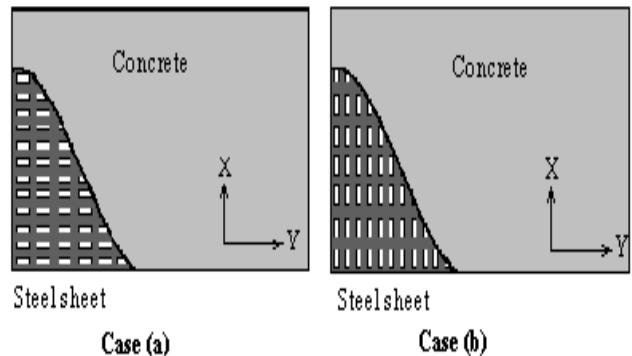


Fig. 20 : Orientation of the corrugation cells.

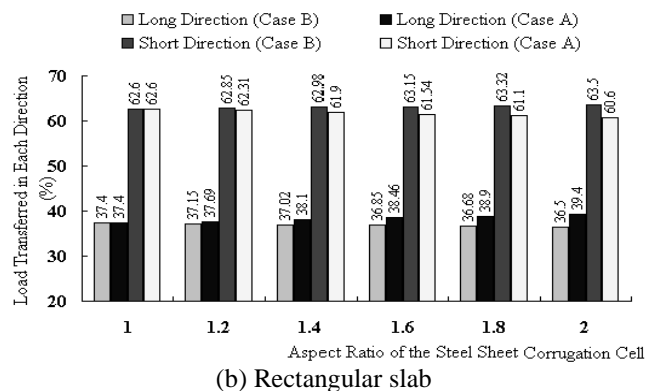
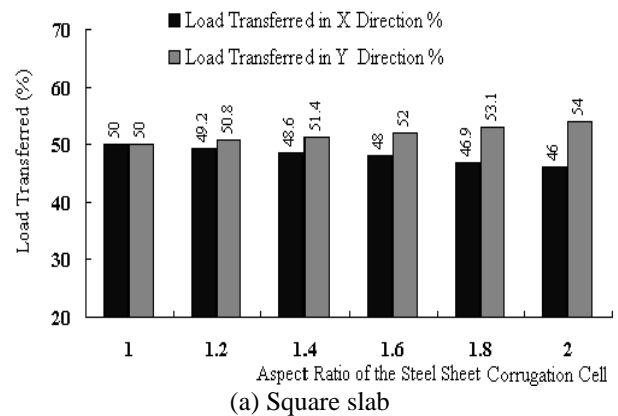


Fig. 21: Load transferred in each direction.

V. RESEARCH HIGHLIGHTS

- 1) The 3-D finite element real shape model was found to be the best modeling technique to represent both waffle and one-way composite slabs.
- 2) In construction stage, waffle composite slab proved its superiority on its one-way analogue by reducing steel sheet stress and the percentage of load transferred in the short direction.
- 3) In composite stage waffle composite slab sustained loads higher than its one-way analogue, and permitted lower deflection value. Steel and concrete stresses in waffle composite slab were lower and better load distribution along the two orthogonal directions was gained. In addition, the way by which waffle steel sheet is formed, prevents the slippage between the sheet and

the concrete. In architectural point of view, the good appearance of the waffle shape saves the cost of constructing artificial ceiling.

- 4) Increasing slab aspect ratio decreased slab deflection and increased steel sheet and concrete stresses in the short direction. Symmetric-fixed boundaries lead to the best behavior of waffle composite slab. The re-entrant waffle corrugated sheet permitted lower deflection, higher steel sheet and concrete stresses than the trapezoidal shape, and it allowed the same percentage of load distribution in the two directions. Square corrugation cell of steel sheet was found the best in both square and rectangular slab. A better behavior of waffle composite slab was obtained when the steel sheet depth was equal to one third of the total slab depth.

VI. CONCLUSION

In this paper we have discussed various factors influence on the structural behavior of a newly developed composite slab type (*waffle composite slab*) reinforced with two-ways corrugated steel sheets. The motivating force to achieve this research is trying to avoid some undesired structural features and executive conditions of the well-known traditional composite slabs reinforced with one-way corrugated steel sheets. In general, the behavior of waffle composite slabs is superior and it is strongly recommended to manufacture new steel sheet configurations to allow using this new developed type of composite slabs.

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