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Analytical Study of Surface Tension Driven Magneto Convection in a Composite Layer Bounded by Adiabatic Boundaries

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Abstract-The problem of surface tension driven magneto-convection is investigated in a two layer system comprising an incompressible electrically conducting fluid saturated porous layer over which lies a layer of the same fluid in the presence of a vertical magnetic field. The lower rigid surface of the porous layer and the upper free surface are considered to be insulating to temperature perturbations. At the upper free surface, the surface tension effects depending on temperature are considered. At the interface, the normal and tangential components of velocity, heat and heat flux are assumed to be continuous. The resulting eigenvalue problem is solved exactly and an analytical expression for the Thermal Marangoni Number is obtained. The effect of variation of the physical parameters Horizontal wave number, Porous parameter, Chandrasekhar number, viscosity ratio and porosity on the Thermal Marangoni Number which is the criteria for the onset of Marangoni convection in the composite layer is investigated in detail.

Keywords: **Marangoni Convection, Porous Parameter, Chandrasekhar Number, Porosity.**

I. INTRODUCTION

Magneto convection in an electrically conducting fluid has been studied extensively by many authors (Busse [1], Chandrasekhar [2], Rudraiah [10], Weiss [17], [18], Knobloch et al [6]). Rudraiah [11] have studied both linear and steady nonlinear magneto convection in a sparsely packed porous medium using Brinkman model with effective viscosity same as fluid viscosity, however the experiments show that the ratio of effective viscosity to fluid viscosity takes the value ranging from 0.1 to 10.0 as given in Givler and Altobelli [5].

Single component convection in composite layers is investigated by Many of the researchers started by Nield,[8] Rudraiah [12], Taslim and Narusawa [15], McKay [7], Chen [3]. I. S. Shiva kumara et. al [13] have investigated the onset of surface tension driven convection in a two layer system comprising an incompressible fluid saturated porous layer over which lies a layer of the same fluid. The critical Marangoni number is obtained for insulating boundaries both by Regular Perturbation technique and also by exact method. They also have compared the results obtained by both the methods and found in agreement.

Double diffusive convection in composite layers has wide applications in crystal growth and solidification of alloys. Inspite of its wide applications not much work has been done in this area. Chen and Chen [4] have considered the problem of onset of finger convection using BJ-slip condition at the interface. The problem of double diffusive convection for a thermohaline system consisting of a horizontal fluid layer above a saturated porous bed has been investigated experimentally by Poulikakos and Kazmierczak [9]. Venkatachalappa et al [16] have investigated the double diffusive convection in composite layer conducive for hydrothermal growth of crystals with the lower boundary rigid and the upper boundary free with deformation. Recently the double diffusive magneto convection in a composite layer bounded by rigid walls is investigated by Sumithra [14] by regular perturbation method is used to find the eigenvalue. Siddheshwar et al [19] have investigated the thermorheological effect on Rayleigh-Bernard and Marangoni Magneto convection in Newtonian liquid numerically for all possible boundary combinations such as rigid- rigid/free- free/rigid- free and isothermal/ adiabatic to know the influence of temperature dependent viscosity and externally applied magnetic field on the on set of convection by higher order Rayleigh-Ritz method. The problem under investigation has many engineering applications like the moisture migration in thermal insulation and stored grain, underground spreading of chemical pollutants, waste and fertilizer migration in saturated soil and geothermal and petroleum reservoirs.

II. FORMULATION OF PROBLEM

We consider a horizontal two - component, electrically conducting fluid saturated isotropic sparsely packed porous layer of thickness d_m underlying a two component fluid layer of thickness d with imposed magnetic field intensity H_0 in the vertical z – direction. The lower surface of the porous layer is rigid and the upper surface of the fluid layer is free with the surface tension effects depending on temperature. Both the boundaries are kept at different constant temperatures. A Cartesian coordinate system is chosen with the origin at the interface between porous and fluid layers and the z – axis, vertically upwards. The continuity, solenoidal property of the magnetic field, momentum energy, magnetic induction equations are,

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\nabla \cdot \vec{H} = 0 \tag{2}$$

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$$\rho_0\left[\frac{\partial\vec{q}}{\partial t}+(\vec{q}\cdot\nabla)\vec{q}\right]=-\nabla P+\mu\nabla^2\vec{q}+\mu_p(\vec{H}\cdot\nabla)\vec{H} \quad (3)$$

$$\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla)T=\kappa\nabla^2 T \quad (4)$$

$$\frac{\partial\vec{H}}{\partial t}=\nabla\times\vec{q}\times\vec{H}+\nu_m\nabla^2\vec{H} \quad (5)$$

For the porous layer,

$$\nabla_m\cdot\vec{q}_m=0 \quad (6)$$

$$\nabla_m\cdot\vec{H}=0 \quad (7)$$

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial\vec{q}_m}{\partial t}+\frac{1}{\varepsilon^2}(\vec{q}_m\cdot\nabla_m)\vec{q}_m\right]=-\nabla_m P_m+\mu_m\nabla^2\vec{q}_m-\frac{\mu}{K}\vec{q}_m+\mu_p(\vec{H}\cdot\nabla_m)\vec{H} \quad (8)$$

$$A\frac{\partial T_m}{\partial t}+(\vec{q}_m\cdot\nabla_m)T_m=\kappa_m\nabla_m^2 T_m \quad (9)$$

$$\varepsilon\frac{\partial\vec{H}}{\partial t}=\nabla_m\times\vec{q}_m\times\vec{H}_m+\nu_{em}\nabla_m^2\vec{H}_m \quad (10)$$

