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Second Order Analysis of Slender Hollow Circular Pier

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Abstract*— Slender member is subjected to axial load and biaxial bending moment and fails due to buckling. This buckling is caused due to slenderness effect also known as 'PΔ' effect. This buckling gives rise to excessive bending moment occurring at a point of maximum deflection. This additional bending moment is considered in second order analysis. The objective of the research reported in this paper is to formulate bending moment equation by using beam column theory and to study the behavior of solid circular section and hollow circular section of bridge pier. The optimization in area of cross section is done by providing a hollow circular section in place of a solid circular section of pier within permissible limits. A comparative study on the cost effectiveness of the area of cross section is done.

***Index Terms*— Slender Column, Buckling, 'PΔ' Effect, Beam-Column, Second Order Analysis, Bridge Pier.**

I. INTRODUCTION

Piers are not only subjected to axial load but also forces in longitudinal direction as well as in transverse direction. These forces cause moment in longitudinal direction and transverse direction at base of pier. Thus, pier is idealized as a column subjected to axial load and biaxial moment. These forces cause the pier to buckle along its height. The moment due to buckling is not considered in first order analysis. In order to get accurate forces one has to go for second order analysis where in the buckling effect is considered. Beam column theory is one of the methods to calculate the bending moment by second order analysis. Iterative neutral axis method is used to design the cross section of pier. In a section subjected to axial load combined with two orthogonal moments, by assuming the neutral axis at certain depth and stress at that point is to be calculated. This stress at neutral axis should be zero or else the procedure is revised for another trail.

II. SECOND ORDER ANALYSIS USING BEAM-COLUMN THEORY

Beams subjected to axial compression with lateral loads act as beam-column. The basic equation for analysis of beam-column can be derived by considering a beam as shown in Figure1.The beam is subjected to an axial compressive force **P** and lateral load of intensity '**q**' which varies with the distance '**x**' along the beam. Consider an element of length 'd_x' between two cross sections taken normal to the original axis of beam as shown in Figure 2. The lateral load has a constant intensity '**q**' over a distance 'd_x' and will be assumed positive when in direction of positive **y** axis which is downward in this case. The shearing force ***V*** and bending moment ***M*** acting on either side of the elements are assumed positive in the downward direction. The relation between load, shear force and bending moment are obtained from the equilibrium of the element in Figure 2. On summing forces in the y direction it gives:

$$-V+qdx+(V+dv)=0$$

Or $$q=-\frac{dV}{dx}$$

(1)

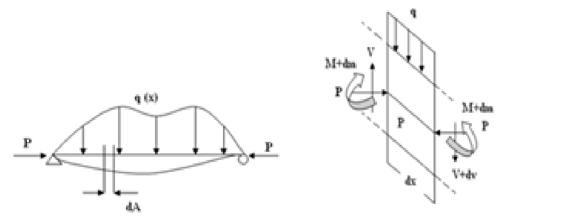

Figure1. General loading beam-column analysis Figure2. Cross section of beam

Fig.1 & 2

Taking the moment about point on beam and assuming that angle between the axis of beam and horizontal axis is small, we obtain,

$$M+ qdx\frac{dx}{2} + (V+ dv) - (M+ dM) + P\frac{dy}{dx}dx = 0$$

If terms of second-degree are neglected, this equation becomes

$$V=\frac{dM}{dx} - P\frac{dy}{dx}$$

(2)

If the effects of shearing deformations and shortening of the beam axis are neglected the expression for the curvature of the axis of the beam is,

$$EI\frac{d^2y}{dx^2} = -M$$

(3)

The quantity ***EI*** represents the flexural rigidity of beam in a plane of bending, i.e. ***XY*** plane, which is assumed to be plane of symmetry. Combining equation (3) with equation (1) and equation (2) we can express the differential equations of the axis of the beam in the following alternate forms:

$$EI\frac{dy}{dx} + P\frac{dy}{dx} = -V$$

(4)

$$EI\frac{dy}{dx} + P\frac{dy}{dx} = q$$

(5)

Equations (1) to (5) are the basic differential equations for bending of beam-column. If the axial force's ***P*** equals zero, these equations reduces to the usual equations for bending by lateral loads only. The nature of the axial forces have significant effect on the deflections and ultimately on the secondary moments.

