ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 8, Issue 3, September 2018

Scattered failure of damaged beam in bending under monotone loading conditions

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Abstract: ***The problem of scattered failure of a beam made of hereditary damaged material in bending for a monotonically time – variant bending moment is formulated and solved. The motion equation of failure front was obtained and analyzed.***

Key words: **Damage, failure front, incubation period, scattered wave.**

I. INTRODUCTION

Design and analysis of constructions and their elements supposes to determine their endurance that is connected with the long – term strength problem. One of the significant factors that influences on load – carrying capability of structural elements is damageability of materials of which the constructions themselves are manufactured. Usually, all possible deficiencies that are available, formed and accumulated in operation process in the bulk of construction's material are determined by the term of damageability. At present there are different theories of damageability and each of them has its own applicability sphere. Failure because of damageability is called scattered, as this fracture is associated with critical density of damage at some point of the body. When this part of the body loses capability of resistance to external load, there happens redistribution of stresses and the failed part of the body extends. Motion of the boundary of extended area called failure front, determines the long – term failure process.

II. PROBLEM STATEMENT

For statistical loading, the solution of the problem of initiation and extension of failure area of a beam bending with regard to damageability process within in the framework of kinematic theory of damageability is given in [1], within hereditary theory of damageability in [2].

In this paper, within hereditary theory of damageability [4], the solution of the problem of scattered failure of flexible monotonically time – variant bending moment of a rectangular cross – section beam is given in quasistatic statement.

As pure bending state is present in the beam, then the only nonzero stress is the longitudinal stress σ expressed by the bending moment M by the known formula:

$$\sigma = \frac{M}{J} y_0; \qquad J = \frac{a h_0^3}{12} \quad (1)$$

Manuscript received: 25 August 2018
Manuscript received in revised form: 20 September 2018
Manuscript accepted: 06 October 2018
Manuscript Available online: 10 October 2018

Where J is the inertia moment of the cross section, h_0 is the half – width of the beam, a is its width.

III. PROBLEM SOLUTION

Direct the axis x_0 along the initial neutral axis of the beam, the axis y_0 perpendicular to it (fig.1)

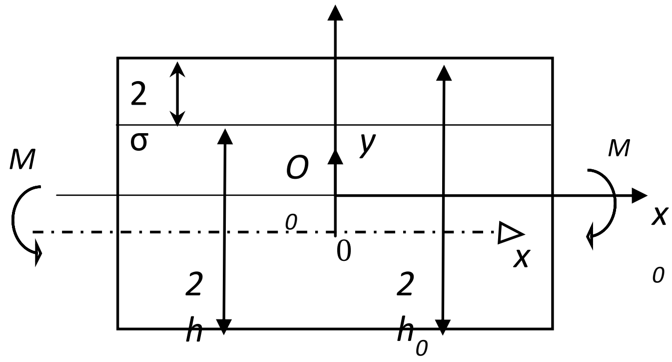

Fig 1. Scattered failure of the beam in bending

In the extension area of the beam $y_0 > 0$ the stress is positive. It attains the greatest value for $y_0 = h_0$ and it equals to

$$\sigma_{\max} = \frac{12M}{a h_0^2}.$$

Use the failure criterion given in [4]:

$$\sigma_i + Q^* \sigma_i = \sigma_n \quad (2)$$

Where $\sigma_i = \sqrt{\frac{2}{3}} \sigma$ is stress intensity, σ_n is the ultimate momentary strength, Q^* is the integral operator of damageability [4] that under monotone loading is an integral operator of hereditary type:

$$Q^* \sigma_i = \int_0^t Q(t - \tau) \sigma_i(\tau) d\tau. \quad (3)$$

Taking into account representation for load intensity in failure criterion (2) for initial failure time t_0, i.e. for the incubation period we get the equation

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$$M(t_0)+\int_0^{t_0} Q(t_0-\tau)M(\tau)d\tau=\frac{a\sigma_n h_0^2}{4\sqrt{6}}.$$

(4)

We represent the bending moment in the form $M(t)=M_0\cdot f(t)$, where M_0 is a dimension factor, $f(t)$ is a dimensionless time function. We also introduce the denotation $g=\dfrac{a\sigma_n h_0^2}{4\sqrt{6}M_0}$. Then equation (4) takes the form:

$$f(t_0)+\int_0^{t_0} Q(t_0-\tau)f(\tau)d\tau=g.$$

(5)

