Integrated Water Management for a Multipurpose Project

Archana K. Chowdhary, Keerti K. Chowdhary, R. K. Shrivastava

Abstract—Surface water availability shows temporal fluctuations in terms of floods and droughts and ground water availability shows mainly spatial variability in terms of quality and quantity due to the hydrologic setting, boundary conditions and aquifer properties. To face this challenge a new holistic system approach, relying on integrated use of surface and ground water resource is needed to overcome the current fragmental management of water. A regional integrated use model is developed for a multipurpose Mahanadi Reservoir Project (MRP) with the aim of exploring the capacity of Genetic Algorithm to derive optimal operating strategies of the system. The study focused on four reservoirs within Mahanadi and Pairi basin located in Raipur (Chhattisgarh). The integration of the reservoir operation for canal release, ground water pumping & crop water allocation during different periods of crop season is achieved through the objective of maximizing the sum of relative yields of crops over a year considering three sets of constraints mass balance at the reservoir, Soil moisture balance for individual crops and governing equations for ground water flow. An integrated use policy is termed stable when the policy results in a negligible change in the ground water storage over a normal year.

Keywords—Genetic Algorithm, Reservoir Operations, Optimization, Hydrologic Setting, Aquifer Properties, Linear Programming, Stable Operating Policy.

I. INTRODUCTION

A critical problem that mankind has to face and cope with is how to manage the intensifying competition for water among the expanding urban centers, the agricultural sector and in stream water uses dictated by environmental concerns. Confronted with the prospect of heightened competition for available water and the increased difficulties in constructing new large-scale water plants, water planners must depend more and more on better management of existing projects through basin-wide strategies that include integrated utilization of surface and ground water. Todd (1959) defined this process as integrated use. Lettenmaier & Burges (1982) distinguished integrated use, which deals with short-term use, from the long-term discharging and recharging process known as cycle storage.

Until the late fifties, planning for management and development of surface and groundwater were dealt with separately, as if they were unrelated systems. Although the adverse effects have long been evident, it is only in recent years that integrated use is being considered as an important water management practice. In general terms, integrated use implies the planned and coordinated management of surface and groundwater, so as to maximize the efficient use of total water resources. Because of the interrelationship existing between surface and subsurface water, it is possible to store during critical periods the surplus of one to tide over the deficit of the other. Thus groundwater may be used to supplement surface water supply, to cope with peak demands for municipal and irrigation purposes, or to meet deficits in years of low rainfall. On the other hand, surplus surface water may be used in overdraft areas to increase the groundwater storage by artificial recharge. Moreover surface water, groundwater or both, depending on the surplus available, can be moved from water-plentiful to water-deficit areas through canals and other distribution systems. On the whole the integrated system, correctly managed, will yield more water than separately managed surface and groundwater systems.

In other words, integrated use of surface water and ground water offers a great potential for enhanced and assured water supplies at minimum cost. There are several ways of making combined use of integrated use of surface water and ground water. It can take the room of full utilization of surface water supplies supplemented by ground water or the direct use of ground water during period of low canal supplies or canal closures. Such combined use as is now practiced was not planned earlier but came into being out of necessity. Based on the technique used, integrated use models developed earlier may be classified as simulations models, dynamics programming models, linear programming model, hierarchical optimization model, non linear programming models and others.

Simulation approach provided a framework for conceptualizing, analyzing and evaluating stream-aquifer systems. Since the governing partial differential equations for complex heterogeneous ground water and stream-aquifer systems are not amenable to closed form analytical solution, various numerical models using finite difference or finite element methods have been used for solution (Latif. M James, 1991; Chaves – Morales et al., 1992).

Dynamic programming (DP) has been used because of its advantages in modeling sequential decision making processes, and applicability to nonlinear systems, ability to incorporate stochasticity of hydrologic processes and obtain global optimality even for complex policies (Onta et al., 1991). However the “curse of dimensionality” seems to be the major reason for limited use of DP in integrated use studies.

Linear programming (LP) has been the most widely used optimization technique in integrated use modeling. Nieswand and Granstrom (1971) developed a set of chance constrained linear programming models for the integrated use of surface water and ground water for the Mullica River basin in New Jersey, Vedula and Majumdar (2005) developed a deterministic linear programming model for deriving a stable operating policy for integrated use of surface water and ground water for the reservoir command.
area in Karnataka. Belaine et al. (1999) presented a simulation/optimization model that integrates linear reservoir decision rules, detailed simulations of stream/aquifer flows, integrated use of surface and ground water, and delivery via branching canals to water users. Reservoir storage and branching canal flows and interflows are represented using embedded continuity equations. Results of application indicate that more the detail used to represent the physical system, the better is the integrated management. Azaiez and Hariga (2001) developed a model for a multi-reservoir system, where the inflow to the main reservoir and the demand for irrigation water at local areas are stochastic.

