ISSN: 2277-3754
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 1, Issue 3, March 2012

A Solution to the Inverse Problem of Impact Force Determination from Structural Responses

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***Abstract*— Problem of determining the force acting on a structure from measurements of the response of the structure due to this force is an inverse problem. In most practical situations, it is difficult to perform direct measurements of the external forces acting on an existing structure. In such cases, the structure response measured using sensors can be used for determining the position and magnitude of the exciting forces. Unfortunately, the results from the inverse process are often highly sensitive to noise in the measurements of response and errors in the model of the structure leading to ill conditioning. The forces obtained by such inverse process are prone to errors. Ill conditioning of the frequency response function (FRF) matrix causes measurement errors to be magnified significantly and makes its inversion difficult. This paper presents the identification of impact force history acting on a cantilever beam. The acceleration response is used as input for prediction. The force history prediction algorithm is developed in both time and frequency domains to determine the impact force amplitude. In the time domain the beam response due to the impact load is first predicted. The sum of mean square errors between the predicted and measured response is then defined as the objective function. The optimization problem is thereby constructed and is then solved for the amplitude of the impact force. The accelerance method is used in the frequency domain.. A Cantilever beam is introduced to experimentally verify the force predictive model. Results show that the predictive model is feasible and applicable to arbitrary structures.**

Index Terms— **Force identification, Frequency response function, Inverse problem, Least square**

I. INTRODUCTION

The determination of the location and magnitude of self-generated or input forces on an arbitrary structure may prove to be very important for the proper evaluation at the design and modification phases as well as in the case of control and fatigue life predictions. The location of the input excitation forces can reveal the possible causes of vibration, while the force amplitude determines the severity of the vibration condition. The identification of the input forces has also attracted a great deal of interest in machine health monitoring and trouble shooting. The impact force identification has long been an important issue for both researchers and engineers because in many practical applications, such as cutting forces of machine tools, reaction forces of engine mounts and supporting forces of bearings[1] etc., it is difficult and sometimes impossible to directly measure the dynamic forces that are acting on a vibrating structure hence, it could be beneficial to compute the time history and the location of the applied forces indirectly, using structural response measurement together with a dynamic model of the structure. The common problems encountered during inverse force determination are discussed below.

The solutions of the inverse problems lack in the existence, the uniqueness or the stability. Such inverse problems are called ill-posed. If a solution for a posed problem does not have uniqueness, it is not possible to obtain a reliable solution without additional information, since the solution can be false. Even when the existence and the uniqueness are guaranteed, most inverse problems suffer from the lack of stability of solution. To overcome the difficulty of the loss of the stability, inverse analysis schemes incorporating regularization are applied. The method, which transforms an ill-posed problem to the form of a well-posed problem by providing additional information about the sought solution, is called the regularization process.

The inverse problem of force determination has interested many researchers. Steven [2] has given an excellent survey of the literature on the force identification. Also Chan *et al* [3] and Zhu and Law [4] have presented a theoretical background of various moving force identification methods. Gombi and Ramakrishna [5],[6] have developed force identification algorithms for the determination of harmonic force on cantilever and simply supported beams. Wang *et al* [7] and Thite and Thompson [8] developed a prediction algorithm for unknown impact and harmonic forces. These models could estimate force amplitude and its location simultaneously, but it is time consuming and sensitive to location of measurement. Zhu and Lu [9] presented a time domain method to identify both concentrated and distributed loads on beam and plate structures. Doyle [10],[11] presented a series of papers to determine the impact force for beam and plate types of structures subjected to transverse impacts both in time and frequency domains. Recently Bennani *et al* presented transfer function based method to identify the location of an impact force on a circular plate [12],[13].

In this paper, the identification of impact force acting on a cantilever beam is addressed. It is assumed that the acceleration response of the structure to the impact force at various combinations of selected locations is known, and the magnitude of the unknown impact force is sought for. The impact force has been predicted in both time and frequency domains by formulating the problem as an optimization problem using the least square approach that is to minimize the errors between the predicted and measured responses.

II. IMPACT FORCE DETERMINATION TECHNIQUES

When forces on a structure cannot be measured directly, it is possible in principle to estimate these forces from vibration responses (such as strain, displacements or accelerations) measured from a remotely located vibration sensor. The response of a structure to an impact force can often be considered to be linearly dependent on the impact force. This is the case, for example, when the structure can be considered

IJEIT
ISSN: 2277-3754
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 1, Issue 3, March 2012

to be linearly elastic during the impact process and when the deformation of the structure can be considered to be small enough to neglect geometric nonlinearity. In such cases the response a(t) at a point of the structure can be related to the impact force f(t) by a linear convolution integral as

$$a(t) = \int_{-\infty}^{\infty} f(t-\tau)h(\tau)d\tau \qquad (1)$$

Where h(t) is the impulse response function of the linear system and it is assumed that f(t)=h(t)= a(t)=0 for t<0.