Where the symbols in the above equations have the following meaning $\vec{q}=(u,v,w)$ is the velocity vector, $\vec{H}$ is the magnetic field, t is the time, μ is the fluid viscosity, $P=p+\frac{\mu_p H^2}{2}$ is the total pressure, ρ_0 is the fluid density, μ_p is the magnetic permeability, $A=\frac{(\rho_0 C_p)_m}{(\rho C_p)_f}$ is the ratio of heat capacities, C_p is the specific heat, K is the permeability of the porous medium, T is the temperature, κ is the thermal diffusivity of the fluid, $\nu_m=\frac{1}{\mu_p\sigma}$ is the magnetic viscosity, σ is the electrical conductivity, ε is the porosity, $\nu_{em}=\frac{\nu_m}{\varepsilon}$ is the effective magnetic viscosity and the subscripts m and f refer to the porous medium and the fluid respectively.The basic steady state is assumed to the quiescent and we consider the solution of the form,

$$\left[u,v,w,P,T,\vec{H}\right]=\left[0,0,0,P_b(z),T_b(z),H_0(z)\right] \quad (11)$$

in the fluid layer and in the porous layer

$$\left[u_m,v_m,w_m,P_m,T_m\right]=\left[0,0,0,P_{mb}(z_m),T_{mb}(z_m)\right] \quad (12)$$

Where the subscript 'b' denotes the basic state. The temperature distributions $T_b(z)$, $T_{mb}(z_m)$, are found to be

$$T_b(z)=T_0-\frac{(T_0-T_u)z}{d} \text{ in } 0\le z\le d \quad (13)$$

$$T_{mb}(z_m)=T_0-\frac{(T_l-T_0)z_m}{d_m} \text{ in } 0\le z_m\le d_m \quad (14)$$

Where $T_0=\frac{\kappa d_m T_u+\kappa_m d T_l}{\kappa d_m+\kappa_m d}$ is the interface temperature In order to investigate the stability of the basic solution, infinitesimal disturbances are introduced in the form,

$$\left[\vec{q},P,T,\vec{H}\right]=\left[0,P_b(z),T_b(z),H_0(z)\right]+\left[\vec{q}',P',\theta,\vec{H}'\right] \quad (15)$$

And

$$\left[\vec{q}_m,P_m,T_m,\vec{H}\right]=\left[0,P_{mb}(z_m),T_{mb}(z_m),H_0(z_m)\right]+\left[\vec{q}_m',P_m',\theta_m,\vec{H}'\right] \quad (16)$$

Where the primed quantities are the perturbed ones over their equilibrium counterparts. Now equations (15) and (16) are substituted into the equations (1) to (10) and are liberalized in the usual manner. Next, the pressure term is eliminated from equation (3) and equation (8) by taking curl twice on these two equations and only the vertical component is retained. The variables are then non-dimensionalised using d, $\frac{d^2}{\kappa}$, $\frac{\kappa}{d}$, T_0-T_u and H_0 as the units of length, time velocity, temperature, and the magnetic field in the fluid layer and d_m, $\frac{d_m^2}{\kappa_m}$, $\frac{\kappa_m}{d_m}$, T_l-T_0 as the corresponding characteristic quantities in the porous layer. Note that the separate length scales are chosen for the two layers so that each layer is of unit depth..In this manner the detailed flow fields in both the fluid and porous layers can be clearly obtained for all the

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depth ratios $\hat{d} = \frac{d_m}{d}$. The dimensionless equations for the perturbed variables are given by,

$$\frac{1}{\Pr}\frac{\partial \nabla^2 w}{\partial t} = \nabla^4 w + Q\tau_{fm}\frac{\partial \nabla^2 H_z}{\partial z} \quad (17)$$

$$\frac{\partial \theta}{\partial t} = w + \nabla^2 \theta \quad (18)$$

$$\frac{\partial H_z}{\partial t} = \frac{\partial w}{\partial z} + \tau_{fm}\nabla^2 H_z \quad (19)$$

$$\frac{\beta^2}{\Pr_m}\frac{\partial \nabla_m^2 w_m}{\partial t} = \hat{\mu}\beta^2\nabla_m^4 w_m - \nabla_m^2 w_m + \beta^2 Q_m \tau_{mm}\frac{\partial \nabla_m^2 H_{zm}}{\partial z_m} \quad (20)$$

$$\frac{\partial \theta_m}{\partial t} = w_m + \nabla_m^2 \theta_m \quad (21)$$

$$\varepsilon\frac{\partial H_{zm}}{\partial t} = \frac{\partial w_m}{\partial z} + \tau_{mm}\nabla_m^2 H_{zm} \quad (22)$$

For the fluid layer $\Pr = \frac{\nu}{\kappa}$ is the Prandtl number, $Q = \frac{\mu_p H_0^2 d^2}{\mu\kappa\tau_{fm}}$ is the Chandrasekhar number, $\tau_{fm} = \frac{\nu_{mv}}{\kappa}$ is the diffusivity ratio. For the porous layer, $\Pr_m = \frac{\varepsilon\nu_m}{\kappa_m}$ is the Prandtl number, $\beta^2 = \frac{K}{d_m^2} = Da$ is the Darcy number, $\hat{\mu} = \frac{\nu_m}{\nu}$ is the viscosity ratio, $Q_m = \frac{\mu_p H_0^2 d_m^2}{\mu\kappa_m\tau_{mm}} = Q\varepsilon\hat{d}^2$ is the Chandrasekhar number $\tau_{mm} = \frac{\nu_{em}}{\kappa_m}$, in the porous layer.