Barlow et al. (2003) developed integrated-management models that couple a numerical simulation with linear optimization to evaluate trade-offs between groundwater withdrawals and stream flow depletions for alluvial-valley, stream-aquifer systems representative of those of the northeastern United States. Rao et al. (2004) developed a regional integrated use model for real deltaic aquifer system, irrigated from a diversion system, with some reference to hydrogeoclimatic conditions prevalent in the east coastal deltas of India. The combined simulation optimization model proposed in this study is solved as a nonlinear, non convex combinatorial problem using a simulated annealing algorithm and an existing sharp interface model. Marino (2001) discussed simulation and optimization models and decision- support tools that have proven to be valuable in the planning and management of regional water supplies.

Despite the fact that most integrated use management problems are nonlinear in nature, application of nonlinear programming (NLP) has been rather limited. This may be because of the complexity and the slow rate of convergence of the NLP algorithms, difficulty in considering stochasticity and possibility of getting a local instead of global optimal solution (Yeh, 1992). Genetic algorithm (GA) is a search procedure which combines an artificial survival of the fittest with genetic operators abstracted from nature, used successfully in optimization of water systems (Wang, 1991; Ritzel et at, 1994; Mckinney and Lin 1994; Aly and Peralta, 1999; Patrick Reed and David E Goldberg 2000). Artificial Neural Network (ANN) specially the feed forward network has been successfully used for water resources variables modeling and prediction (Coulibaly et al 1999; Maceir and Dandy, 2000). ANN models have been developed for water table depth modeling (Paulin Coulibaly, 2001). Choice of method depends upon type of system, the availability of data and on the specified objectives and constraints (Yeh 1985).

In the early stages dynamic programming was applied for integrated use of surface and ground water. Unfortunately the use of dynamic programming restricts the specification of the ground water and surface water systems to low dimensional representations of the system. The limitation of simulation approach is that it only allows comparison of specific strategies; it does not allow direct maximization or minimization of a particular objective.

Non linear programming algorithms arrive at only locally optimal solution to most ground water management problems. Genetic Algorithm provides an alternative to gradient based techniques for solving complex and highly non linear optimization problem.

A study of existing shows that there is no single comprehensive model developed for irrigation of multiple crops in which reservoir operation and irrigation allocation decisions at field level are integrated, and in which a integrated use policy for the irrigated area, apportioning the surface and ground water components, taking into account the distributed parameter characteristics of the aquifer and the soil moisture dynamics of the crop level is embedded. The present study is an attempt in this direction and in the identification of a stable integrated use policy for canal command areas of MRP Complex in Chhattisgarh State.

II. OBJECTIVES OF THE STUDY
The specific objectives of the present study are:
1) to develop a mathematical programming model to determine a stable integrated use policy for irrigation in a reservoir–aquifer system for multiple crops in a reservoirs canal command area, for a normal year,
2) to extract the optimal temporal crop water allocation pattern from the identified stable policy for application to a given year, using simulation, and
3) to apply the model, derive a stable integrated policy for an existing canal command area, and apply the policy over a period of time to examine the aquifer response (storage) for stability.

A. Model Formulation

1. Linear Programming Model
The system is characterized by three main components: the reservoir, the irrigated area and the underlying aquifer with the associated dynamic relationships determining the interactions among them. The integration of decision making in reservoir operation, ground water pumping and crop water allocations during the different periods of crop season is achieved through optimization with the objective of maximizing the sum of relative yields of crops over a year, and considering three sets of constraints: mass balance at the reservoirs, soil moisture continuity for individual crops, and governing equations for ground water flow. These sets of constraints are linked together appropriately by additional constraints. The reservoir release and the ground water are optimally allocated to achieve the maximum annual relative yield of crops.
A two-dimensional, isotropic, homogeneous unconfined aquifer is considered. The aquifer response is modeled through the use of a finite difference ground water model. To specify the initial conditions at each node in the study area, ground water contours are drawn from the data of observation wells in and around the study area, and the initial conditions are specified accordingly. The recharge to the aquifer from an element consists of the recharge due to rainfall, canal seepage and the deep percolation from the root zone of the crop grown in the element. The water logging condition in the study area is avoided by imposing an upper bound on the ground water level at each node.

An integrated use policy is defined by specifying the ratio of the annual allocation of surface water to that of ground water pumping at the crop level for entire irrigated area. The integrated use model is run for different predetermined ratios of annual surface and ground water applications at the crop level (i.e. for different integrated use policies). Ground water balance components over the entire study area are computed for each of these runs, and an examination of annual ground water balance is made in each case. The policy for which the annual change in ground water storage is negligible for a normal year is considered as the “stable policy”. The derived stable integrated use policy aids in planning the total crop water allocation for irrigation in the study area.

### 2. Ground Water Model

The purpose of this model is to incorporate the dynamic response of the aquifer to ground water extraction for irrigation and ground water recharge due to deep percolation from irrigated area, rainfall and canal seepage. The ground water response is modeled using the finite difference method.