The basic scheme for deconvolution is to discretize the integral Equation (1) in to algebraic equations in-the time domain as

$$a(t) = f(t) * h(t) \qquad (2)$$

Where '*' symbol indicates a mathematical convolution. The convolution symbol is a short hand way of writing the convolution integral.

In the frequency domain, the companion relation to Equation (1) can be obtained by taking the Fourier transforms of the variables, i.e.

$$\{A_j(w)\} = \{F_i(w)\} * [H_{ij}(w)] \qquad (3)$$

Where

$$\{A_j(w)\} \equiv [A_1(w), A_2(w), \ldots\ldots, A_n(w)]^T \qquad (4)$$

$$\{F_i(w)\} \equiv [F_1(w), F_2(w), \ldots\ldots, F_n(w)]^T \qquad (5)$$

$$[H_{ij}(w)] = \begin{bmatrix} H_{11}(w) & H_{12}(w) & . & H_{1m}(w) \\ H_{21}(w) & H_{22}(w) & \cdot & H_{2m}(w) \\ . & . & & . \\ H_{n1}(w) & H_{n1}(w) & \cdot & H_{nm}(w) \end{bmatrix} \qquad (6)$$

The functions $A_j(\omega), F_i(\omega), H_{ij}(\omega)$ denote the Fourier transform of the $a_j(t)$, f_i (t) and $h_{ij}(t)$, respectively, and the super script T indicates the transpose of the matrix. The ω is the radian frequency. Hence convolution in time domain is transformed in to multiplication in the frequency domain.

The only measurable quantity in the force identification problems is a (t) or A (ω). This vibration signal is affected by both the input force and the transfer function of the system. The fact that the input force f (t) and the transfer function h (t) are inextricably woven together i.e. they cannot be separated, creates the main problem. The obvious solution is to measure the transfer function h (t) of the structure ahead of the time. Then the input force can be calculated from:

$$F(w) = A(w)/H(w) \qquad (7)$$

The time history f (t) can be determined from inverse Fourier transform of F (ω). But in reality, the practical application of Equation (7) is not nearly as neat and clean as the mathematics suggest. The principle difficulty is with the way that noise or errors affect the calculation of the forces. Structures and also their transfer functions are usually resonant in nature. The magnitude of the frequency domain FRF function I H (ω) I consists of a number of resonant peaks separated by valleys or anti resonance regions as shown in Fig.1. The magnitude of I H (ω) I in this anti–resonance frequency range is very small and experimentally measured values of I H (ω) I in these frequency ranges can be heavily corrupted by noise. The same is true for the measured response signal A (ω). When F (ω) is calculated from Equation (7), it can be so distorted by noise that the time domain force waveform f (t) calculated from it will not be accurate at all.

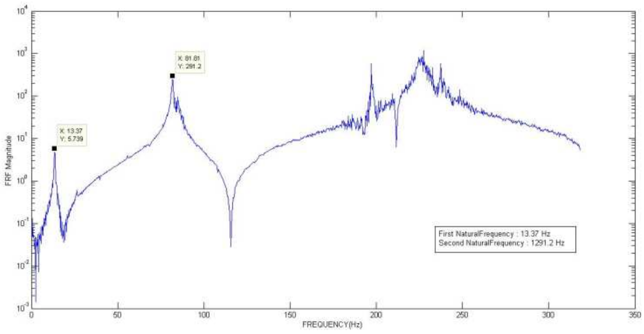

Fig.1. The magnitude of the frequency domain FRF function I H (ω) I

III. HANDLING THE ILL-POSEDNESS OF THE INVERSE PROBLEM AND REGULARIZATION

The inverse problem of solving Equation (3) for {F} may be ill-posed or ill-conditioned in the following sense: The algebraic Equation (3) can be indeterminate or inconsistent depending on the combination of the locations to measure the response, which means that the uniqueness or existence of the solution is not satisfied.

1. In practice the vector {A} and the matrix [H] cannot be free from errors such as errors due to limitations of the measuring instruments, measurement noises, quantization errors associated with digital sampling, and numerical errors of the discrete Fourier transformation. The uniqueness or existence of the solution might be violated due to such errors.
2. If the condition number cond (H) is large enough, the stability of the solution may be poor because of the expansion of errors involved in {A} and [H].