We make the normal mode expansion and seek solutions for the dependent variables in the fluid and porous layers according to

$$\begin{bmatrix} w \\ \theta \\ H \end{bmatrix} = \begin{bmatrix} W(z) \\ \Theta(z) \\ H(z) \end{bmatrix} f(x,y)e^{nt} \quad (23)$$

and

$$\begin{bmatrix} w_m \\ \theta_m \\ H_m \end{bmatrix} = \begin{bmatrix} W_m(z_m) \\ \Theta_m(z_m) \\ H_m(z_m) \end{bmatrix} f(x_m,y_m)e^{n_m t} \quad (24)$$

With $\nabla_2^2 f + a^2 f = 0$ and $\nabla_{2m}^2 f_m + a_m^2 f_m = 0$, where a and a_m are the non-dimensional horizontal wave numbers, n and n_m are the frequencies. Since the dimensional horizontal wave numbers must be the same for the fluid and porous layers, we must have $\frac{a}{d} = \frac{a_m}{d_m}$ and hence $a_m = \hat{d}a$ .Substituting equations (23) and (24) into the equations (17) to (22) and denoting the differential operator $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial z_m}$ by D and D_m respectively, an eigenvalue problem consisting of the following ordinary differential equations is obtained,

In $0 \le z \le 1$,

$$\left(D^2 - a^2 + \frac{n}{\Pr}\right)\left(D^2 - a^2\right)W = -Q\tau_{fm}D\left(D^2 - a^2\right)H \quad (25)$$

$$\left(D^2 - a^2 + n\right)\Theta + W = 0 \quad (26)$$

$$\left[\tau_{fm}\left(D^2 - a^2\right) + n\right]H + DW = 0 \quad (27)$$

In $-1 \le z_m \le 0$

$$\left[\left(D_m^2 - a_m^2\right)\hat{\mu}\beta^2 + \frac{n_m\beta^2}{\Pr_m} - 1\right]\left(D_m^2 - a_m^2\right)W_m = -Q_m\tau_{mm}\beta^2 D_m\left(D_m^2 - a_m^2\right)H_m \quad (28)$$

$$\left(D_m^2 - a_m^2 + n_m\right)\Theta_m + W_m = 0 \quad (29)$$

$$\left[\tau_{mm}\left(D_m^2 - a_m^2\right) + n_m\varepsilon\right]H_{mz} + DW_m = 0 \quad (30)$$

It is known that the principle of exchange of instabilities holds for double diffusive magneto convection in both fluid and porous layers separately for certain choice of parameters. Therefore, we assume that the principle of exchange of instabilities holds even for the composite layers. In other words, it is assumed that the onset of convection is in the form of steady convection and accordingly we take $n = n_m = 0$. And eliminating the magnetic field in equations (25) and (28) from equations (27) and (30) we get, in $0 \le z \le 1$.

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$$\left(D^2-a^2\right)^2 W = QD^2W \qquad (31)$$

$$\left(D^2-a^2\right)\Theta + W = 0 \qquad (32)$$

In $-1 \le z_m \le 0$

$$\left[\left(D_m^2-a_m^2\right)\hat{\mu}\beta^2 - 1\right]\left(D_m^2-a_m^2\right)W_m = \beta^2 Q_m D_m^2 W_m \qquad (33)$$

$$\left(D_m^2-a_m^2\right)\Theta_m + W_m = 0 \qquad (34)$$

Thus we note that, in total we have a 12th order ordinary differential equation and we need 12 boundary conditions to solve them.

III. BOUNDARY CONDITIONS

The bottom boundary is assumed to be rigid and insulating to temperature so that at $z_m = -d_m$

$$w_m = 0, \frac{\partial w_m}{\partial z_m} = 0, \frac{\partial T_m}{\partial z_m} = 0 \qquad (35)$$

The upper boundary is assumed to be free, insulating temperature so the appropriate boundary conditions at $z = d$ are,

$$w = 0,\ \hat{\mu}\frac{\partial^2 w}{\partial z^2} = -\frac{\partial \sigma_t}{\partial T}\left[\nabla_2^2 T\right], \frac{\partial T}{\partial z} = 0$$

Where $\sigma_t = \sigma_0 - \sigma_T\, T,$ is the Surface tension, here

$$\sigma_T = -\left(\frac{\partial \sigma_t}{\partial T}\right)_{T=T_0}$$

At the interface (i.e., at $z = 0, z_m = 0$), the normal component of velocity, tangential velocity, temperature, heat flux are continuous and respectively yield following Nield [8],

$$w = w_m, \frac{\partial w}{\partial z} = \frac{\partial w_m}{\partial z_m}, T = T_m, \kappa\frac{\partial T}{\partial z} = \kappa_m \frac{\partial T_m}{\partial z_m} \qquad (36)$$