The governing equation for a two-dimensional, unsteady flow in an isotropic, homogeneous, unconfined aquifer is given by Yeh (1992)

\[
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \left( T \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( T \frac{\partial h}{\partial y} \right) = S_y \frac{\partial h}{\partial t} + Q_P - Q_R
\]

where \( h \) is the ground water level in m; \( T \) the transmissivity in m²/day; \( S_y \) the specific yield; \( Q_P \) the pumping rate per unit area in m³/(day m²); \( Q_R \) the recharge rate per unit area in m³/(day m²); \( x \) and \( y \) are the Cartesian coordinates in plan, and \( t \) is the time in days.

### B. Crop yield optimization

Integrated use modeling for irrigation requires interfacing of reservoir operation, soil moisture accounting and ground water balance. The relative yield of a crop as a function of deficits suffered in the individual growth stages is given by

\[
y = \frac{y_{\text{max}}}{1 - \sum_{g=1}^{NGS} ky_g \left( 1 - \frac{AET}{PET} \right)}
\]

Where \( y \) is the actual yield of the crop; \( y_{\text{max}} \) the maximum yield of the crop; \( g \) the growth stage index; \( NGS \) the number of growth stages within the growing season of the crop; \( ky_g \) the yield response factor for the growth stage \( g \); \( AET \) the actual and \( PET \) the potential evapotranspiration.
The objective function used for the overall integrated use model is
\[
\text{maximize } \sum_{c=1}^{NC} \left[ 1 - \sum_{g=1}^{NGS_c} k_y^c \left( 1 - \sum_{reg} \frac{AET_c}{PET} \right) \right] 
\]

Where \( c \) is the crop index; \( g \) the growth stage of the crop; \( k_y^c \) the yield response factor for the growth stage \( g \) of the crop \( c \); \( NGS_c \) the number of growth stages of the crop, and \( NC \) the number of crops.

The optimization is to maximize sum of relative yields of crops in the reservoir command area.

The relative yield of a crop, \( y = y_{\text{max}} \), in Eq. (2) would be equal to one if the volume of water available for the season is greater than or equal to the total crop water requirement in all the periods, thus permitting irrigation allocation to individual crops such that \( AET = PET \).

C. Soil Moisture Balance

The different elements considered in conceptualizing the soil moisture balance are shown in Fig. 2. At the beginning of the first period of the season the soil moisture is assumed to be at field capacity, for all crops
\[
SM_{t=1}^c = SM_{\text{max}}^c \quad \forall c
\]

Where \( SM_{t=1}^c \) is the available soil moisture (soil moisture above the permanent wilting point) at the beginning of the first period for the crop \( c \); \( SM_{\text{max}}^c \) is the available soil moisture at the field capacity for the crop \( c \).

The soil moisture balance equation for a given crop \( c \) for any time period \( t \) is given by
\[
SM_{t+1}^c = SM_{t}^c + D_t^c + x_c^t + xg_t^c + Rain_t - AET_t^c + SM_{\text{max}}(D_t^c - D_t^c) - DP_t^c \quad \forall c, t
\]

where, \( SM_t^c \) is the available soil moisture at the beginning of the period \( t \) for the crop \( c \); \( D_t^c \) the average root depth during the period \( t \) for crop \( c \); \( x_c^t \) the irrigation allocation from surface water to crop \( c \) in period \( t \); \( xg_t^c \) the irrigation allocation from ground water to crop \( c \) in period \( t \); \( Rain_t \) the rainfall in period \( t \), assuming that all the rain would contribute to enrich the soil moisture; \( AET_t^c \) the actual evapotranspiration during period \( t \) for crop \( c \); \( DP_t^c \) the deep percolation during the period \( t \) for the crop \( c \).

The available soil moisture in any time period \( t \) for crop \( c \) should not exceed the maximum corresponding to the field capacity of the soil
\[
SM_t^c \leq SM_{\text{max}}^c \quad \forall c, t
\]
\[
AET_t^c \leq PET_t^c \quad \forall c, t
\]

Where \( PET_t^c \) is the potential evapotranspiration during period \( t \) for crop \( c \).

The linear relationship between \( AET/PET \) and the soil moisture is maintained as per the following constraint
\[
AET_t^c \leq \left( \frac{SM_{t}^c + x_c^t + xg_t^c + Rain_t}{SM_{\text{max}}^c} \right) PET_t^c \quad \forall c, t
\]

The following constraints are imposed to see that whenever the deep percolation exists, the available soil moisture in the root zone of the crop at the end of the time period is at field capacity. In other words \( DP > 0 \) only when \( SM_{t+1}^c = SM_{\text{max}}^c \) for any \( t \). This is achieved by introducing integer variables, \( \lambda \)
\[
DP_t^c \leq \lambda_c^t \quad \forall c, t
\]
\[
\lambda_c^t \leq \frac{SM_{t+1}^c}{SM_{\text{max}}^c} \quad \forall c, t
\]

Where \( \lambda_c^t \) is a binary (0 or 1) variable and \( G \) is an arbitrarily large number 2.5

D. Reservoir Mass Balance

The effectiveness of an operation policy of a reservoir system depends on the information of the actual quantity of
water, which would be available in the reservoirs. The planning and design of storage structures rely basically upon the observations of elevation-storage levels and outflows from reservoir using mass balance approach.