In order to improve the first two of the ill-posedness listed above, we have adopted the least squares method based on the singular value decomposition (SVD) to solve Equation (3).

The SVD of the coefficient matrix [H] is given by:

$$[H] = [U]\,[\Sigma]\,[V]^H \qquad (8)$$

Where [U] and [V] are m x m and n x n unitary matrices respectively and the superscript H indicates the conjugate transpose of the matrix. In addition, [Σ] is an m x n real matrix in the form:

$$[\Sigma] = \begin{bmatrix} \sigma_1 & 0 & \cdot & 0 \\ 0 & \sigma_2 & \cdot & 0 \\ 0 & 0 & \cdot & \sigma_n \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (9)$$

Where σ_i, i=1, 2… n are the singular values of [H] such that $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \ldots\ldots\ldots \geq \sigma_n \geq 0$.

According to the theory of SVD, the least square solution $\bar{F}$ of Equation (3), in the sense that both the Euclidean norms $\| \bar{F} \|_2$ and $\| H\bar{F} - A \|_2$ are minimum, is given by

$$\{\bar{F}\} = [H] + \{A\} \qquad (10)$$

ISSN: 2277-3754
International Journal of Engineering and Innovative Technology (IJEIT)
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Where $[H]^+$ denotes Moore-Penrose generalized inverse.

The least square solution (10) does exist uniquely, which means that the existence and uniqueness of the solution are satisfied by imposing two restrictions:

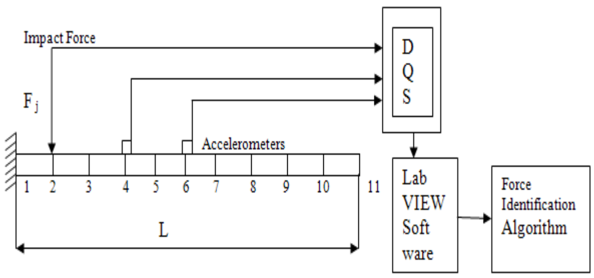
1. $\| \bar{F} \|_2$ is minimum, that is, the energy of the impact force is minimum and,
2. $\| H\bar{F} - A \|_2$ is minimum, that is, the length of the residual vector is minimum.

Next, in order to improve the third ill-conditioned aspect, the method of 'rank decision' is adopted. The condition number of [H] is expressed in terms of the maximum singular value σ_1 and the minimum non-zero singular value σ_r (r is the rank of [H]), as cond (H) = σ_1 / σ_r (11)

IV. EXPERIMENTAL SET-UP

Fig.2 shows the schematic arrangement of the experimental set up. A flat, made of Aluminum with 25mm width and 3mm thickness, having a length of 400mm was fixed at one end with bolts and nuts to ensure the perfect boundary condition. The cantilever beam is divided into 10 equal parts of 40mm each with 11 nodes on it. The accelerometers (PCB Piezotronics make) are mounted on the beam at specified nodes, and the impact is given with an impact hammer (PCB Piezotronics make) with a rubber tip at various other nodes. The acceleration response is collected through a Data Acquisition system (DQS) and Signal Conditioners [NI USB 6229]. The software Lab VIEW 8.6 version is used for further signal processing.

2(a)



2(b)

Fig.2. Schematic arrangement of the experiment

V. RESULTS AND DISCUSSION

Using the theoretical formulation developed in the previous section, a general algorithm is written in MATLAB (Version R2008b) to predict the force history in both time and frequency domain using the measured acceleration response. In order to investigate the practical aspects and accuracy of the present approach in estimating the unknown input forces, experimental case studies are performed first and the required response and force data for various combinations of sensor and impact locations are collected. In the present paper, the influence of the selection of the sensor location and the presence of noise in the measurements, on the quality of the force predictions is shown. Results in both time and frequency domain are compared.

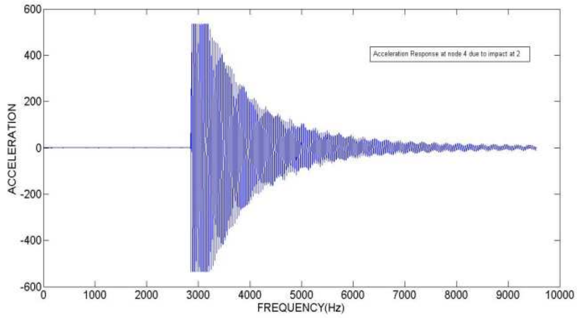


Fig.3. Acceleration response at node 4 due to impact at node 2

In the time domain, using the acceleration response at locations 4 and 6, and impact at node 2, the impulse function is constructed. Now for a new impact at node 2 the responses at 8 and 10 are recorded. Using these responses and adopting the least square error approach, the force history at node 2 is predicted, as shown in Fig.4.