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III. ITERATIVE NEUTRAL AXIS METHOD

Iterative neutral axis method is used for design of slender member which are subjected to axial load and biaxial moment. In this method, some percentage of steel is assumed and the moment of inertia of full section is calculated. Then inclination of neutral axis is calculated. Then, moment of inertia and eccentricity of cracked section is computed. Compute stress at neutral axis, if it is zero, and if stresses at extreme fibers are within permissible limit, the assumed percentage of steel is acceptable otherwise the neutral axis has to be shifted and same procedure has to be carried out.

IV. THEOROTICAL FORMULATION

A. Trapezoidal load throughout the height of pier:-

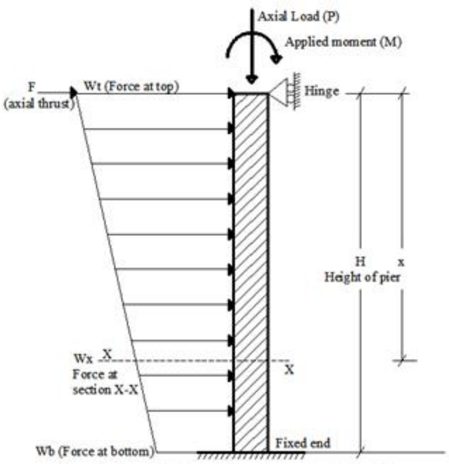

Figure 3 Trapezoidal force acting on complete height of pier

$$W_x = W_T - \left[\frac{\left(W_T - W_B\right)x}{H}\right]$$

(6)

1. First order analysis of pier:-

Let 'M_x' be the bending moment at a general section '*XX*' at a distance 'x' from top of pier,

$$\therefore M_x = -M - Fx + R_T - \frac{W_T x^2}{2} + \frac{(W_T - W_B)x^3}{6H}$$

(7)

$$R_T = \frac{3M}{2H} + F + \frac{11W_T H}{40} + \frac{W_B H}{10}$$

$$M_x = -M + \frac{3M}{2H}x + \frac{11W_T H}{40}x + \frac{W_B H}{10} - \frac{W_T x^2}{2} + \frac{(W_T - W_B)x^3}{6H}$$

(8)

2. Second order analysis of pier:-

Considering the same values used in first order analysis as given above: Substituting constant k_w in equation (6)

$$k_w = \frac{\left(W_T - W_B\right)}{H}$$

$$W_x = W_T - k_w x$$

Bending moment at a general section 'x' is given by

$$M_x = Py - M_A + (R_T - F)x - \frac{W_T x^2}{2} + \frac{k_w x^3}{6}$$

(9)

$$y = \textit{complementary solution} + \textit{particular}$$

$$y_c = A\sin(\alpha x) + B\cos(\alpha x)$$

$$\backslash y_p = -\frac{k_w}{6P}x^3 + \frac{W_T}{2P}x^2 + \frac{x}{P}\left[F - R_T + \frac{k_w}{\alpha^2}\right] + \frac{1}{P}\left[M_A - \frac{W_T}{\alpha^2}\right]$$

Complete solution,

$$y = A\sin(\alpha x) + B\cos(\alpha x)$$

$$-\frac{k_w}{6P}x^3 + \frac{W_T}{2P}x^2 + \frac{x}{P}\left[F - R_T + \frac{k_w}{\alpha^2}\right] + \frac{1}{P}\left[M_A - \frac{W_T}{\alpha^2}\right]$$

(10)

On substituting the boundary condition, $x=0, y=0$

In equation (10) we get,

$$B = -\frac{1}{P}\left[M_A - \frac{W_T}{\alpha^2}\right]$$

On substituting boundary condition, $x=H, y=0$ and $x=H$

$$\frac{\partial y}{\partial x} = 0$$

In equation (10) we get,

$$A = \frac{1}{\sin(\alpha H)}\left[\frac{k_w}{6P}H^3 - B\cos(\alpha H) - \frac{W_T}{2P}H^2 - \frac{H}{P}\left[F - R_T + \frac{k_w}{\alpha^2}\right] - \frac{1}{P}\left[M_A - \frac{W_T}{\alpha^2}\right]\right]$$