1. Mass Balance Equation for Ravishankar Reservoir

If the evaporation losses are expressed as a function of storage, a mass balance equation can be stated as given in Loucks et al. (1981). This mass balance equation involves releases, inflow, storage and losses through the reservoir during the period t and is expressed in volume units as

\[ I_{r,t} + \sum_{e} [S_{o,r,t} - r_{o,r,t} - d_{r,t} + R_{t,r} - R_{t,o} - O_{t,o} + O_{t,r} + A_{t,r}] = R_{t+1,r} + O_{t+1,o} - O_{t,o} + O_{t,r} + A_{t,r} \] (11)

where \( t \) represents time period in month of a year, \( I_{r,t} \) is inflow into Ravishankar reservoir during time period \( t \), \( S_{o,r,t} \) is storage in Ravishankar reservoir at the end of time period \( t \), \( S_{r,t} \) is storage in Ravishankar reservoir at the beginning of time period \( t \), \( R_{t,r} \) is total release from Ravishankar reservoir during time period \( t \), \( R_{t,d} \) and \( R_{t,m} \) are releases from Dudhawa and Murumsilli reservoirs during time period \( t \) respectively, \( O_{t,o} \) and \( O_{t,r} \) are spills from Dudhawa, Murumsilli and Ravishankar reservoirs respectively during time period \( t \), \( A_{t,o} \) is reservoir water surface area corresponding to dead storage volume for Ravishankar reservoir, \( e_{r} \) is evaporation rate for time period \( t \) in depth units from Ravishankar reservoir, \( a_{e} \) is reservoir water storage area per unit volume of active storage above \( A_{t,o} \) for Ravishankar reservoir. The values of \( A_{t,o} \) and \( A_{t,d} \) are the reservoir continuity equation (i.e., Eq. 11) are 31.88 Mm³ and 0.08*10³ m³ respectively. Similarly the mass balance can be worked for Dudhawa, Murumsilli, and Sondur Reservoir.

Inflow series of the reservoirs in MRP are computed based on the observed data using mass balance equation. Based on the AIC, AR(1) model each for Ravishankar, Dudhawa, Murumsilli, and Sondur reservoirs is found appropriate to generate inflow series.

### Table 1 Area-Elevation and Storage-Elevation Equations for Reservoirs in MRP

<table>
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<td>( A_r = 0.09x^2 + 4x + 31.88 ) (Mm³)</td>
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Note: DSL = Dead storage level

### E. Linking Constraints

The dynamic relationships among the three main components of the integrated use system – the reservoir, the aquifer and the irrigated area are incorporated by imposing the appropriate linkages as constraints in the model. The reservoir release is related to the surface water allocations to the crops. The ground water pumping is related to the ground water allocation to the crop grown in the element. The conveyance loss in canals is assumed as a fraction of the release from the reservoir. The entire amount of canal seepage is assumed to contribute to the ground water recharge, and is assumed distributed uniformly among the elements through which the canal is running.

The release from the reservoir in a given period after losses should equal to the sum of the irrigation allocations from the surface reservoir to all crops in the period time period \( t \) respectively, \( O_{t,o} \) and \( O_{t,r} \) are spills from Dudhawa, Murumsilli and Ravishankar reservoirs respectively during time period \( t \), \( A_{t,o} \) is reservoir water surface area corresponding to dead storage volume for Ravishankar reservoir, \( e_{r} \) is evaporation rate for time period \( t \) in depth units from Ravishankar reservoir, \( a_{e} \) is reservoir water storage area per unit volume of active storage above \( A_{t,o} \) for Ravishankar reservoir. The values of \( A_{t,o} \) and \( A_{t,d} \) are the reservoir continuity equation (i.e., Eq. 11) are 31.88 Mm³ and 0.08*10³ m³ respectively. Similarly the mass balance can be worked for Dudhawa, Murumsilli, and Sondur Reservoir.

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The release from the reservoir in a given period after losses should equal to the sum of the irrigation allocations from the surface reservoir to all crops in the period

\[
\eta R_t = \sum_{c=1}^{N_c} x_c A_{c,r} \text{ t} \] (12)

Where \( \eta \) is the conveyance efficiency factor; \( R_t \) the reservoir release in period \( t \), in Mm³; \( x_c \) is the irrigation allocation from surface water to crop \( c \) in period \( t \), in mm; \( A_{c,r} \) the area of the crop \( c \), in km².