For the same data the force history is reconstructed in the frequency domain, using the accelerance method as shown in Fig.5.The results obtained in the frequency domain method gives more accurate predictions when compared with the measured one.

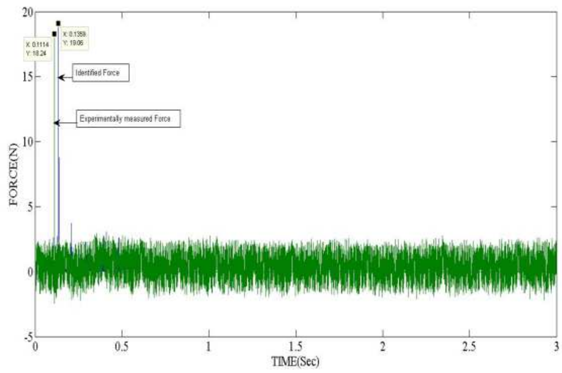


Fig.4. Comparison of Identified force at 2 with the measured force in time domain

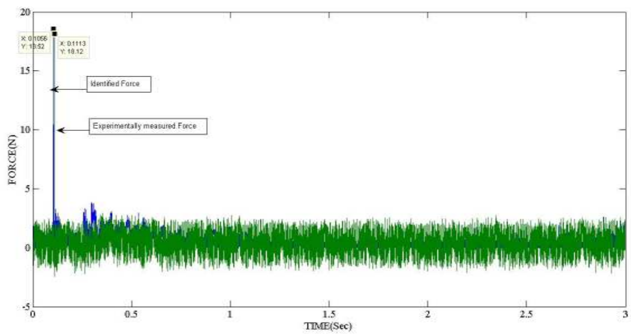


Fig.5. Comparison of Identified force at 2 with the measured Force by accelerence method

ISSN: 2277-3754
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To show the influence of the choice of sensor location and presence of noise in the measurements, first the FRF matrix $[H_{25}\ H_{45}\ H_{65}]^T$ is constructed from the acceleration responses at nodes 2, 4 and 6 with an impact at node 5. The force history is reconstructed at node 5 using the acceleration responses at 6, 8 and 10, as shown in Fig.6. Even though the results obtained are highly oscillatory, the location and magnitude of the reconstructed impact force has been perfectly matching.

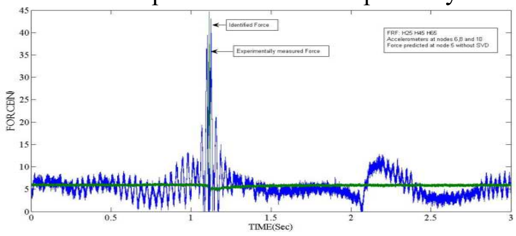

Fig.6. Comparison of Identified force at 5 using FRF $[H_{25}\ H_{45}\ H_{65}]^{T}$ with the measured force.

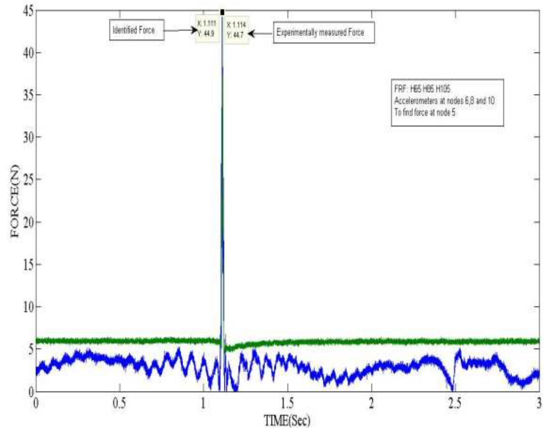

Fig.7 Comparison of Identified force at 5 using FRF $[H_{45}\ H_{65}\ H_{105}]$ T with the measured force

Next, the second set of FRF matrix $[H_{45}\ H_{65}\ H_{105}]^T$ is constructed from the responses at nodes 4, 6 and 10 with an impact at node 5. The force history at node 5 is once again reconstructed from the responses at 6, 8 and 10 as shown in Fig.7. The results show excellent close agreement with the actual measured force history.

It was observed that, using the first set of FRF matrix, the results obtained were highly oscillatory. This could be due to the ill-conditioning of the FRF matrix because of improper placement of accelerometers and also due to noise in the measured data. To reduce this effect, the regularization technique, SVD is applied to this FRF matrix at every frequency, before it is inverted. Fig.8 exhibits excellent improvement in the results, reducing the oscillations considerably and improving the accuracy of location and magnitude.

