

Optimization of a Control Loop Using Adaptive Method

K.Prabhu, Dr. V. Murali Bhaskaran

Abstract -- Continuous Stirred Tank Reactor (CSTR) plays an important role in the processing industries. It helps for maintaining the temperature of the liquid in the reactors. This paper deals with the comparison of adaptive control and conventional PID control in CSTR process. In the type of adaptive control, Model reference adaptive control (MRAC) method is used. The adaptation law is developed by MIT rule. The MIT rule gives the controller parameters (θ_1 and θ_2) and is used to adjust the controller gain. A simulation is made using MATLAB and the results were analysed. Hence it proves that the response of adaptive control is better than the conventional PID controller.

Keywords— CSTR, MRAC, MIT rule, PID controller.

I. INTRODUCTION

Most process especially chemical processes are nonlinear in some aspects. The process varies with load or can change with time, which is non-stationary. The above aspects require the settings of the controller revised and also we need a control mechanism that will improve the response affected by some disturbances. The proposed method called adaptive control is developed to maintain the liquid temperature of a continuous stirred tank reactor. This method automatically detects the changes that occur in gain or dead time of the process and readjust the PID control mode settings, thereby optimizing the response of control loop. If the process nonlinearities are compensated by controller function, then the controller was a nonlinear controller. The analysis includes two methods like conventional PID and Adaptive control method. The simulation result shows the best improvement in the transient response characteristics. The main objective is to maintain the temperature of liquid in CSTR. Adaptive control method gives the satisfactory result than conventional PID controller. Adaptive method has a wide number of applications when the plant exhibits non-linear behavior and when the plant is unknown. This gives the controller to maintain a desired point of performance in spite of any noise or fluctuation in the process. The conventional PID controllers have the less capability to solve these problems.

II. MODEL REFERENCE ADAPTIVE CONTROL

We postulate a reference model with tells us how the controlled process output ideally should respond to the command signal (set point). The model output is compared to the actual process output. This is the possible way to give specification for a servo problem. The difference between two outputs is used through a computer to adjust the controller gain in such a way as to minimize the integral square error.

$$\text{Minimized ISE} = \int_0^t [e_{\text{MR}}(t)]^2 dt$$

The MRAC composed of two loops. The inner loop is an ordinary feedback loop. The outer loop includes the adaptation mechanism and also looks like feedback loop. The model output place the role of the set point while the process output is the actual measurement. The key problem is to design the adaptation mechanism in such a way as to provide a stable system [15].

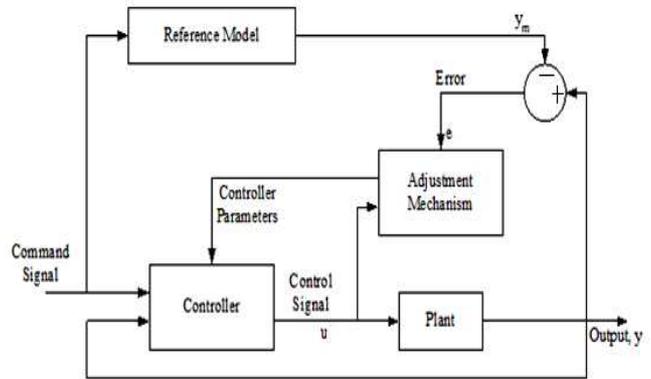


Fig 1. Model Reference Adaptive Controller

There are two ways for adjusting the parameters, the gradient method or Lyapunov method. The adaptation law utilizes the error between the process and the model output. These parameters are varied so as to minimize the error between the process and the reference model.

III. MATHEMATICAL MODELING

The CSTR reactor is shown in Fig 2. The mathematical model of the reactor comes from energy balance. An exothermic reaction $A \rightarrow B$ takes place in the reactor, which is in turn cooled by a coolant that flows through a jacket around the reactor. The jacket is assumed to be perfectly mixed. Heat transfer takes place through the reactor wall into jacket. The main objective is to maintain the temperature of the reacting mixture at desired value. The manipulated variable is the coolant temperature [16].