As a Kernel of damageability operator we accept:

$$Q(t)=me^{-\lambda t}$$

Taking it into account in (5) and considering the time t and parameter λ nondimensionalized with respect to the coefficient m, we get:

$$f(t_0)+\int_0^{t_0} e^{-\lambda(t_0-\tau)}\cdot f(\tau)d\tau=g.$$

(6)

In further analysis the function $f(t)$ is given and we accept it in the form:

$$f(t)=e^{-\beta t}. \qquad (7)$$

Allowing for this kind, equation (6) will be:

$$e^{-\beta t_0}+\int_0^{t_0} e^{-\lambda(t-\tau)}e^{-\beta\tau}d\tau=g.$$

(8)

Whence we can easily get the following transcendental algebraic equation for t_0, duration of incubation period, the latent failure time:

$$(\delta-\beta+1)e^{-\beta t_0}-e^{-\lambda t_0}=g(\delta-\beta).$$

(9)

If we accept $\lambda=0$, then from (9) for t_0 we find the explicit expression:

$$t_0=\frac{1}{\beta}\ln\frac{1-\beta}{1-g\beta}. \qquad (10)$$

It should be noted that existence of incubation period, i.e. fulfillment of the condition t >0, imposes certain restrictions on interaction of power and geometrical parameters of the problem.

As the surface layer $y=h$ fails, there happens redistribution of stresses with the next unloading of the newly formed surface layer, and so on. Thus, extended part of the failure area is shaped, and in the figure it is given by a dotted line. The neutral part of undischarged part is displaced in lateral direction. Along to it we direct the axis x, and the axis y perpendicular to it. We denote by 2δ the width of the failed part that corresponds to the moment of time t.

Following the technique given in [1], based on the failure criterion (2) we obtain the following differential equation of the failure front that determines the half – width $h(t)$ of undamaged part of the beam at moment of time t:

$$\frac{f(t)}{h^2(t)}+\int_0^t Q(t-\tau)\frac{f(\tau)[2h(t)-h(\tau)]}{h^3(\tau)}d\tau=g. \qquad (11)$$

For further analysis we accept $f(t)=e^{-\beta t}$, $Q(t)=m$. Taking the same previous nondimensionalization, from (11) we get:

$$\frac{e^{-\beta t}}{h^2(t)}+\int_0^t \frac{2h(t)-h(\tau)}{h^3(\tau)}e^{-\beta t}d\tau=g. \qquad (12)$$

Introduce it in the form of Volterra integral equation of second kind:

$$\frac{1}{h^2(t)}+\int_0^t e^{\beta(t-\tau)}\cdot\frac{2h(t)-h(\tau)}{h^3(\tau)}d\tau=ge^{\beta t}. \qquad (13)$$

The sought – for function $h(t)$ has the structure:

$$h(t)=\begin{cases}1; t\le t_0\\ h(t); t>t_0\end{cases} \qquad (14)$$

where t_0 is determined by formula (9).

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Naturally, by its way the function $h(t)$ is a monotonically decreasing function. By representation (14), integral equation (12) of motion of failure front is representable in the form:

$$\frac{1}{h^2(t)}+\int_0^{t_0} e^{\beta(t-\tau)}\cdot\frac{2h(t)-h_0}{h_0^3}\,d\tau+\int_0^{t} e^{\beta(t-\tau)}\cdot\frac{2h(t)-h(\tau)}{h^3(\tau)}\,d\tau=ge^{\beta t}$$

Allowing for $t \succ t_0$, the first integral term of this equation in the explicit form looks like:

$$\int_0^{t} e^{\beta(t-\tau)}\cdot\frac{2h(t)-h_0}{h_0^3}\,d\tau=\frac{2h(t)-h_0}{\beta h_0^3}\cdot e^{\beta t}\left(1-e^{-\beta t_0}\right).$$