(11)

On substituting the values of constants in the deflection equation (10)

$$R_T = \frac{1}{\frac{1}{P}\left[\frac{\tan(\alpha H)}{\alpha} - H\right]} \times$$

$$\left[\frac{W_T}{P}H\left\{\frac{\tan(\alpha H)}{\alpha} - \frac{H}{2}\right\} - B\left\{\sin(\alpha H)\tan(\alpha H) + \cos(\alpha H)\right\} - \frac{k_w H}{P\alpha}\left\{\frac{H\tan(\alpha H)}{2} + \frac{1}{\alpha}\right\} + \frac{\tan(\alpha H)}{\alpha P}\left\{F + \frac{k_w}{\alpha^2}\right\} + \frac{k_w}{6P}H^3 + \frac{W_T}{P\alpha^2} - \frac{HF}{P} - \frac{M}{P}\right]$$

(12)

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B Trapezoidal load at a general height of the pier:

This analysis is done in two parts:

i. For axial compressive load and lateral forces

ii. For axial compressive load and bending moment (applied moment) at top of pier.

1. For axial compressive load and lateral forces:

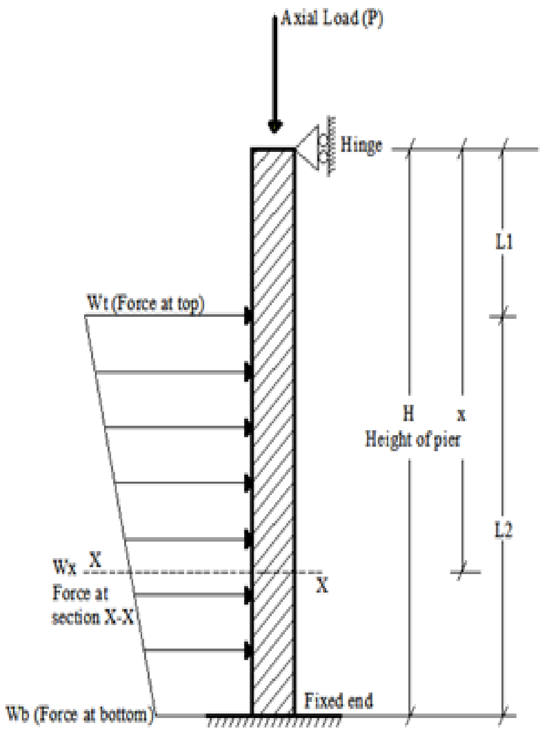

Figure 4 Trapezoidal force acting on partial height of pier

$$W_x = W_B + \frac{\left(W_T - W_B\right)\left(L-x\right)}{\left(L-l\right)}$$

Second order analysis for axial load and trapezoidal lateral load by using Beam-Column Analysis:

$$M_x = Py - M_B + \left(W_x - R_{T1}\right) x \qquad (13)$$

We have, $$EI_x \frac{\partial^2 y}{\partial x^2} = -M_x$$

$$EI_x \frac{\partial^2 x}{\partial y^2} = -R_{T1} - Py \qquad (14)$$

$$y = \textit{complementary solution} + \textit{particular solution}$$

$$y = y_c + y_p$$

$$y_c = A\cos(\alpha x) + B\sin(\alpha x)$$

$$y_p = ax + b$$

On solving the above equation (14) for constants we get

$$y = A\cos\left(\alpha x\right) + B\sin\left(\alpha x\right) - \frac{R_{T1}}{P} x \qquad (15)$$

$$y = C\cos\left(\alpha x\right) + D\sin\left(\alpha x\right) + \frac{M_B}{P} + \frac{x}{P}\left(R_{T1} - W_x\right) \qquad (16)$$