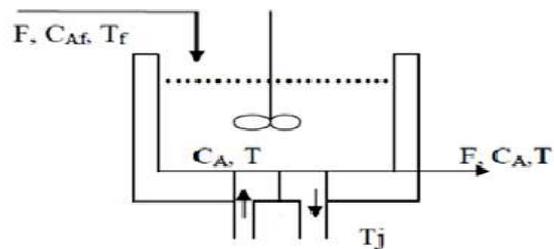


Fig. 2. Continuous Stirred Tank Reactor with Cooling Jacket

The CSTR has three input signals: C_{Af} —Concentration of feed stream, T_f —Inlet feed stream temperature, T_j —Jacket

coolant temperature. The two output signals: C_A - Concentration of A in reactor tank, T - Reactor Temperature.

A. Overall material balance

Let us now identify the state variables for the CSTR. The CSTR process is modeled using energy conservation principle.

By applying energy balance equation,

Rate of energy accumulation = total energy input – total energy output

$$\frac{dV\rho}{dt} = F_{in}\rho_{in} - F_{out}\rho_{out}$$

Energy Balance assuming constant Cp

$$V\rho C_p \frac{dT}{dt} = F\rho C_p(T_f - T) + (-\Delta H)Vr - UA(T - T_j)$$

B. Steady State Solution

The steady state solution is obtained when $dC_A/dt=0$ and $dT/dt=0$, that is

$$f_1(C_A, T) = \frac{dC_A}{dt} = 0 = \frac{F}{V}(C_{Af} - C_A) - K_0 e^{(-E/RT)} C_A$$

$$f_2(C_A, T) = \frac{dT}{dt} = 0 = \frac{F}{V}(T_f - T) + \left(\frac{-\Delta H}{\rho C_p} \right) K_0 e^{(-E/RT)} C_A - \frac{UA}{(V\rho C_p)}(T - T_j)$$

To solve these two equations, all parameters and variables are specified in the Table I.

TABLE I. Reactor Parameter's value

Reactor parameters	Values
$F/V, hr^{-1}$	4
K_0, hr^{-1}	15×10^{12}
$(-\Delta H), BTU/lbmol$	40000
$E, BTU/lbmol$	33500
$\rho C_p, BTU/ft^3$	54.65
$T_f, ^\circ C$	70
$C_{Af}, lbmol/ft^3$	0.132
UA/V	122.1
$T_j, ^\circ C$	60

C. Linearization of Dynamic Equation

Linearization is the process by which we approximate nonlinear systems with linear ones.

The stability of the non-linear equation can be found by using the following state space equation:

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

Next, determining the Eigen values of the system matrix A (state space)

The non-linear dynamic equations are

$$F_1(C_A, T) = \frac{dC_A}{dt} = \frac{F}{V}(C_{Af} - C_A) - K_0 e^{(-E/RT)} C_A$$

$$F_2(C_A, T) = \frac{dT}{dt} = \frac{F}{V}(T_f - T) - \left(\frac{\Delta H}{\rho C_p} \right) K_0 e^{(-E/RT)} C_A - \left(\frac{UA}{V\rho C_p} \right) (T - T_j)$$

Let us determine the state and input variables in the form of a deviation variable:

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} C_A - C_{As} \\ T - T_s \end{bmatrix}$$

$$U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} T_j - T_{js} \\ T_f - T_{fs} \\ C_{Af} - C_{Afs} \\ F - F_s \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ UA \\ V\rho C_p \end{bmatrix} \quad C = [0 \ 1] \quad D = [0]$$

$$A = \begin{bmatrix} -\frac{F}{V} - K_0 e^{(-\frac{E}{RT})} & -K_0 e^{(-\frac{E}{RT})} \left(\frac{E}{RT^2} \right) C_A \\ -\left(\frac{\Delta H}{\rho C_p} \right) K_0 e^{(-\frac{E}{RT})} & -\frac{F}{V} - \frac{UA}{V\rho C_p} + \left(\frac{-\Delta H}{\rho C_p} \right) K_0 e^{(-\frac{E}{RT})} \left(\frac{E}{RT^2} \right) C_A \end{bmatrix}$$

D. Stability Analysis

While designing a control system, we are seriously concerned about its stability characteristics. A dynamic system is considered to be stable if for every bounded input it produces a bounded output, regardless of its initial state. The steady state operating point is $C_{As} = 7.5938$, $T_s = 313.17$. The stability of particular operating point is determined by finding the A -matrix for that particular operating point and finding the Eigen values of the A -matrix [12].