We note that two more velocity conditions are required at $z = 0$ Since we have used the Darcy-Brinkman equations of motion for the flow through the porous medium, the physically feasible boundary conditions on velocity are the following, at $z = 0$ and $z_m = 0$

$$P_m - 2\mu_m \frac{\partial w_m}{\partial z_m} = P - 2\mu\frac{\partial w}{\partial z} \qquad (37)$$

Which will reduce to

$$\mu\left(3\nabla_2^2 + \frac{\partial^2}{\partial z^2}\right)\frac{\partial w}{\partial z} = -\frac{\mu_m}{K}\frac{\partial w_m}{\partial z_m} + \mu_m\beta^2\left(3\nabla_{2m}^2 + \frac{\partial^2}{\partial z_m^2}\right)\frac{\partial w_m}{\partial z_m}$$

The other appropriate velocity boundary condition at the interface $z = 0, z_m = 0$ can be,

$$\mu\left(-\frac{\partial^2 w}{\partial z^2} + \nabla_2^2 w\right) = \mu_m\left(-\frac{\partial^2 w_m}{\partial z_m^2} + \nabla_{2m}^2 w_m\right) \qquad (38)$$

All the Twelve boundary conditions equations (35) to (38) are non-dimensionalised and are subjected to Normal mode expansion and are given by

$$W(1) = 0,\ D^2W(1) + M\, a^2\Theta(1) = 0,\ D\Theta(1) = 0,$$

$$\hat{T}W(0) = W_m(0), \quad \hat{T}\hat{d}DW(0) = D_m W_m(0),$$

$$\hat{T}\hat{d}^2\left(D^2+a^2\right)W(0) = \hat{\mu}\left(D_m^2 + a_m^2\right)W_m(0)$$

$$\hat{T}\hat{d}^3\beta^2\left(D^3W(0) - 3a^2DW(0)\right)$$
$$= -D_m W_m(0) + \hat{\mu}\beta^2\left(D_m^3 W_m(0) - 3a_m^2 D_m W_m(0)\right)$$

$$\Theta(0) = \hat{T}\Theta_m(0), \quad D\Theta(0) = D_m\Theta_m(0),$$

$$w_m(-1) = 0, \quad D_m w_m(-1) = 0,\ D_m\Theta_m(-1) = 0$$

(39)

The equations (31) to (34) are to be solved with respect to the boundary conditions equation (39).

IV. EXACT SOLUTION

The solutions of the equations (31) and (33) are independent of Θ and Θ_m can be solved and expressions for W and W_m can be obtained as,

$$W(z) = A_1 Cosh(\delta z) + A_2 Sinh(\delta z) + A_3 Cosh(\xi z) + A_4 Sinh(\xi z) \qquad (40)$$

$$W_m(z_m) = A_5 Cosh(C_4 z_m) + A_6 Sinh(C_4 z_m) + A_7 Cosh(C_5 z_m) + A_8 Sinh(C_5 z_m) \qquad (41)$$

Where $\delta = \dfrac{\sqrt{Q} - \sqrt{Q+4a^2}}{2}, \zeta = \dfrac{\sqrt{Q} + \sqrt{Q+4a^2}}{2}$

and $C_4 = \sqrt{\dfrac{C_1 + C_3}{2}}, C_5 = \sqrt{\dfrac{C_1 - C_3}{2}}$ and A_1, A_2, A_3, A_4 and A_5, A_6, A_7, A_8 are constants to be

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determined using the velocity boundary conditions of equation (39) and the expressions for $W(z)$ and $W_m(z)$ are

$$W(z) = A_1\begin{bmatrix} Cosh(\delta z) + a_1 Sinh(\delta z) \\ + a_2 Cosh(\xi z) + a_3 Sinh(\xi z) \end{bmatrix} \quad (42)$$

$$W_m(z_m) = A_1\begin{bmatrix} a_5 Cosh(C_4 z_m) + a_6 Sinh(C_4 z_m) \\ + a_7 Cosh(C_5 z_m) + a_8 Sinh(C_5 z_m) \end{bmatrix} \quad (43)$$

And now consider Heat equations (32) and (34), the expressions for Θ and Θ_m are obtained as,

$$\Theta(z) = A_1[a_8\, Coshaz + a_9 Sinhaz + f(z)] \quad (44)$$

$$\Theta_m(z) = A_1[a_{10}\, Cosha_m z_m + a_{11} Sinha_m z_m + f_m(z_m)] \quad (45)$$

Where $$f(z) = -\begin{bmatrix} \frac{1}{\delta^2 - a^2}(a_1 Sinh\delta z + Cosh\delta z) \\ + \frac{1}{\xi^2 - a^2}(a_3 Sinh\xi z + a_2\, Cosh\xi z) \end{bmatrix}$$

$$f_m(z_m) = -\begin{bmatrix} \frac{1}{c_4^2 - a_m{}^2}(a_4 Coshc_4 z_m + a_5\, Sinhc_4 z_m) \\ + \frac{1}{c_5^2 - a_m{}^2}(a_6 Coshc_5 z_m + a_7\, Sinhc_5 z_m) \end{bmatrix}$$