For those elements, the recharge due to rainfall is assumed as a known fraction of the rainfall occurring in the element in a given period \( t \). For the elements over the irrigated area, the recharge will be accounted in the soil moisture balance equation through the deep percolation term (Eq. (5)).

The recharge to the aquifer from the element \( e \) during the time period \( t \) is given by

\[
Q_{s,e}^{b} = \delta \frac{\text{Rain}_{e} \text{ Canal reach}_{e}}{1000 \Delta t} + \frac{1 - \delta}{1000 \Delta t} \text{ DP}_{e} \text{ t} \] (13)
Where \( Q_{R}^{e} \) is the recharge rate from the element during the period \( t \), in m³/(dayₘ²).

\[ \theta = 0 \] if any crop is grown in the element \( e \), \( \theta = 1 \) if no crop is grown in the element \( e \), \( c_{D} \) is the rainfall recharge coefficient, \( R_{a}^{i} \) is the rainfall during the time period \( t \) in mm, \( \Delta t \) the time step in days, \( R_{c}^{e} \) the recharge from canal seepage for element \( e \) in time \( t \) in m.

The ground water pumping \( Q_{p}^{e} \) in any element \( e \) during time period \( t \), in m³/(dayₘ²), is equal to the rate of irrigation application from ground water to the crop in element \( e \) during the period \( t \) is given by

\[ Q_{p}^{e} = \frac{1}{1000} \frac{R_{c}^{e}}{a_{s}^{e}} \quad \forall \ e, t \]  

The recharge from canal seepage is assumed as a fraction of the release from the reservoir and is assumed to be distributed uniformly among all the elements through which canals are running. The recharge to the aquifer from canal seepage for element \( e \) (among those through which the canals pass) during time period \( t \) is calculated as

\[ \text{Canal reach}^{e} = \frac{1}{2} \frac{Q_{R}^{e}}{a_{s}^{e}} \]  

Where \( R_{i} \) is the release from the reservoir in time period \( t \), in Mm³; \( NE \) the number of elements through which the canals are running; \( A_{e} \) the area of the element \( e \), in km².

F. Integrated use policy

Integrated use model is run for different pre-determined ratios of annual surface and ground water applications at the crop level. Each of these runs is associated with a integrated use policy identified by the ratio of the annual surface water to the ground water application. A 75:25 policy refers to one in which 75% of the annual irrigation application at the crop level comes from surface source and 25% from ground water pumping. Ground water balance components over the entire study area are computed for each run and the ground water balance examined.

1. Ground water balance of the study area

The annual ground water balance of the study area can be written as

\[ \Delta S = R_{r} + R_{c} + R_{w} - O - B - P \]  

where \( \Delta S \) is the net change in ground water storage; \( R_{r} \) the rainfall recharge; \( R_{c} \) the recharge from irrigated area; \( R_{w} \) the recharge due to canal seepage; \( P \) the annual ground water pumping; \( O \) the ground water net out flow to surroundings; \( B \) the base flow to the river.

2. Stable policy specification

The integrated use optimization model is run for a normal year for different assumed integrated policies. Against the allocations obtained by solving the optimization model, a complete water balance is prepared for the study area and the change in the annual ground water level computed, for each integrated use policy.

The particular policy for which the annual change in ground water storage is negligible when the model is run for a normal year is identified as a “stable policy”. A higher proportion of ground water allocation (compared to the stable policy) may increase the annual crop yield, but at the expense of a declining water table. On the other hand, an increase in the proportion of surface water allocation causes an increase in the ground water levels, which may lead to undesirable water logging conditions in the long run. Thus the stable integrated use policy as identified above will be useful as a planning aid in irrigation allocation.

G. Simulation

A simulation model is developed to simulate the performance of the system when operated with the stable integrated use policy for those years, for which historical data of rainfall and in flows to the reservoir are available.

1. Implementation of the stable policy

For a given year, the surface water and ground water components of the irrigated water supply, for each crop in each period of the year under consideration, are computed making use of the parameters \( r_{i} \) and \( a_{s}^{e} \) through simulation.

The actual evapotranspiration \( AET_{c}^{i} \) of a crop \( c \) in period \( t \) in a year is computed as

\[ AET_{c}^{i} = a_{s}^{e} \cdot PET_{c}^{i} \]  

Where \( PET_{c}^{i} \) is already known. The net irrigation requirement is then assumed to be equal to

\[ [AET_{c}^{i} - Rain_{t}] \]  

ignoring the net contribution from soil moisture

\[ x_{i}^{c} + xg_{i}^{c} = AET_{c}^{i} - Rain_{t} \]  

The ratio of surface to total irrigation application in a period, \( r_{i} \) being known, \( x_{i}^{c} \) and \( xg_{i}^{c} \) are computed individually

\[ x_{i}^{c} = r_{i} \left( x_{i}^{c} + xg_{i}^{c} \right) \]  

\[ xg_{i}^{c} = \left( 1 - r_{i} \right) \left( x_{i}^{c} + xg_{i}^{c} \right) \]  

The demands from the reservoir and aquifer for each crop at the crop level are \( x_{i}^{c} + xg_{i}^{c} \) respectively. The total demand from the surface reservoir is calculated by summing up the demands from each crop and dividing by the conveyance factor. The reservoir is operated following the standard operating policy.