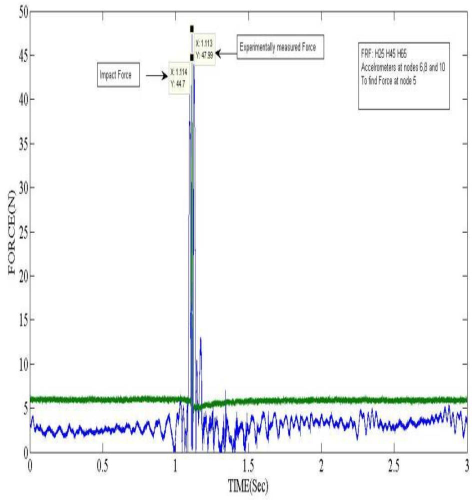

Fig.8. Comparison of Identified force at node 5 using SVD on FRF $[H_{25}\ H_{45}\ H_{65}]$ T with the measured force

VI. Conclusion

This paper proposes a method of inverse analysis of impact force to measure the magnitude and location of the impact force acting on a body of arbitrary shape. With the prior knowledge of the structural information and the location of the impact force when an accelerometer is used as a sensor to measure the structural response due to impact force, the magnitude of the impact force can be determined effectively by the solution of the formulated optimization problem. The developed method was verified in both time and frequency domain through experiments of impact of a cantilever beam. The main limitation of the method studied here seems to be the ill-conditioning property of the FRF matrix during its inversion. This is mainly due to the improper placement of sensors and noise in the measurements. Even though the estimated force in frequency domain is quite good, it can be further improved by using a low pass filter to remove the noise at low frequencies. The application of the regularization technique SVD has improved the predicted results considerably.

Acknowledgment

This research work is carried out under the financial assistance from AICTE, New Delhi under the research project "Inverse force identification in complex structures and machines". (F.No. 8023 /BOR/ RID/RPS-120/2007-08) under RPS scheme. Authors acknowledge the AICTE, New Delhi and JNNCE Shimoga.

References

[1] N. Okubo, S. Tanabe and T.Tatsuno "Identification of forces generated by a machine under operating condition", Proceedings of the 3rd International Modal analysis Conference, 920-927(1997).

[2] Stevens K.K., "Force identification problems-an overview", Proceedings of the 1987 SEM conference on Experimental Mechanics838-844 (1987).

[3] T.H.T. Chan, L. Yu, S.S. Law and T.H. Yung: "Moving force identification studies-I; theory", Journal of Sound and Vibration, Vol. 247 (2001), p. 59-76.

[4] X.Q. Zhu, S.S. Law: "Moving Load identification on multi-span continuous bridges with elastic bearings", Mechanical Systems and Signal Processing, Vol. 20 (2006), p. 1759-1782.

ISSN: 2277-3754
International Journal of Engineering and Innovative Technology (IJEIT)
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[5] Gombi S. L and D.S. Ramakrishna: "Inverse Identification of impact force on a Cantilever beam using Inverse Technique", Proceedings of the International conference on Smart Materials and Nono Technology in engineering at Shenzhen, China, 28 (2011).

[6] Gombi S. L and D.S. Ramakrishna: "Inverse Identification of impact forces acting on arbitrary structures", Proceedings of the International conference on Experimental Mechanics 2010,100 (2010).

[7] B.T. Wang, C.H. Chiu: "Determination of unknown impact force acting on a simply supported beam", Mechanical System and Signal Processing, Vol. 17 (2003), p. 683-704.

[8] A.N. Thite, D.J. Thompson: "Selection of response measurement location to improve inverse force determination", Applied Acoustics, Vol. 67 (2006), p. 797-818.

[9] J. Zhu, Z. Lu: "A time domain method for identifying dynamic loads on continuous systems", Journal of Sound and Vibration, Vol. 148 (1991), p. 137-146.

[10] J. F. Doyle: "An experimental method for determining the dynamic contact law", Experimental Mechanics, Vol. 24 (1) (1984), p. 10-16.

[11]J. F. Doyle: "Determining the contact force during the transverse impact of plates", Experimental Mechanics, Vol. 27 (10) (1997), p. 68.

[12] E.Jacquelin, A. Bennani, P Hamelin: "Force reconstruction: analysis and regularization of a deconvolution problem", Journal of Sound and Vibration, Vol. 265 (2003), p. 81-107.

[13] Z. Boukriya, P. Perrotin, A.Bennani: "Experimental Impact Force location using inverse problems: application for a circular plate", International Journal of Mechanics, Vol. 5 (1) (2011), p. 48-55.

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