$$A = \begin{bmatrix} -7.3929 & -0.014674 \\ 2622.9 & 4.7534 \end{bmatrix}$$

To find the Eigen values using Mat lab command

```
>>A=[-7.3929 -0.014674; 2622.9 4.7534]
```

```
>>Y=eig(A)
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>>Y= -1.3134
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-1.3567
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Both the values are negative. Hence the system is stable.

IV. ADAPTATION LAW

The adaptation law gives to find a set of parameters that minimize the error between the plant and the model outputs. Hence the parameters of the controller are adjusted until the error becomes zero. A number of adaptation laws have been developed. The two main types are the Gradient and the Lyapunov approach. Here the Gradient approach (MIT Rule) was used to develop the adaptation law [20].

A. MIT Rule

The MIT rule is the original approach to model reference adaptive control. The name is derived from the fact that it was developed at the Instrumentation Laboratory (now the Draper Laboratory) at Massachusetts Institute of Technology (MIT), U.S.A.

To present the MIT rule, we will consider a closed loop system in which the controller has one adjustable parameter. The desired closed loop response is specified by a model output Y_M . The error (e) is difference between the output of the system (Y) and the output of the reference model (Y_M).

The Modeling error e is given by equation

$$e = Y - Y_M$$

One possibility is to adjust parameters in such a way that the loss function $J(\theta)$ is minimized.

$$J(\theta) = \frac{1}{2} e^2$$

To make J small, it is reasonable to change the parameters in the direction of negative gradient of J . That is,

$$\frac{d\theta}{dt} = -\gamma \frac{\delta J}{\delta \theta} = -\gamma e \frac{\delta e}{\delta \theta}$$

This is the celebrated MIT rule. The partial derivative

$\frac{\delta e}{\delta \theta}$ is called the sensitivity derivative of the system, tells how the error is influenced by the adjustable parameter.

γ is called adaptation gain.

B. Adaptive MIT (AMIT) Algorithm

For designing the control systems, many of the advanced control techniques are based on the understanding of the system. If the process is not known then we call that as a “black-box” model. In most of the situations, we know something about the process but which is not sure whether it is correct or not. This is called as a “grey-box” model. If we know the process information, then it is called as a “white-box” model. Based on a priori knowledge, the process is modelled as second order [8]. The transfer function of the process is represented as given below

$$\frac{Y}{U} = \frac{K}{s^2 + a_1 s + a_2}$$

Where K , a_1 and a_2 are positive and are the process parameters

The control law is given by

$$U = \theta_1 U_c - \theta_2 Y$$

The closed-loop transfer function related to the output and input with the controller in the loop is given by equation

$$\frac{Y}{U_c} = \frac{K \theta_1}{s^2 + a_1 s + (a_2 + K \theta_2)}$$

Where U_c is the command signal (reference input). The controller parameters are updated by the adaptation mechanism such that the process output follows the model output equation.

$$\frac{Y_M}{U_c} = \frac{K_M}{s^2 + A_1 s + A_2}$$

Where K_M , A_1 and A_2 are the reference model parameters.