The inflow to the reservoir during period \( t \), \( Q_{i} \) is added to the beginning of the period of storage, \( SA \), and \( SA_{a} \) and the evaporation rate for the period. The availability of water for release from the reservoir for each period \( t \) is calculated by

\[ \text{avail}_{i} = SA_{a} + Q_{i} - SA_{a+1} - \text{Evap}_{i} \]  

If the availability is less than the total demand from the reservoir, the entire available volume of water is released. The deficit is distributed among all the crops in the ratio of their demands. If the availability equals or exceeds the demand (plus any overflow that may have to be spilled with end of period storage set to the maximum), the final reservoir storage at the end of the period \( t \) is calculated after the release, as

\[ SA_{a+1} = SA_{a} + Q_{i} - \text{Evap}_{i} - \text{Rel}_{i} \]  

Where \( \text{Rel}_{i} \) is the reservoir release made in period \( t \).
The evaporation loss from the reservoir calculated earlier is corrected iteratively till the evaporation loss calculated in two successive iterations converges (within the limits of tolerance, set at 0.1% in the present case).

With known rainfall values and surface and ground water applications to each crop, the actual evapotranspiration values for each crop are reset using the relationship

\[ AET^e_t = PET^e_t \text{ if } \left( \frac{SM^e_t D^c_t + x^c_t + x^g_t + Rain_t}{SM^e_{max} D^c_t} \right) \geq 1 \]

\[ = \left( \frac{SM^e_t D^c_t + x^c_t + x^g_t + Rain_t}{SM^e_{max} D^c_t} \right) PET^e_t \text{ Otherwise} \]

\[ (23) \]

This is done to maintain an assumed linear relationship between AET and PET. However, the irrigation allocations \( x^c_t \) and \( x^g_t \) as computed earlier are not changed.

Now that \( AET^e_t, x^c_t, x^g_t \), and \( Rain_t \), are known the end of the period soil moisture condition, \( SM_{t+1} \), is computed from

\[ SM^e_{t+1} D^c_{t+1} = SM^e_t D^c_t + x^c_t + x^g_t + Rain_t - AET^e_t + SM^e_{max} \left( D^c_t - D^c_{max} \right) \forall c, t \] \[ (25) \]

The end of the period soil moisture \( SM^e_{t+1} \) is connected for deep percolation, \( D^c_t \), if necessary, using the criterion that

\[ SM^e_{t+1} D^c_{t+1} = SM^e_{max} D^c_{max} \text{ then } D^c_{t+1} = SM^e_{max} D^c_{max} \]

\[ \text{and } SM^e_{t+1} = SM^e_{max} \text{ else } D^c_{t+1} = 0 \forall c, t \]

\[ (26) \]

Knowing the ground water application to each crop and the deep percolation from each cropped area and the rainfall from the non-irrigated area, the ground water response is simulated by solving the ground water balance equations given by

\[ [A][h] = [B][h]_0 + [F] \]

\[ (28) \]

Knowing the ground water levels at each node for each time period, the annual average ground water change is calculated to examine the annual ground water balance.

II. Genetic Algorithm Model

GAs are combinatorial optimization methods that search for solutions using an analogy between optimization and natural selection. The methodology of GAs involves coding, fitness function computation, and operations of reproduction, crossover, and mutation (Goldberg 2000). The advantages of GAs are that they (1) work with coding of the parameter set but not with the parameters themselves, (2) search from a population of points, not a single point, (3) use objective function information itself but not any derivatives, and (4) use probabilistic transitions rules but not deterministic rules. A constrained problem is converted into unconstrained problem in a GA by introducing a penalty function

\[ F_i = f(x) + \varepsilon \sum_{j=1}^{n} \delta_j \phi_j \]

where \( F_i \) = fitness value; \( f(x) \) = Objective function value; \( \kappa \) = number of constraints; \( \varepsilon = -1 \) for maximization and +1 for minimization; \( \delta_j \) = penalty coefficient; and \( \phi_j \) = amount of violation.

In this study a GA based model is formulated to allocate the surface and ground water available for each season optimally between different crops for each time period of different growth stages. The objective is to maximize the sum of the relative yields of all crops, given inputs of reservoir storage at the beginning and end of the season, inflow, rainfall on the irrigated area and crop water requirements assessed from potential evapotranspiration. The model also takes into account the intra-seasonal competition for water among multiple crops, soil moisture dynamics for each cropped area, and the heterogeneous nature of the soil and crop response to the level of irrigation applied. For the present study, in the case of the LP model, the following assumptions are essential: (1) Crop root growth is linear; (2) reservoir elevation versus storage curve is linear; and (3) the relation between the ratio of the actual evapotranspiration (AET) to the potential evapotranspiration (PET) and the corresponding soil moisture content is linear. All these assumptions are essential for LP but are not required in a GA. However, these assumptions are followed for the GA also, to make a comparison between the GA and LP models possible.

