

Modeling Uncertainty in Risk Assessment Using Double Monte Carlo Method

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Abstract- The main aim of risk assessment is to determine the potential detriment to human health from exposure to a substance or activity that under plausible circumstances can cause to human health. Risk assessment models involve inputs which may not be precisely known. The uncertainty of the inputs gets propagated to the output risk. So we need to quantify the uncertainty so as to be aware of the risk involved in any decision making process. Uncertainties can be modeled and analyzed using different theories, viz. Probability theory, Possibility theory, Evidence theory etc. Modeling of an uncertain parameter depends on the nature of the information available. In this paper we have considered uncertainty quantification of parameters in the case of radiological risk assessment. We have analyzed the propagation of the risk both in terms of probability and possibility theory. An advanced method of probabilistic risk assessment (PRAs) viz. Double Monte Carlo method is discussed in this paper. A case study is carried out with this method and compared with the results taking the parameters of the input distribution of the model as Fuzzy number.

Key Words- Double Monte Carlo, Fuzzy number, Uncertainty, Variability.

I. INTRODUCTION

Uncertainty analysis is a systematic study in which “a neighborhood of alternative assumptions is selected and the corresponding interval of inferences is identified” [5]. Uncertainty plays a critical role in the analysis for a wide and diverse set in various fields. Ideals and concepts of uncertainty have long been associated with gambling and games. The Greek in the 4th century BC were the first recorded civilization to have considered uncertainty. There are two kinds of uncertainty. One kind arises as variability (or Aleatory uncertainty) resulting from inherent variability, natural stochasticity, environmental or structural variation across space or through time, manufacturing or genetic heterogeneity among components or individuals, and variety of other sources of randomness. The standard representation of variability is the probability distribution function. Another kind is called Epistemic Uncertainty. This is defined as uncertainty which arises from incompleteness of knowledge about the world. Sources of epistemic Uncertainty include measurement uncertainty, small sample size, detection limits and data censoring, ignorance about the details of the physical mechanisms and processes involved and other imperfection in scientific understanding.

These two kinds of Uncertainty can propagate through various mathematical expressions with different

calculation methods. Probability Bounds Analysis (PBA) is related to one of these methods. It is a combination of probability theory and Interval Analysis. Probability Theory is used to propagate Aleatory Uncertainty (or variability) and Interval Analysis is used to propagate Epistemic Uncertainty (or Uncertainty). Probabilistic approaches characterize the uncertainty in the parameter by a probability distribution.

If the input variables of the risk assessment model consist of variability and uncertainties, two interpretations are generally proposed for the distribution of the input variable. First, Uncertainty regarding variability may be viewed in terms of probability regarding frequencies. Second variability is described by frequency distributions, and that uncertainty in general, including sampling error, measurement error, and estimates based upon judgment, is described by probability distribution. The most widely used method in PRA is Monte Carlo Analysis (MCA), which is a means of quantifying uncertainty or variability in a probabilistic framework using computer simulation. One of the advanced modeling approaches that may be used to conduct PRA studies is Two-dimensional Monte Carlo analysis (2D MCA). A 2D MCA is a term used to describe a model that simulates both uncertainty and variability in one and more input variables. The uncertainty is characterized by a p-box in the case of the

Double Monte Carlo method.

Another method to propagate the uncertainty is possibility (Fuzzy) method. Fuzzy set Theory (Zadeh, 1965) provides a methodology for handling uncertainty in the absence of complete and precise data. A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the Fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the Fuzzy set. Thus, individuals may belong to the Fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. These membership grades are very often represented by real-number values ranging in the closed interval between 0 and 1.

Exposure Pathways:

Radiological Risk means, risk associated with the release of radio nuclides when radioactive materials are released into the environment. There are various pathways through which radio nuclides can reach human being namely inhalation, ingestion through drinking water and through contaminate food. Radioactive materials can flow

accidentally or intentionally. Accidental releases have occurred many times at commercial nuclear power plants and nuclear waste disposal sites. When radioactive materials are released in to the environment, radio nuclides will move into the body by inhalation and ingestion, which can cause internal exposure.

An exposure pathway is any route that a chemical may travel from an environmental source to a receptor. An exposure pathway has five main parts: [13]

- A chemical source
- A release mechanism(e.g., leaking, leaching, wind erosion)
- A transport and/or exposure medium(e.g., air, water, soil, sediment, food),
- An exposure point with receptors present or potentially present(actual location where exposure is possible),and
- A route of entry (inhalation, ingestion, dermal contact).

In this case study we used the ingestion path way only.