On substituting the boundary condition,

$$x=0, y=0 \ \& \ x = 0, \frac{\partial y}{\partial x} = 0$$

In equation (16), we get

$$D = -\frac{M_B}{P}$$

$$C = -\frac{1}{P\alpha}\left(R_{T1} - W_x\right)$$

$$y = \frac{1}{P}\left(R_{T1} - W_x\right)\left\{x - \frac{\sin(\alpha x)}{\alpha}\right\} + \frac{M_B}{P}\left\{1 - \cos(\alpha x)\right\} \qquad (17)$$

On substituting the boundary condition, $x=0, y=0$

In equation (15), we get

$$B=0$$

$$y = A\sin(\alpha x) - \frac{R_{T1}}{P} x$$

(18) At

$x=a$,

Deflection and slope of the column remains same on both side

Comparing equation (17) and (18)

$$A\sin(\alpha a) - \frac{R_{T1}}{P} a = \frac{1}{P}\left(R_{T1} - W_x\right)\left\{X - \frac{\sin(\alpha X)}{\alpha}\right\} + \frac{M_B}{P}\left\{1 - \cos(\alpha X)\right\}$$

On solving above equation we get

$$A\sin(\alpha a) - \frac{R_{T1}}{P}\left\{a + \left[X - \frac{\sin(\alpha X)}{\alpha}\right] - H\left[1 - \cos(\alpha X)\right]\right\} = \frac{W_x}{P}\left\{X\left[1 - \cos(\alpha X)\right] - \left[X - \frac{\sin(\alpha X)}{\alpha}\right]\right\} \qquad (19)$$

Now comparing slopes of equations (17) and (18)

$$A\alpha\cos(\alpha a) - \frac{R_{T1}}{P} = C\alpha\cos(\alpha X) - D\alpha\sin(\alpha X) + \frac{1}{P}\left(R_{T1} - W_x\right)$$

On solving above equation we get

$$A\alpha\cos(\alpha a) - \frac{R_{T1}}{P}\left[2 - \cos(\alpha X) - H\alpha\sin(\alpha X)\right] = \frac{W_x}{P}\left\{\cos(\alpha X) + X\alpha\sin(\alpha X) - 1\right\} \qquad (20)$$

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Solving equation (19) and (20) simultaneously we get value of ‘ R_{T1} ’

$$R_{T1} = \frac{\left[W_x \left\{ \left(\frac{\alpha}{\tan(\alpha a)} \left(X \left(1-\cos(\alpha X) - \left(X - \frac{\sin(\alpha X)}{\alpha} \right) \right) \right) - \left(\cos(\alpha X) + \alpha X \sin(\alpha X) - 1 \right) \right) \right\} \right]}{\left[\left\{ 2-\cos(\alpha X) - H\alpha \sin(\alpha X) \right\} - \frac{\alpha}{\tan(\alpha a)} \left\{ a + \left(X - \frac{\sin(\alpha X)}{\alpha} \right) - H \left(1-\cos(\alpha X) \right) \right\} \right]}$$

(21)

Since, the resultant force ‘ W_x ’ is varying along the column from l_1 to l_2. on integrating R_{T1} along the length we can get value of reactions and substituting this value we can calculate other constants of integration a, c, and d respectively. and substituting the all values the deflections at any general point can be calculated.

2. For axial compressive load and bending moment:

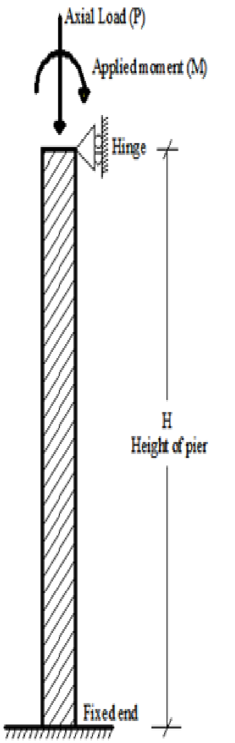

Figure 5 Axial load and bending moment acting on pier

At a general section ‘*XX*’ at a distance ‘*x*’ from top, bending moment is given by

$$M_x = Py - M_A + R_{T2}x$$

$$y = y_c + y_p$$

$$y_c = A\cos(\alpha x) + B\sin(\alpha x)$$

$$y_p = -\frac{R_{T2}}{P}x + \frac{M_A}{P}$$

$$y = A\sin(\alpha x) + B\cos(\alpha x) - \frac{R_{T2}}{P}x + \frac{M_A}{P}$$

(22)