III. MODEL APPLICATION

A. MRP Complex

The study area is located in the Mahanadi basin (Fig 3) Chhattisgarh extends over an area of 141,589km² lies between east longitudes 80°30’ to 86°50’ and north latitudes 19°21’ to 23°35’. The basin has a culturable area of about 79,900km², normal annual rainfall is 141.7cm. Mahanadi reservoir project (MRP) is situated in the upper reaches of river Mahanadi. MRP comprises of four reservoirs, one major diversion work, and two feeder canals along with two service canals. Three reservoirs namely Ravishankar, Dudhawa and Murumsilli are located in Mahanadi basin and Sondur reservoir is located in Pairi basin. Inter basin transfer of water from Pairi basin to Mahanadi basin is facilitated by Sondur Feeder Canal (SFC). The MRP system is planned to provide irrigation intensity of 135%(100% Kharif season and 35% Rabi season) with irrigation potential of 276260 hectares in Kharif season and 96691 hectares in Rabi season.
We selected time series of monthly water table depth records obtained for thirty four observation wells located in the Chhattisgarh plain, near the Mahanadi River (Figure 1). Daily averaged and daily minimum, maximum, and mean temperature series were available for the period 1996-2006. The monthly averaged data are less noisy than raw daily or weekly records and seem more appropriate for long-term or seasonal forecasting [World Meteorological Organization (WMO), 1994].

**B. Crops and Cropping Pattern**

The principal crops with growth stages and the cropping pattern for the area irrigated by the reservoir canals is shown in table2 & table3. A water year has two principal crop seasons: Kharif and Rabi. The actual extent of cropping as per records varied from year to year, but the areas considered in the present study fairly represent the average picture. Table 4 shows the cropping pattern and crop areas in percentage of culturable command area under Kharif and Rabi crops in MRP command area.

The target intensities of irrigation for Kharif season and Rabi season are taken as 100% and 35% respectively (Govt. of M. P., Irr. Dept. Report, 1990).

### Table 2 Crop Stages and Their Duration with Crop Coefficients ($K_c$) for Rice

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Crop</th>
<th>Crop duration (Days)</th>
<th>Sowing date</th>
<th>First and second month $K_c$</th>
<th>Mid season stage duration</th>
<th>Late season stage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Days $K_c$ Days $K_c$</td>
<td>Days $K_c$ Days $K_c$</td>
</tr>
<tr>
<td>1</td>
<td>Rice HYV1</td>
<td>120</td>
<td>July 16-31</td>
<td>1.10</td>
<td>30</td>
<td>1.05</td>
</tr>
<tr>
<td>2</td>
<td>Rice HYV2</td>
<td>135</td>
<td>July 16-31</td>
<td>1.10</td>
<td>45</td>
<td>1.05</td>
</tr>
<tr>
<td>3</td>
<td>Rice HYV3</td>
<td>135</td>
<td>Aug 01-15</td>
<td>1.10</td>
<td>45</td>
<td>1.05</td>
</tr>
<tr>
<td>4</td>
<td>Rice 2 b1</td>
<td>135</td>
<td>June 16-30</td>
<td>1.10</td>
<td>45</td>
<td>1.05</td>
</tr>
<tr>
<td>5</td>
<td>Rice 3 b1</td>
<td>135</td>
<td>July 01-15</td>
<td>1.10</td>
<td>45</td>
<td>1.05</td>
</tr>
</tbody>
</table>

### Table 3 Crop Stages and Their Duration with Crop Coefficients ($K_c$)

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Crop</th>
<th>Crop duration (Days)</th>
<th>Sowing date</th>
<th>Initial stage duration</th>
<th>Development stage duration (Days)</th>
<th>Mid season stage duration</th>
<th>Late season stage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Day $K_c$ Day $K_c$ Day $K_c$</td>
<td>Day $K_c$ Day $K_c$</td>
<td>Day $K_c$ Day $K_c$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Wheat 2mv</td>
<td>135</td>
<td>Nov 16-30</td>
<td>20 0.3 7</td>
<td>25</td>
<td>60</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>Wheat 3mv</td>
<td>120</td>
<td>Dec 01-15</td>
<td>15 0.3 25</td>
<td>50</td>
<td>50</td>
<td>1.1</td>
</tr>
</tbody>
</table>
Table 4

<table>
<thead>
<tr>
<th>Season</th>
<th>Crop variety</th>
<th>Irrigated crop area as % of culturable commanded area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kharif</td>
<td>Rice HYV1 (High yield variety) (Transplanted)</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Rice HYV2 (Transplanted)</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Rice HYV3 (Transplanted)</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Rice 2 Bi (Broadcast)</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Rice 3 Bi (Broadcast)</td>
<td>20</td>
</tr>
<tr>
<td>Rabi</td>
<td>Wheat 2 mesican variety (mv)</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Wheat 3 mesican variety (mv)</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Gram</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Green gram</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Linseed</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Vegetables</td>
<td>3</td>
</tr>
</tbody>
</table>

Potential evapotranspiration of crops.