II. DOUBLE MONTE CARLO METHOD

Monte Carlo simulation (MCS) is one of several techniques currently employed to carry out risk assessments. In the 1940s MCS, originated at Los Alamos from the work of Ulam, von Neumann and Fermi, as a random sampling technique for solving difficult deterministic equations. An overview of the history of Monte Carlo Simulation is given by Rugen and Callahan (1996). Since then Monte Carlo methods have continued to evolve and due to advances in computing they can now be used in many applications. Double Monte Carlo Method or Two-dimensional Monte Carlo simulation (2D MCS) is used in a wide range of applications including human health risk assessment, avian risk assessment, environmental flood risk assessment, and microbial risk assessment. Two-dimensional Monte Carlo simulation (2D MCS) or Two-phase Monte Carlo simulation [11] is an extension of Monte Carlo simulation. In 2D MCS there are two loops (as opposed to just one in MCS) allowing variability and uncertainty to be modeled separately. The variability is modeled in the inner loop and the uncertainty in the outer loop.

2D MCS can produce bounds on the output of a particular model at any credible level and it takes into account parameter uncertainty for each random quantity in the model. One major advantage of Monte Carlo method is that it can be used as a sensitivity analysis by making adjustments to the model and then comparing the results from each adjustment to see the effect of changes. Model uncertainty can also be included by setting up different models and comparing or enveloping the results from each of them. Also, 2D MCS can be implemented with copulas to take account of any known correlations between the random quantities in the model. In

environmental risk assessment problems, a two-phase simulation approach was adopted in considering epistemic and aleatory uncertainty. Treatment of epistemic and aleatory uncertainties in the simulation approaches is carried out by sampling epistemic variables in the outer loop and aleatory variables in the inner loop. For a problem of second order random variable, the epistemic uncertainty in the parameters of the distribution is sampled first and later the uncertainty in the distribution is propagated. The procedure for carrying out the two-phase Monte Carlo is explained below (See fig.1 in appendix).

1. Information regarding PDF of elements in the model and the uncertainty in the parameters of PDF is obtained.

2. Distributions for PDF parameters of components are first sampled by any sampling approach. This action takes place in the first loop of a two-loop sampling, as shown in fig.1. The first loop or outer loop focuses on epistemic uncertainty, and the second loop of inner loop focuses on aleatory uncertainty.

3. Epistemic variables are treated as constant inside the second loop, that is, the sampled values from step 2 are passed on the second loop. Now in the second loop, simulation is carried out by sampling aleatory variables.

4. Step 3 is repeated for sufficient number of iterations and required measures of uncertainty are obtained from the simulation results. This is one time execution of the inner loop and the uncertainty from randomness is obtained in this step.

5. Check for the number of times the first loop has to be executed. If it is less than the predetermined numbers of iterations, then go to the step 2, in which sampling is done again for epistemic parameters and subsequently enter the second loop.

6. After sufficient number of iteration of the outer loop, the summarized results look like a family of curves. Each cumulative probability curves of these denotes the uncertainty due to randomness, whereas the spread is due to epistemic uncertainty in the parameters of PDFs.

III. A CASE STUDY ON RADIOLOGICAL RISK OF THE RADIONUCLIDE CS-137

We have considered a case of Radiological Risk due to the radionuclide Cesium-137 (Cs-137) through the pathways of ingestion. The uncertain parameters of the risk model are food intake, food activity, and water intake and water activity. For this case study, we considered some hypothetical data which are available in terms of Minimum, Most likely and maximum values as shown in Table 1 below (see appendix). Using this data, we calculated the radiological risk for the radionuclide Cs-137 by two approaches, viz., Fuzzy set and 2DMCS.

IV. RISK CALCULATION USING FUZZY SET METHOD

Data including intake of food items, activity of food items, water intake, water activity, risk factor (for the radionuclide Cs-137) are given in table 1. Here we take the most likely value as core and the min and max value is taking as end points of the support of the triangular Fuzzy number (TFN). Here we used the Risk Calc Software for calculation.

A. RISK DUE TO INGESTION OF CONTAMINATED FOOD

$$\text{Risk (/Yr)} = \text{Activity On Food Items (Bq/Kg)} \times \text{Intake Food(Kg/Yr)} \times \text{Risk Factor(/Bq)}$$

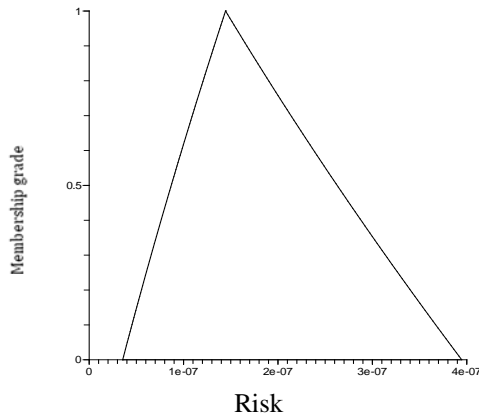


Fig. 2 Risk due to ingestion of contaminated food

The resultant Fuzzy number gives the core value as 1.444559e-07 and the support of the Fuzzy number is [3.549942e-08, 3.945852e-07].