On substituting the boundary condition, $x=0, y=0$

In equation (22), we get

$$B = -\frac{M_A}{P}$$

On substituting the boundary condition, $x=H, y=0$

In equation (22), we get

$$A\sin(\alpha H) - \frac{R_{T2}}{P}H = -\frac{M_A}{P}(1-\cos(\alpha H))$$

(23)

On substituting the boundary condition, $x=H \quad \frac{\partial y}{\partial x} = 0$

In equation (22), we get

$$A\alpha\cos(\alpha H) - \frac{R_{T2}}{P} = -\frac{M_A}{P}\alpha\sin(\alpha H)$$

(24)

On solving above equations (23) and (24) simultaneously we get

$$R_{T2} = \frac{\left[\frac{M_A}{P} \alpha \left\{ \sin(\alpha H) - \frac{1}{\tan(\alpha H)} \left(1-\cos(\alpha H) \right) \right\} \right]}{\left[1 - \frac{\alpha H}{\tan(\alpha H)} \right]} \quad (25)$$

From above equations (21) and (25)

We get total reaction ‘ R_T ’ for the column.

C. Validation for the bending moment equation:

At, x = 0, hinged support

x = H, fixed support

For solid pier ***d*** = 3.0 m

For hollow pier, External diameter = 3.0 m,

Internal diameter = 2.4 m.

Span of bridge = 30m.

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The values for base moment obtained by theoretically and by computer application (STAAD) are compared.

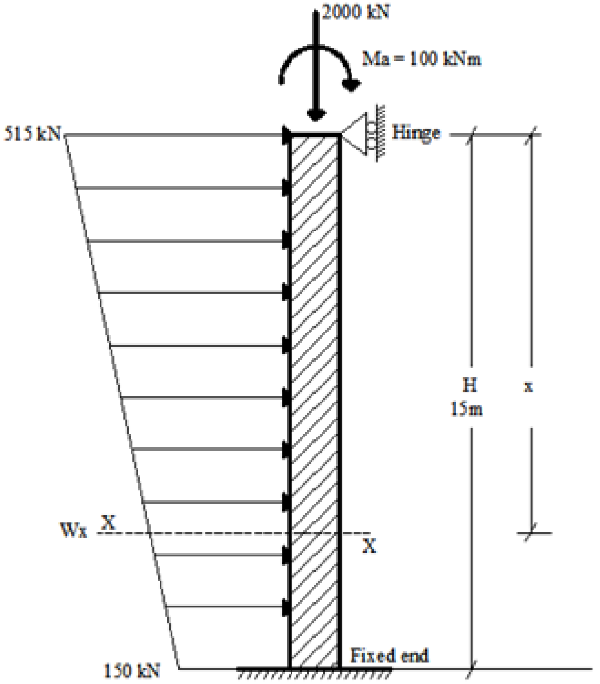


Figure 6 Trapezoidal force acting on complete height of pier

V. PARAMETRIC STUDY

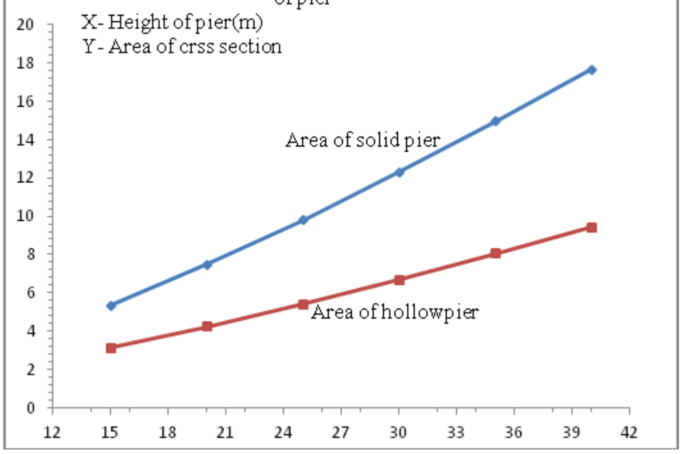
Forces on pier are calculated as specified in IRC and the maximum moment is calculated in ***Table 1*** shown below. Using combined stress equation and keeping the stress constant, behavior of a solid circular and hollow circular section is studied. The percentage saving in area of cross section when hollow circular section is used in place of solid circular section is plotted for different heights of pier. The variation in area of cross section and moment of inertia for different bending moments is studied.