V. THERMAL MARANGONI NUMBER

The expressions of $\Theta(1)$ and $W(1)$ are substituted in equation $(39)^2$ and an expression for Thermal Marangoni number M is obtained as

$$M = -\frac{\begin{bmatrix} \delta^2\, Cosh\delta + a_1\, \delta^2 Sinh\delta \\ + a_2 \xi^2 Cosh\xi + a_3 \xi^2 Sinh\xi \end{bmatrix}}{a^2\begin{bmatrix} a_8 Cosha + a_9 Sinha - \frac{(Cosh\delta + a_1 Sinh\delta)}{(\delta^2 - a^2)} \\ -\frac{(a_2 Cosh\xi + a_3 Sinh\xi)}{(\xi^2 - a^2)} \end{bmatrix}} \quad (46)$$

Where

$$a_1 = \frac{a_2\,\Delta_{32} + \Delta_{34}}{\Delta_{33}}$$

$$a_2 = \frac{-\Delta_{36}}{\Delta_{35}}$$

$$a_3 = \frac{a_2\,\Delta_{12} + \Delta_{13} - a_1\,\Delta_{25}}{\Delta_{26}}$$

$$a_4 = \frac{a_2\,\Delta_7 + \Delta_8}{\Delta_9}$$

$$a_5 = \frac{a_1\Delta_{16} + a_3\Delta_{17}}{\Delta_{18}}$$

$$a_6 = \hat{T}(1 + a_2) - a_4$$

$$a_7 = \frac{a_1\Delta_3 + a_3\Delta_4 - a_5\Delta_5}{\Delta_8}$$

$$a_8 = \hat{T}a_{10} + \Delta_{38}$$

$$a_9 = \frac{a_{11}a_m + \Delta_{39}}{a}$$

$$a_{10} = \frac{a_{11}a_m\, Cosha_m - \Delta_{40}}{a_m\; Sinha_m}$$

$$a_{11} = \frac{\Delta_{45}}{\Delta_{44}}$$

$$\Delta_1 = c_4^2 + a_m^2$$

$$\Delta_2 = c_5^2 + a_m^2$$

$$\Delta_3 = \hat{T}\,\beta^2 \hat{d}^3(\delta^3 - 3a^2\delta)$$

$$\Delta_4 = \hat{T}\,\beta^2 \hat{d}^3(\xi^3 - 3a^2\xi)$$

$$\Delta_5 = -c_4 + \beta^2\hat{\mu}\, c_4^3 - 3a_m^2\, c_4$$

$$\Delta_6 = -c_5 + \beta^2\hat{\mu}\, c_5^3 - 3a_m^2\, c_5$$

$$\Delta_7 = \hat{T}\left\{(\xi^2 + a^2)\hat{d}^2 - \hat{\mu}\,\Delta_2\right\}$$

$$\Delta_8 = \hat{T}\left\{\hat{d}^2\,\delta^2 - \hat{\mu}\,\Delta_2 + \hat{d}^2\, a^2\right\} \quad \Delta_9 = \hat{\mu}(\Delta_1 - \Delta_2)$$

$$\Delta_{10} = Coshc_4 - Coshc_5 \quad \Delta_{11} = c_4\, Sinhc_4 - c_5\, Sinhc_5$$

$$\Delta_{12} = \frac{\Delta_7\,\Delta_{10}}{\Delta_9} + \hat{T}\; Coshc_5 \quad \Delta_{13} = \frac{\Delta_8\,\Delta_{10}}{\Delta_9} + \hat{T}\; Coshc_5$$

$$\Delta_{14} = \frac{\Delta_7\,\Delta_{11}}{\Delta_9} + c_5\,\hat{T}\; Sinhc_5$$

$$\Delta_{15} = \frac{\Delta_8\,\Delta_{11}}{\Delta_9} + c_5\,\hat{T}\; Sinhc_5 \quad \Delta_{16} = \hat{T}\hat{d}\delta - \frac{c_5\,\Delta_3}{\Delta_8}$$

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$$\Delta_{17} = \hat{T}\hat{d}\xi - \frac{c_5\,\Delta_4}{\Delta_8}$$

$$\Delta_{18} = c_4 - \frac{c_5\,\Delta_5}{\Delta_8}$$

$$\Delta_{19} = Sinhc_4 - \frac{\Delta_5}{\Delta_8}Sinhc_5$$

$$\Delta_{20} = \frac{\Delta_3}{\Delta_8}Sinhc_5$$

$$\Delta_{21} = \frac{\Delta_4}{\Delta_8}Sinhc_5 \quad \Delta_{22} = c_4\,Coshc_4 - \frac{\Delta_5}{\Delta_8}c_5\,Coshc_5$$