The reference evapotranspiration, ET₀, is being evaluated using ANN. The potential evapotranspiration PETₜ, of a crop c for the period t is then determined by

\[ \text{PET}_t^c = k_t^c \text{ET}_0 \]  \hspace{1cm} (29)

Where \( k_t^c \) is the crop factor for the crop c in the period t.

C. Crop Root Depth

The root depth for a given period is taken as the average of the beginning and end of the period root depths. The maximum root depth for all crops is assumed as 120 cm.

1. Soil moisture

Table 5

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Crop</th>
<th>Monthly crop coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Jan</td>
</tr>
<tr>
<td>1</td>
<td>Rice HYV1</td>
<td>1.10</td>
</tr>
<tr>
<td>2</td>
<td>Rice HYV2</td>
<td>1.10</td>
</tr>
<tr>
<td>3</td>
<td>Rice HYV3</td>
<td>1.10</td>
</tr>
</tbody>
</table>
Since GA is dependent on various parameters such as population, generations, cross over and mutation probabilities various combinations are tried. It is found that the approximate parameters for number of generations, population size, cross over probability and mutation probabilities are 200, 50, 0.7, and 0.01 respectively. The results obtained are presented in terms of total fitness function values in Fig.4 and number of generations in Fig.5. Termination criterion is set to 200 generations of GA simulation.

![Fig 4: Comparison Fitness Function Values for Various Crossover and Mutation Probabilities](image-url)
AET values obtained with the GA allocation model are also compared with those of a LP model in Fig. 4 for Rice HYV1, Rice HYV2, Rice HYV3, Rice 2BI, and Rice 3BI in kharif season. Fig. 5 presents similar comparison of AET values for Wheat 2MV, Wheat 3MV, gram, and green gram in rabi season. It is observed from Figs. 4 and 5 that AET values obtained by the GA and the LP compared well for most of the periods. The computational time required for the GA is practically insignificant compared to the time required for LP. In addition all the assumptions required for the LP model (stated earlier) are not required for the GA thus rendering the GA more realistic.

From this study, it is apparent that the GA performs well and is efficient when compared with LP. However, the LP model contains a very simple optimization approach due to the required but unrealistic simplificative hypotheses of linearity (e.g., the relation between AET/PET and the corresponding soil moisture content is linear). The GA model proposed in this study can be further improved by incorporating nonlinear constraints to overcome the simplifications inherent in the LP model.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Sum of relative yields using LP</th>
<th>Sum of relative yields using GA</th>
<th>GW storage change (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>73:27</td>
<td>5.784</td>
<td>5.721</td>
<td>-2.35</td>
</tr>
<tr>
<td>75:25</td>
<td>5.624</td>
<td>5.595</td>
<td>+0.89</td>
</tr>
<tr>
<td>85:15</td>
<td>4.024</td>
<td>3.955</td>
<td>+24.85</td>
</tr>
</tbody>
</table>

Without any restriction on the groundwater pumping the objective function value, which is the sum of the relative yields of crops, obtained is 6.513. The resulting annual ground water storage change is -21.55 mm, which is high for a stable condition. Therefore, the integrated use model is run for different pre-determined ratios of annual surface and ground water applications and the results analyzed. A 75:25 policy refers to a case where 75% of the annual irrigation application at the crop level comes from surface source and 25% through ground water pumping. Ground water balance components over the entire study area are calculated for each of these runs as mentioned earlier.

IV. CONCLUSION

The integrated water management model is developed to optimize relative yield from a specified cropping pattern by using both Genetic Algorithm (GA) and Linear Programming (LP). In Genetic Algorithm value of fitness function is equal to objective function. Penalty function approach is used to convert the constrained problem into an unconstrained problem with a reasonable penalty function. It is observed from the results that solutions obtained by both GA and LP are reasonably close proving that GA can be used for integrated use of surface and ground water modeling. However results obtained from GA can be further refined for a number of factors such as penalty function values, mutation and crossover probabilities, generation and population. It is noticed, that as ground water allocation is reduced, the sum of relative yield decreases indicating that the crops get decreasing amount of water for their requirement. The 75:25 policies, however, resulted in an annual change in ground water storage of +0.89 mm which is considered negligible. Thus 75:35 policy is considered as the “stable policy”. The sum of relative yields corresponding to this policy is 5.624. Ground water simulation and optimization techniques can be used together to explore management options. The problem of optimizing integrated use of surface water and ground water is quite complex and can be handled more conveniently with help of tools of systems engineering.

REFERENCES