Then the equation of failure front will be written as:

$$\frac{1}{h^2(t)}+\frac{2\left(1-e^{-\beta t_0}\right)}{\beta h_0^3}\left(2h(t)-h_0\right)e^{\beta t}+\int_0^{t} e^{\beta(t-\tau)}\cdot\frac{2h(t)-h(\tau)}{h^3(\tau)}\,d\tau=ge^{\beta t}.$$

Nondimensionalize the sought – for function of the half – width $h(t)$ with respect to its initial value h_0. Then the equation of failure front in dimensionless quantities has the same form that the above one has, but wherein we should assume $h_0=1$:

$$\frac{1}{h^2(t)}+\frac{2\left(1-e^{-\beta t_0}\right)}{\beta}\left(2h(t)-1\right)e^{\beta t}+\int_0^{t} e^{\beta(t-\tau)}\cdot\frac{2h(t)-h(\tau)}{h^3(\tau)}\,d\tau=g_*e^{\beta t}, \quad (15)$$

Where $g_*=gh_0^2$.

For numerical realization we use the step by step method. Let Δt be a time step, then we denote $t_n=t_0+n\Delta t$, and also $h_n=h(t_n)$. Taking this into account in (15) and representing the integral term by the Sympson formula, we get:

$$\frac{1}{h_n^2}+\frac{2\left(1-e^{-\beta t_0}\right)}{\beta}\left(2h_n-1\right)e^{\beta t_n}+\sum_{k=0}^{n} æ_k\, e^{\beta(t_n-t_k)}\cdot\frac{2h_n-h_k}{h_k^3}\Delta t=g_*e^{\beta t_n} \quad (16)$$

Where

$$æ_k=\begin{cases}\frac{1}{2};k=0;n\\ 1;k=\overline{1;n-1}\end{cases}$$

There with t_0 is assumed to be given and is determined by the formula given in the first part of the paper. Write out for illustration, the first two equations from (16) corresponding to $n=1$ and $n=2$:

For $n=1$:

$$\frac{1}{h_1^2}+\frac{2\left(1-e^{-\beta t_0}\right)}{\beta}\left(2h_1-1\right)e^{\beta(t_0+\Delta t)}+e^{\beta\Delta t}\left(h_1-\frac{1}{2}\right)\Delta t+\frac{\Delta t}{2h_1^2}=g_*e^{\beta(t_0+\Delta t)} \quad (17)$$

For $n=2$:

$$\frac{1}{h_2^2}+\frac{2\left(1-e^{-\beta t_0}\right)}{\beta}\left(2h_2-1\right)e^{\beta(t_0+2\Delta t)}+e^{2\beta\Delta t}\left(h_2-\frac{1}{2}\right)\Delta t+$$

$$+e^{\beta\cdot\Delta t}\frac{2h_2-h_1}{h_1^3}\Delta t+\frac{\Delta t}{2h_2^2}=g_*e^{\beta(t_0+2\Delta t)} \quad (18)$$

As is seen from the given formulas, on every time step we get a cubic equation with respect to the sought – for quantity h_k.

Sufficient exactness of the accepted time step was found by estimating the errors of values of duration of incubation period t_0 determined by the explicit formula (10) and by applying the step-by-step method to equation (8).

The curve of motion of failure front in cited dimensionless quantities for $g_*=gh_0^2=11$, $\beta=0{,}1$ is given in figure 2.

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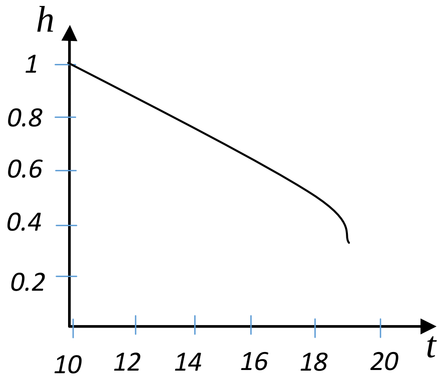

Fig 2. Curve of failure front

As it follows from the figure, the failure front moves with increasing velocity. Then for the time of loss of load – carrying ability, i.e. exhaustion of endurance may be accepted the time when the velocity of the motion of failure front attains the given limiting value.

IV. CONCLUSION

The problem on failure of a scattered beam made of hereditary damaged material in bending for a monotonically time – variant bending moment was stated and solved. The equation of motion of failure front was revealed.

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