$$\Delta_{23} = \frac{\Delta_3}{\Delta_8}c_5\,Coshc_5$$

$$\Delta_{24} = \frac{\Delta_4}{\Delta_8}c_5\,Coshc_5$$

$$\Delta_{25} = \Delta_{20} + \frac{\Delta_{16}\,\Delta_{19}}{\Delta_{18}}$$

$$\Delta_{26} = \Delta_{21} + \frac{\Delta_{17}\,\Delta_{19}}{\Delta_{18}}$$

$$\Delta_{27} = \Delta_{23} + \frac{\Delta_{16}\,\Delta_{22}}{\Delta_{18}}$$

$$\Delta_{28} = \Delta_{24} + \frac{\Delta_{17}\,\Delta_{22}}{\Delta_{18}} \quad \Delta_{29} = Sinh\delta - \frac{\Delta_{25}}{\Delta_{26}}Sinh\xi$$

$$\Delta_{30} = Cosh\xi + \frac{\Delta_{12}}{\Delta_{26}}Sinh\xi$$

$$\Delta_{31} = Cosh\delta + \frac{\Delta_{13}}{\Delta_{26}}Sinh\xi \quad \Delta_{32} = \Delta_{14} - \frac{\Delta_{12}\,\Delta_{28}}{\Delta_{26}}$$

$$\Delta_{33} = \Delta_{27} - \frac{\Delta_{25}\,\Delta_{28}}{\Delta_{26}}$$

$$\Delta_{34} = \Delta_{15} - \frac{\Delta_{13}\,\Delta_{28}}{\Delta_{26}}$$

$$\Delta_{35} = \Delta_{30} + \frac{\Delta_{32}\,\Delta_{29}}{\Delta_{33}}$$

$$\Delta_{36} = \Delta_{31} + \frac{\Delta_{34}\,\Delta_{29}}{\Delta_{33}}$$

$$\Delta_{37} = -\frac{\delta}{\delta^2 - a^2}\left(Sinh\delta + A_1\,Cosh\delta\right) - \frac{\xi}{\xi^2 - a^2}\left(A_2 Sinh\xi + A_3\,Cosh\xi\right)$$

$$\Delta_{38} = \frac{1}{\delta^2 - a^2} + \frac{A_2}{\xi^2 - a^2} - \hat{T}\left(\frac{A_4}{c_4^2 - a_m^2} - \frac{A_6}{c_5^2 - a_m^2}\right)$$

$$\Delta_{39} = \frac{\delta A_1}{\delta^2 - a^2} + \frac{\xi A_3}{\xi^2 - a^2} - \frac{A_5 c_4}{c_4^2 - a_m^2} - \frac{c_5 A_7}{c_5^2 - a_m^2}$$

$$\Delta_{40} = \frac{c_4}{c_4^2 - a_m^2}\left(A_5\,Coshc_4 - A_4\,Sinhc_4\right) + \frac{c_5}{c_5^2 - a_m^2}\left(A_7\,Coshc_5 - A_6\,Sinhc_5\right)$$

$$\Delta_{41} = a\hat{T}\,Sinha$$

$$\Delta_{41} = a_m\;Cosha$$

$$\Delta_{43} = a\,Sinha\,\Delta_{38} + \Delta_{39}\,Cosha + \Delta_{37}$$

$$\Delta_{44} = \frac{\Delta_{41}Cosha_m + \Delta_{42}\,Sinha_m}{Sinha_m}$$

$$\Delta_{45} = \frac{\Delta_{41}\,\Delta_{40}}{a_m\,Sinha_m} - \Delta_{43}$$

VI. RESULTS AND DISCUSSIONS

The Thermal Marangoni number M obtained as a function of the parameters is drawn versus the depth ratio $\hat{d}$ and the results are represented graphically showing the effects of the variation of one physical quantity, fixing the other parameters. The fixed values of the parameters are $\hat{T} = 1$, $\phi = 0.9$, $\beta = 0.1$, $Q = 500$, $\hat{\mu} = 2.5$. The effects of the Horizontal wave number a, the porous parameter β, Chandrasekhar number Q, the viscosity ratio $\hat{\mu}$ and the porosity ϕ on the Thermal Marangoni number M are obtained and portrayed in the figures 1 to 5 respectively. The effects of a, horizontal wave number on the Thermal Marangoni number M are shown in Fig. 1. The graph has three converging curves. The line curve is for $a = 1.1$, the big dotted curve is for 1.2 and the small dotted line curve is for 1.3. Since the curves are diverging, it indicates that for larger values of the depth ratio, the increase in the value of horizontal wave number a makes a drastic change in the values of the thermal Marangoni number M. The increase in the values of horizontal wave number a, the Thermal Marangoni number M increases, so the increase in the value of horizontal wave number stabilizes the system. That is the Marangoni convection is delayed.