B. RISK DUE TO INGESTION OF WATER

$$\text{Risk (/yr)} = \text{Water Intake (L/Yr)} \times \text{Water Activity (Bq/L)} \times \text{Risk Factor (/ Bq)}$$

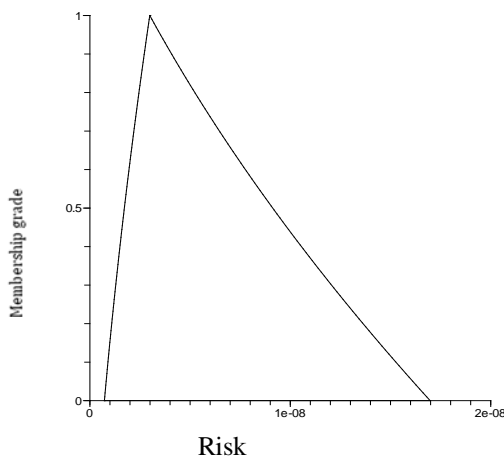


Fig.3 Risk due to ingestion of water

The resultant Fuzzy number gives the core value as 2.988479e-09 and the support of the Fuzzy number is [7.2448e-10, 1.698e-08].

C. TOTAL RISK OF INGESTION FOR THE RADIONUCLIDE CS-137

$$= \text{Risk from food ingestion} + \text{Risk from water}$$

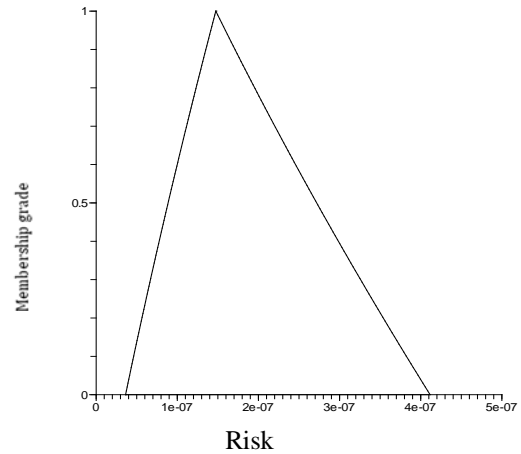


Fig. 4 Total risk for the radionuclide Cs-137

The resultant Fuzzy number gives the core value as 1.474444e-07 and the support of the Fuzzy number is [3.622390399e-08, 4.115652e-07].

V. RISK CALCULATION USING DOUBLE MONTE CARLO METHOD

Data including intake of food items, activity of food items, water intake, water activity, risk factor (for the radionuclide Cs-137) are given in table 1. To incorporate the uncertainty involved in the determination of the most likely value we have considered an interval around it. For our problem we have considered a 90% confidence interval. In this approach the inputs of the risk equation along with the parameters of these variables are described in terms of probability density functions (PDFs). A variable described in this way is called a “Second order random variable.” For this method, we represent each uncertain input as a triangular distribution with support [min, max] where the mode is considered as uniform distribution over the 90% confidence interval of the most likely value. The confidence interval is obtained from the TFN by using á(alpha)-cut method. Table 2 gives (shown in appendix below) the 90% confidence interval of mode of different activities. Here, we use the Ramas Risk Calc. Software for calculation.

A. RISK DUE TO INGESTION OF CONTAMINATED FOOD

$$\text{Risk (/Yr)} = \text{Activity on food items (Bq/Kg)} \times \text{Intake food (Kg/Yr)} \times \text{Risk factor (/Bq)}$$

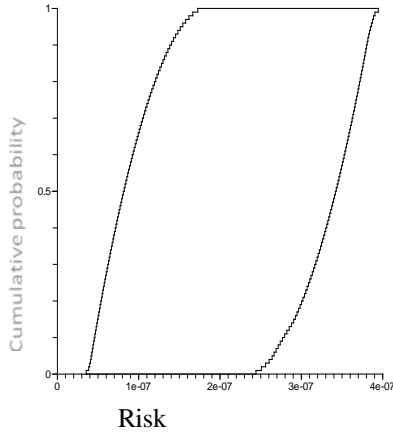


Fig.5 Risk due to ingestion of contaminated food

The resultant P-box gives range = [3.54994e-08, 3.94585e-07], mean = [0.000000135, 0.000000253], and variance = [0, 1.8038695436e-14]