A. Variation of area of cross section for different heights of pier:

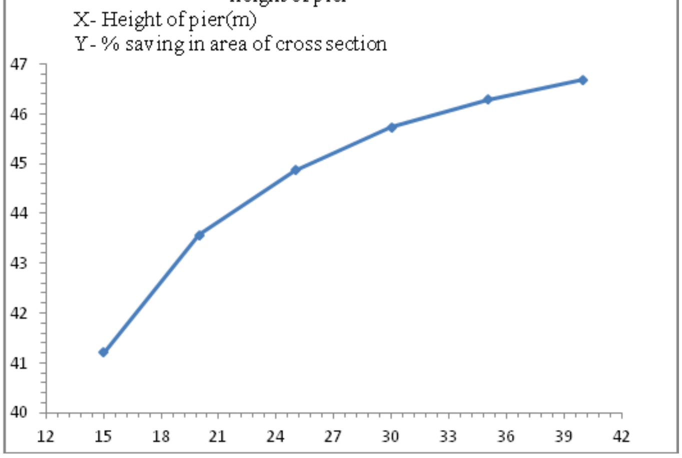
Table I : Indicated percentage saving in area of cross section of pier. (Height ranging from 15m to 40m)

Graph 1: Variation in area of cross section for both cases increases as the height of pier increases. The rate of increase is more in solid section as compared to hollow section. The percentage difference in both cases goes on increasing as the height of the pier increases. The variation for solid as well as hollow section tends to form an inverted parabolic profile.

Graph 2: The percentage saving in area of cross section forms a parabolic profile i.e. it increases with the increase in height. The rate of increase in percentage saving of area is steeper up to 30m height but becomes milder thereafter. The percentage saving in area tends to remain constant after 40m height.



Graph1. Variation of area of cross section for different height of pier



Graph 2. Percentage saving in area of cross section for different height of pier

B. Variation of M.I. for different height of pier:

Table II : Indicates percentage reduction M.I. for a pier ranging from 15m to 40m for critical bending moment (shown below)

Graph 3: Variation in M.I of cross section in both cases increases as the height increases. The rate of increase is steeper in hollow section as compared to solid section. The percentage difference in both cases goes on increasing as the height of the pier increases. The variation profile for solid as well as hollow section tends to form an inverted parabolic profile.

Graph 4: The percentage variation in M.I of cross section forms a parabolic profile i.e. it increases as there in an increase in height. The rate of increase in percentage variation in M.I is fast up to 30m height but after that it goes on reducing. The percentage variation in M.I tends to remain constant after 40m height.

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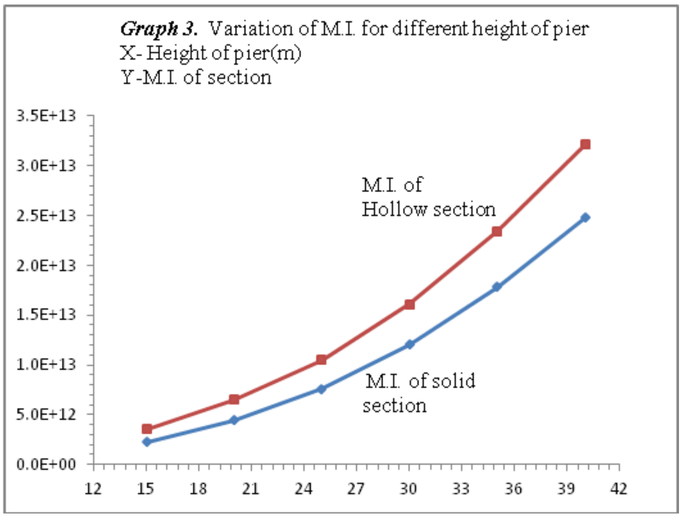