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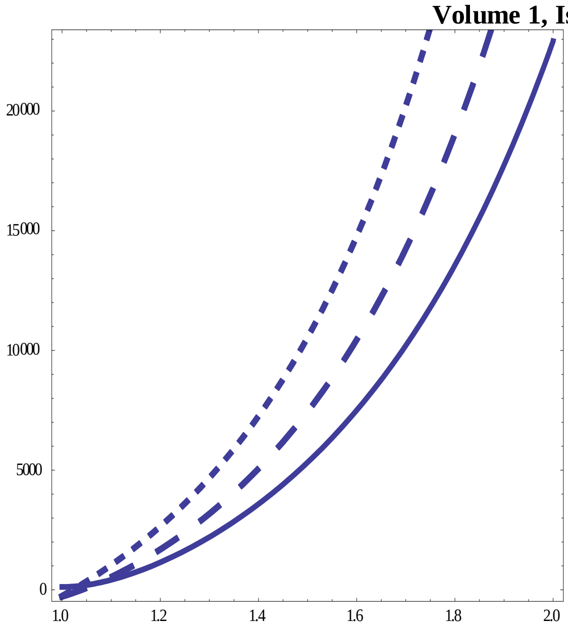

Fig.1. The effects of a, horizontal wave number on the Thermal Marangoni number M for $Q=500, \Phi=1.0, \hat{\mu}=2.5, \beta=0.1$

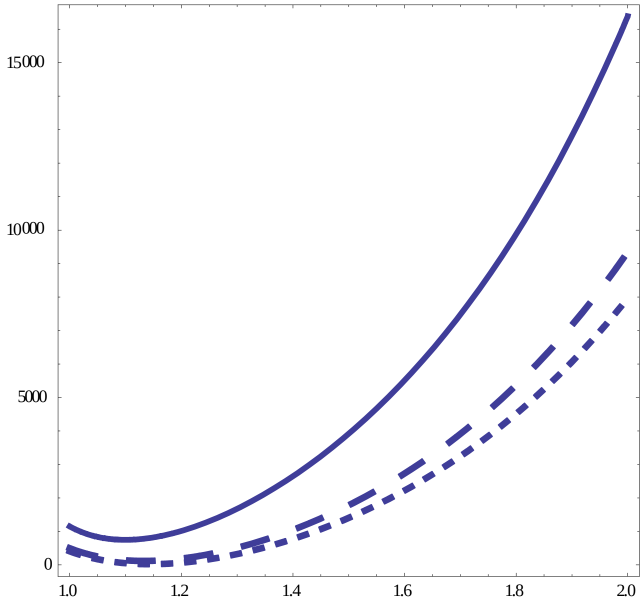

Fig.2. The effects of Porous parameter β on the Thermal Marangoni number M *for* $Q=500, \Phi=1.0, \hat{\mu}=2.5, a=1$

The effects of the porous parameter $\beta=\sqrt{\dfrac{K}{d_m^2}}$ is exhibited in the Fig.2. The graph has three diverging curves. The line curve is for $\beta=0.1$, the big dotted curve is for 0.2 and the small dotted line curve is for 0.3. Since the curves are diverging, it indicates that the increasing values of porous parameter β will affect the onset of Marangoni convection only for larger values of the depth ratio $\hat{d}=\dfrac{d_m}{d}$, that is for porous layer dominant composite systems. From the curves it is evident that for a fixed value of $\hat{d}$, increase in the value of β is to decrease the value of the thermal Marangoni number M i.e., to destabilize the system, so the onset of surface tension driven convection or Marangoni convection is faster, this destabilization may due the presence of magnetic field. In other words increasing the permeability of the porous matrix one can destabilize the fluid layer system in the presence of magnetic field. Figure 3 exhibits the effects of the magnetic field on the onset of convection by the Chandrasekhar number $Q=\dfrac{\mu_p H_0^2 d^2}{\mu\kappa\tau_{fm}}$. As the curves are diverging the effect of the magnetic field is very large for even a small change in the value of the depth ratio. Though the effect of magnetic field is stabilizing in single component, single layer convection problem, here for this set of values of the parameters taken, it is showing dual effect of the Thermal Marangoni number depending on the values of depth ratio.

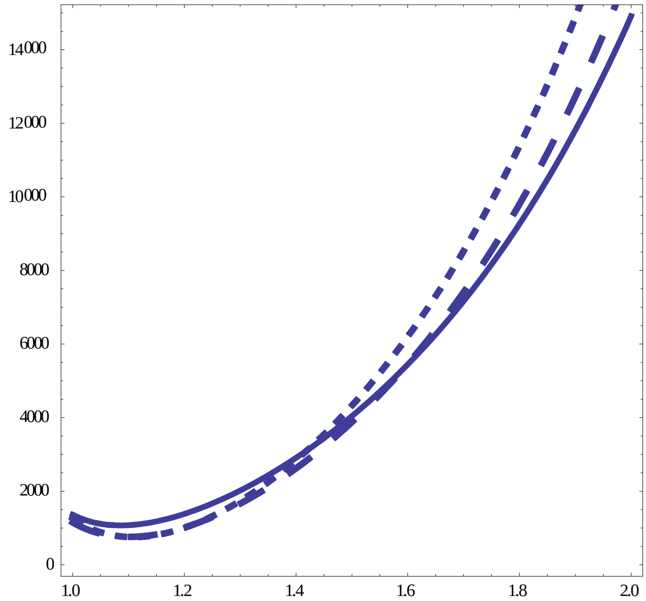

Fig.3. The effects of Chandrasekhar number Q on the Thermal Marangoni number M *for* $\Phi=1.0, \hat{\mu}=2.5, a=1, \beta=0.1$

The line curve is for $Q=300$, the big dotted curve is for 500 and the small dotted line curve is for 700. For values of depth ratio $\hat{d}\le 1.5$, for the fluid layer dominant systems, the increase in the value of Q decreases the thermal Marangoni number M that is to destabilize the system. Where as for values of $\hat{d}>1.5$, porous layer dominant systems, the increase in the value of Q increases the thermal Marangoni number M that is to stabilize the system, so the Marangoni convection or surface tension driven convection can be delayed.