To apply the control law, the sensitivity derivatives are obtained by calculating the partial derivatives of modeling error with respect to the controller parameters. The process parameters K , a_1 & a_2 are not known. These formulas cannot be used directly because the process parameters are not known. One possible approximation is based on the observation when the parameters give perfect model following. We will use the approximation.

$$s^2 + a_1 s + (a_2 + K \theta_2) = s^2 + A_1 s + A_2$$

Which will be reasonable when parameters are close to their correct values? With this approximation we get the following equations for updating the controller parameters:

$$\frac{\partial e}{\partial \theta_1} = \frac{K}{s^2 + A_1 s + A_2} U_c \quad \text{and}$$

$$\frac{\partial e}{\partial \theta_2} = -Y \frac{K}{s^2 + A_1 s + A_2}$$

The controller parameters θ_1 and θ_2 are

$$\theta_1 = -\frac{\gamma'}{s} e \frac{K}{s^2 + A_1 s + A_2} U_c$$

$$\theta_2 = \frac{\gamma'}{s} e \frac{K}{s^2 + A_1 s + A_2} Y$$

$$\text{Where, } \gamma' = \gamma K$$

V. ADAPTIVE CONTROL DESIGN AND SIMULATION

From the CSTR temperature process, we found the transfer function:

$$G_p(s) = \frac{1.478s + 11.02}{s^2 + 3.391s + 3.34}$$

The next step is to define the model transfer function. The standard form of second order system is given by the equation:

$$G_m(s) = \frac{\omega_n^2}{s^2 + 2\omega_n \xi s + \omega_n^2}$$

The required specifications for the temperature control are a maximum Overshoot (M_p) of 2% and a settling time (T_s) of less than 3 seconds. Now, to determine the damping ratio and natural frequency of the system using the equation below.

$$\xi = \frac{\ln\left(\frac{M_p}{100}\right)}{-\pi} \sqrt{\frac{1}{1 + \left[\frac{\ln\left(\frac{M_p}{100}\right)}{-\pi}\right]^2}}$$

Damping ratio

$$(\omega_n) = \frac{4}{\xi T_s}$$

Natural frequency

Now, the damping ratio (ξ) = 0.71 and natural frequency (ω_n) = 2.1834 rad/sec. Hence the transfer function of the model is given:

$$G_m(s) = \frac{4.76}{s^2 + 3.1s + 4.76}$$

Note that we have defined the plant we need to develop a standard controller to compare with the adaptive controller. Controller setting is done using Ziegler-Nichols technique and the best controller parameters are found to be $K_c=9.7$, $K_I=0.82$, $K_d=0.94$.

VI. SIMULATION AND RESULT

The conventional PID controller Simulink model shows in Fig 3. This model has the step input signal, PID controller and transfer function of process plant. The controller output is the combination of step signal and process output signal.

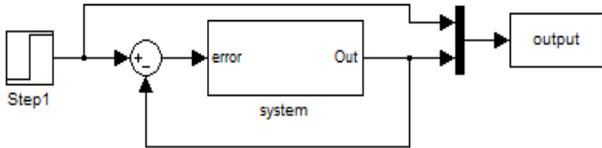


Fig 3. Simulink Model of Conventional PID Controller

The Model Reference Adaptive Controller Simulink model shows in Fig 4. It has the step input signal, reference model transfer function, process transfer function and adaptation gain value. The error signal is produce from the difference between process output and reference model output values ($e = Y_{plant} - Y_{model}$). The controller parameters (θ_1 & θ_2) values are depend on the reference input signal (U_c), transfer function of the reference model, error signal (e) and adaptation gain values.

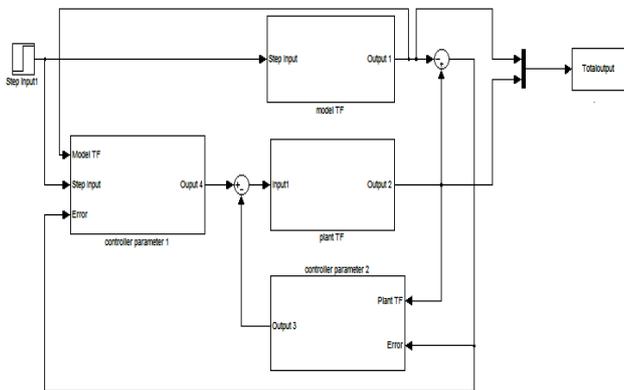


Fig 4. Simulink Model of MRAC with MIT rule

Comparison of Adaptive controller and Conventional controller with a step input

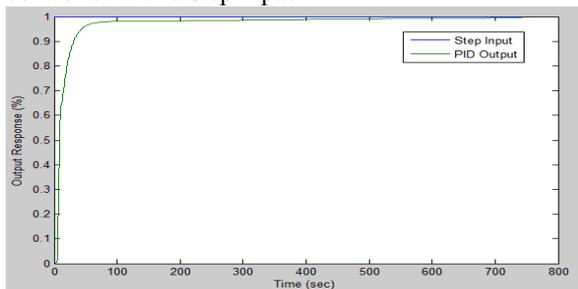


Fig 5. Plant output with Conventional PID Controller for Step Input

Here by using ordinary PID control, the process takes much more time to reach the desired output. That is, the time taken to reach the set value was 630 seconds and the time required for a signal to change from a specified low value to a specified high value was 580 seconds.