B. RISK DUE TO INGESTION OF WATER:

Risk (/yr) = Water Intake (L/Yr) × Water Activity (Bq/L) × Risk Factor (/ Bq)

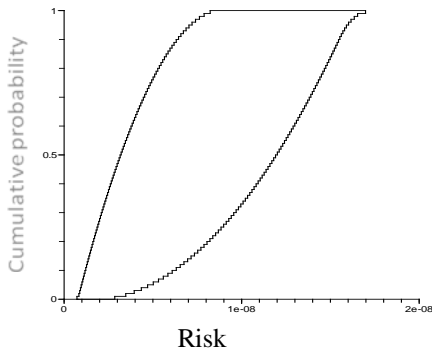


Fig. 6 Risk due to ingestion of water

The resultant P-box gives range = [7.2448e-10, 1.698e-08], mean = [4.2365217e-09, 9.8706131e-09], and variance = [4.4123581e-19, 3.934644e-17]

C. TOTAL RISK FOR THE RADIONUCLIDE CS-137

= Risk from food ingestion + Risk from water

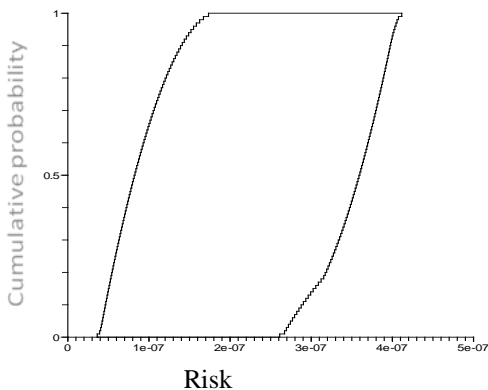


Fig. 7 Total risk for the radionuclide Cs-137

The resultant P-box gives range = [3.62239e-08, 4.11565e-07], mean = [0.00000014, 0.000000263], and variance = [0, 1.9762985137e-14].

VI. CONCLUSION AND COMPARISON OF TOTAL RISK USING DOUBLE MONTE CARLO METHOD WITH FUZZY SET METHOD:

The objective of the uncertainty analysis is to quantify uncertainty in model output. Quantification of uncertainty helps in effective uncertainty management and increases confidence in the results. Many methods are available to propagate the uncertainty. Here, we compare risk for the radionuclide Cesium-137 (Cs-137), using the Fuzzy set method and Double Monte Carlo method. The red fig. indicates the total risk using Double Monte Carlo Method. The Fuzzy set is expressed in terms of possibility and necessity measure and it is represented in the figure in blue. Uncertainty is expressed as lower and upper probability. From the figure we observe that uncertainty in risk calculation is more in case of Double Monte Carlo as compared to fuzzy set method. The range of risk is same in both cases. Considering any fractile, we get an interval which gives the range of risk at that fractile. Fuzzy set method shows that risk is in and around 1.474444e-07, while it may spread over [3.622390399e-08, 4.115652e-07] with different possibilities. The Double Monte Carlo Method shows that the most likely range of risk is [0.00000014, 0.000000263]. Bounds of the risk give the uncertainty involved in the method, and we have seen that in double Monte Carlo method there is more uncertainty than the fuzzy set method.

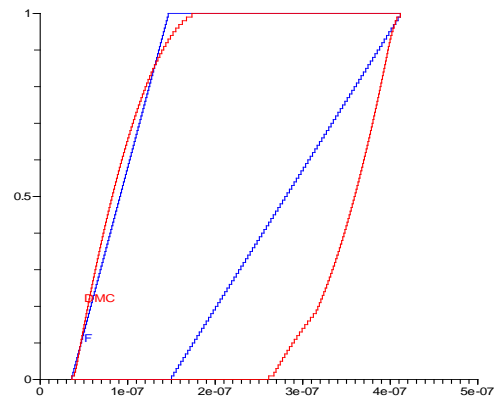


Fig.8 Comparison of total risk Using Double Monte Carlo Method with Fuzzy Set Method

From the above results we see that the range of risk due to ingestion of contaminated food is same in all the method, but the mean which is obtained by triangular Fuzzy Set method is more precise than the other method. From Fig. 8, we observe that the upper probability of risk is almost the same for both the methods. However the same for lower probability is much less in case of Double Monte Carlo method as compared to fuzzy approach. The conclusion regarding risk using Double Monte Carlo method is based on 90% confidence interval of the most likely value. By considering a higher confidence interval we can get a more accurate result.

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