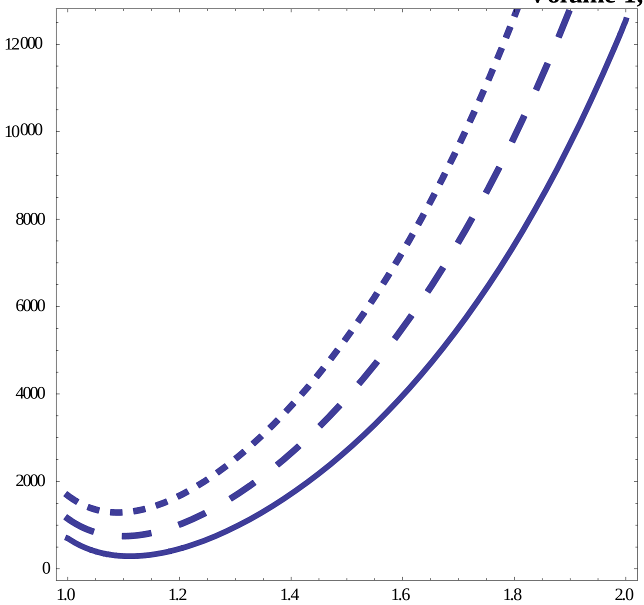

Fig.4. The Effects Of Viscosity Ratio $\hat{\mu}$ On The Thermal Marangoni Number M *For*
$\Phi = 1.0, \hat{\mu} = 2.5, \ a = 1, \beta = 0.1$

The effects of the viscosity ratio $\hat{\mu} = \frac{\mu_m}{\mu}$, which is the ratio of the effective viscosity of the porous matrix to the fluid viscosity are displayed in Fig. 4. The line curve is for $\hat{\mu} = 2.0$, the big dotted curve is for 2.5 and the small dotted line curve is for 3.0. The increasing values of viscosity ratio $\hat{\mu}$ increases the value of the Thermal Marangoni number M i.e., to stabilize the system, so the onset of surface driven magneto - convection is delayed. In other words when the effective viscosity of the porous medium μ_m is made larger than the fluid viscosity μ, the onset of the surface driven magneto - convection in the fluid layer can be delayed. In Fig. 5 the line curve is for $\phi = 0.7$, the big dotted curve is for 0.8 and the small dotted line curve is for 1.0. Fig.5 depict that there is no effect for small values of the depth ratio $\hat{d} = \frac{d_m}{d}$ and a little variation in the curves for the region $\hat{d} \geq 1.25$, The increase in the value of Porosity ϕ is to slightly decrease the value of the thermal Marangoni number M i.e., to destabilize the system, so the onset of surface driven magneto- convection can be made earlier even by increasing the void volume in the porous layer in the presence of the magnetic field, though the effect of magnetic field is stabilizing .

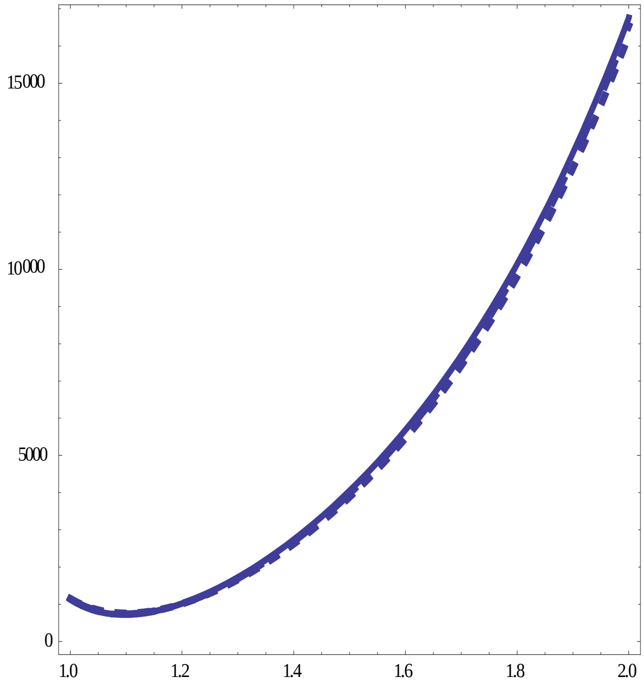

Fig.5. The effects of Porosity ϕ on the Thermal Marangoni number M *for*
$Q = 500, \hat{\mu} = 2.5, \ a = 1, \beta = 0.1$

VII. CONCLUSION

The increase in the values of Horizontal wave number 'a' and Viscosity ratio $\hat{\mu}$ is to increase the thermal Marangoni number M hence their effect is to delay the surface tension driven convection i.e., to stabilize the system. Whereas the increase in the values of the Porous parameter β and Porosity ϕ decrease the Marangoni number M so the effect of these parameters is to destabilize the system. By choosing the appropriate values of the physical parameters, one can control the surface driven convection in the composite layer.

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