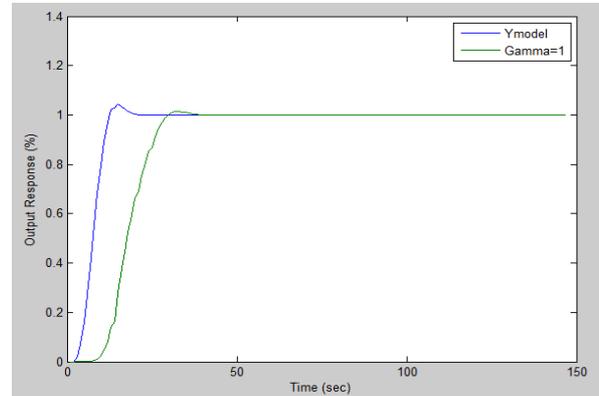


Fig 6. Plant output with Adaptation gain Gamma=1

But by using adaptive control, the time taken to reach the set value was 42 seconds and the time required for a signal to change from a specified low value to a specified high value was 30 seconds for the specified adaptation gain of 1. Therefore, the settling time and rise time in adaptive control method is less than conventional control method. The time domain specifications in adaptive method are improved and it is specified in table II. Hence the adaptive controller is suitable for process control applications than conventional PID controller.

Comparison of Adaptive Controller by Varying the Adaptation Gain

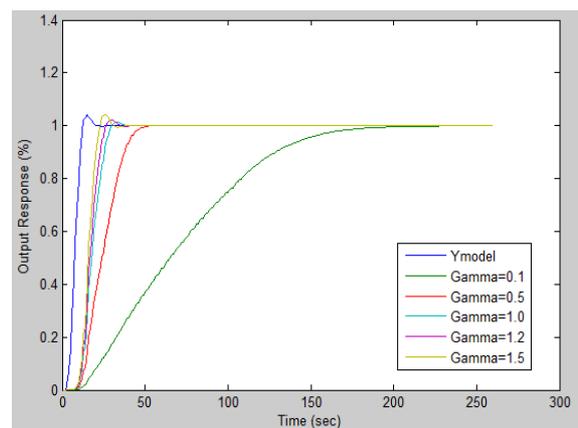


Fig 7. Plant output with Adaptation gain Gamma = 0.1, 0.5, 1.0, 1.2 and 1.5

From Fig 7, it is observed that if the value of adaptation gain is increased then the settling time, peak time and rise time is reduced.

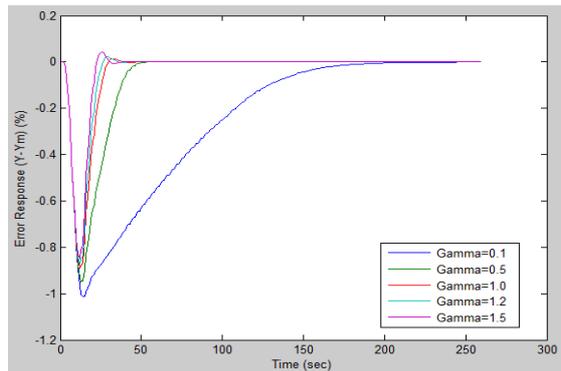


Fig 8. Effect of adaptation gain on output error

Fig 8 shows the effect of adaptation gain on output error (Y-Ym). The error signals for various adaptation gains are mentioned and continuously decreasing.

VII. CONCLUSION

As compared to conventional controllers (PID Controllers), Adaptive Controllers are very effective to handle the situations where the parameter variations and environmental changes occur and it is demonstrated clearly in results. Therefore, the controller parameters are adjusted automatically to give a desired closed loop performance. The adaptive controller maintains constant performance in the presence of disturbances. This paper describes the behavior of a system controlled by model reference adaptive control scheme using MIT rule. The effect of adaptation gain is viewed on the time response characteristic of the second order system. It has been seen that response is very slow with the smaller value of adaptation gain. By increasing the adaptation gain till some extent, the settling time, peak time and rise time was decreased. They are almost constant in time response plot. Table II shows the effect of adaptation gain on different time response specification. Hence for suitable value of adaptation gain in MIT rule, the plant output follows the reference model.

TABLE II. Effect of Adaptation Gain on Time Response Curve

Time Domain Response	PID	MRAC (Adaptation Gain Value)				
		0.1	0.5	1.0	1.2	1.5
Settling Time(sec)	630	220	55	42	37	32
Peak Time(sec)	-	-	-	34	31	27
Rise Time(sec)	580	190	50	30	28	24

VIII. FUTURE SCOPE

Temperature control plays a major role in CSTR process. It is very difficult to control such slow process. Change in operating point due to some environmental conditions is the common problem for these types of applications. We know

the fact that, the stability gets affected due to changes in the operating point. It is to be extended by rectifying this problem and maintain the stability of the given system. Hence the response of a process is automatically improved by considering these types of problem. The solution to this problem is by adding another method called gain scheduling method. This method is used to avoid the stability problem by scheduling the adaptive gain values. This is our future work.

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