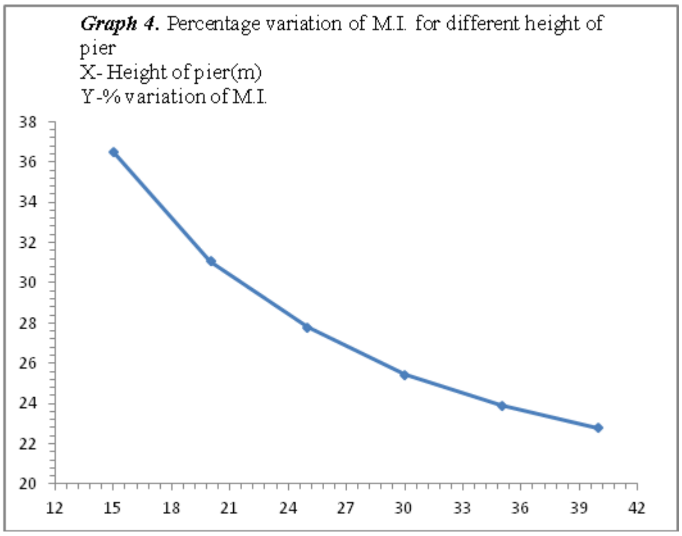

Table I : Variation of area of cross section for different height of pier

Height (*m*)	M max (*KN m*)	D *solid* (*mm*)	D *external* (*mm*)	d *internal* (*mm*)	A *solid* (*sq m*)	A *hollow* (*sq m*)	% saving in area of c/s
15	6823.125	2605	3329	2663	5.33	3.13	41.21
20	12238.36	3087	3865	3092	7.48	4.22	43.57
25	19027.08	3528	4366	3493	9.77	5.39	44.87
30	27536.75	3956	4857	3886	12.29	6.67	45.73
35	37607.71	4363	5329	4263	14.94	8.03	46.29
40	48917.91	4742	5771	4617	17.65	9.41	46.68

Table II: Variation of M.I. for different height of pier

		Solid pier	Hollow pier				
Height (*m*)	Moment (*KN m*)	D *solid* (*mm*)	D *external* (mm)	d *internal* (mm)	M.I. solid	M.I. hollow	% variation in M.I
15	6823.125	2605	3329	2663	2.26E+12	3.56E+12	36.51
20	12238.36	3087	3865	3092	4.46E+12	6.47E+12	31.07
25	19027.08	3528	4366	3493	7.61E+12	1.05E+13	27.77
30	27536.75	3956	4857	3886	1.20E+13	1.61E+13	25.44
35	37607.71	4363	5329	4263	1.78E+13	2.34E+13	23.91
40	48917.91	4742	5771	4617	2.48E+13	3.21E+13	22.78

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VI. ANALYSIS AND RESULT

i. The critical B.M occurs at the base of the pier i.e. base moment and critical B.M. is same.
ii. The difference between moments by first order analysis and second order analysis increases as the height of pier increases.
iii. As the height of pier goes on increasing the percentage saving in area of cross section of pier tends to form a straight line i.e. it becomes milder.

VII. CONCLUSION

i. As the height of the pier increases the B.M. value also increases. The critical B.M occurs at the base of the pier i.e. base moment and critical B.M. is same.
ii. The rate of increase in percentage saving of cross sectional area is fast for small B.M. values i.e. height of pier up to 25m and above this value it becomes milder.
iii. Percentage saving in area of cross section is more for slender pier up to height 30m and after that it goes on decreasing as the height of pier increases. After 35m the percentage saving in area is not more than 1%.
iv. The difference in area of solid section and hollow section goes on increasing as the height of pier increases. The rate of increase in area of solid section is faster than the increase in area of hollow section.
v. Percentage variation of M.I of section forms a parabolic profile in which, the rate of increase is more up to 30m and goes on decreasing as the height of pier increases.
vi. The M.I of the section increases as there is an increase in height but the rate of increase is more for solid section than the hollow section.
vii. As the height of pier increases the hollow circular section proves to be economical as compared to solid circular section of pier.

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