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Generalized S-Variation Inequalities for Fuzzy SM-Iteration Scheme Mappings in Hilbert Spaces

Buthainah A.A.Ahmed, Manar Falih Dheyab
Department of Mathematics, College of Science, University of Baghdad, Iraq

***Abstract*— The idea of this paper, is study strongly convergence for our new iterative scheme under generalized S-variation inequality for two fuzzy mappings.**

***Index Terms*— fuzzy mapping, fuzzy fixed point and variation equality.**

I. INTRODUCTION

The concept of fuzzy set was introduced by L.Zadeh [5] in (1965). After that a lot of work has been done regarding fuzzy set and fuzzy mappings. The concept of fuzzy mapping was first introduced by Heilpern [4]. In this paper, we introduce generalized S-variation inequality for two fuzzy mappings and study strongly convergence for fuzzy SM-Iteration scheme.

II. PRELIMINARIES

In this section, we recall some basic definitions and preliminaries that will be needed in this paper.

Definition 2.1[5]: Let H be a Hilbert space and F(H) be a collection of all fuzzy sets in H . Let A $\in F(H)$ and $\alpha \in$ [0, 1] the $\alpha - level\ set\ of\ A,$ denoted by A_α is defined by

A_α= {x : A(x) $\geq \alpha$ } if $\alpha \in [0,1]$

$A_0 = \overline{\{x : A(x) > \alpha\}}$ Where $\overline{B}$ denotes the closure of a set B.

Definition 2.2[4]: A fuzzy set A is said to be an approximate quantity i$f\ and\ only\ if\ A_\alpha$ is compact and convex for each $\alpha \in [0,1]$, $and\ sup_{x\in X}$ A(x) = 1 .When A is an approximate quantity and A(x_0)=1 for some $x_0 \in$ H , A is identified with an approximate of x_0 .The collection of all fuzzy sets in H is denoted by F(H) and W(H) is the sub collection of all approximate quantities.

Definition 2.3[5]. Let A, B $\in W(H)$. An approximate quantity A is said to be more accurate than B (denoted by A$\subset B)\ if\ and\ only\ if\ A(x) \leq B(x),\ \forall\, x \in H.$

Definition 2.4[4]: A mapping T from the set H into W (H) is said to be fuzzy mapping.

Definition 2.5[4]: The point x $\in H$ is called fixed point for the fuzzy mapping T if {x}⊂Tx. If $x_\alpha \subset Tx$ is called fuzzy fixed point of T.

III. BASIC RESULTS AND FUZZY ITERATION

Definition 3.1[1]: A fuzzy mapping T: H$\rightarrow W(H)$ is called:

1. Fuzzy strong monotone, if for all u ,v $\in H$ and for x $\in [T(u)]_\alpha\ and\ y \in [T(v)]_\alpha$, there exists a constant $\mu \in (0,1)$ such that $\langle u - v, x - y\rangle \geq \mu\|x - y\|^2$.
2. Fuzzy Lipchitz continuous, if for all u ,v $\in H$ and for x $\in [T(u)]_\alpha$

$and\ y \in [T(v)]_\alpha$, there exists a constant $\beta \in (0,1)$ such that

$$\|u - v\| \leq \beta\|x - y\|.$$

We note that for $\beta = 1$, the fuzzy mapping T: H$\rightarrow W(H)$ is called non-expansive.

Definition 3.2: A fuzzy mapping T: H$\rightarrow W(H)$ is called Fuzzy D- Lipchitz continuous if there exists a constant $\xi \in (0,1)$ such that

$D(T(u), T(v)) \leq \xi\|u - v\|, \quad \forall\, u, v \in H.$

Definition 3.3[2]: A mapping $g: H \rightarrow H$ is called:

1. Strong monotone, if for all u ,v $\in H,$ there exists a constant $\sigma > 0$ such that

$\langle u - v, g(u) - g(v)\rangle \geq \sigma\|u - v\|^2$.

2.Lipchitz continuous, if for all u ,v $\in H$, there exists a constant $\zeta > 0$ such that $\|g(u) - g(v)\| \leq \zeta\|u - v\|$

Definition 3.4[2],[3]: 1.for all u$, v, z \in H$, the operator $P_{K(u)}$ satisfies the condition $\|P_{K(u)}(z) - P_{K(v)}(z)\| \leq \gamma\|u - v\|$, Where $\gamma > 0\ is\ a\ constant.$

2.Let us define $P_{K(u)}(z)$, the projection of z on $K(u)$, to be a point in $K(u)$ such that

$\|P_{K(u)}(z) - z\| = inf_{v\in K(u)}\|z - v\|$.

Definition 3.5: Let H be a real Hilbert space, whose norm and inner product are denoted by $\|.\|$ and $\langle .,.\rangle$ respectively. Let T,S: H$\rightarrow$W(H) be a fuzzy mappings and h, g: H$\rightarrow H$ be a single valued operators. Given a point -to-set mapping K: t$\rightarrow K(t)$ which associates a closed convex set K(t) with any element t $\in H$, we consider the problem of finding t,w,x $\in H$ such that w $\in [T(t)]_\alpha$, x $\in [S(t)]_\alpha$, $g(t) \in K(t)$ and

$$\langle w + h(x), v - g(t) \rangle \geq 0 \quad , \forall\, v \in K(t) \; \ldots..(3.1)$$

The problem (3.1) is called the generalized S-variation inequality for fuzzy mappings.

Lemma 3.6: Let K(t) be a closed non-empty convex set in H. Then (t , w, x) is a solution of (3.1) if and only if (t , w) satisfies the relation

$g(t) = P_{K(t)}[g(t) - \rho(w + h(x)],$
$for\ w \in [T(t)]_\alpha$, where $\rho > 0$ is a constant.

Proof: Trivial

Theorem 3.7:_Let K(t) be a closed convex set in H. Let the fuzzy mapping T: H→ $W(H)$ be a fuzzy strong monotone with constant $\mu \in (0,1)$ and fuzzy D-Lipchitz continuous. Let the operator $g: H \to H$ be a strongly monotone with constant $\sigma > 0$ and Lipchitz continuous with constant $\zeta > 0$. Assume that the operator $h: H \to H$ is Lipchitz continuous with constant $\eta > 0$ and S: H→ $W(H)$ is fuzzy D-Lipchitz continuous with constant $\varrho > 0$. If

$$\left|\rho - \frac{\mu - (1-k)\eta\varrho}{\xi^2 - \eta^2\varrho^2}\right| < \frac{[\mu - (1-k)\eta\varrho]^2 - (\xi^2 - \eta^2\varrho^2)(2k - k^2)}{\xi^2 - \eta^2\varrho^2}, \ldots (3.2)$$

$$\rho\eta\varrho < 1 - \cdots \ldots \ldots \ldots . \ldots \ldots \ldots ..(3.3)$$

$$k = \gamma + 2(1 - 2\sigma + \zeta^2) \ldots \ldots \ldots \ldots \ldots ..(3.4)$$

Then, there exists a solution t , w , x $\in H$ such that $w \in [T(t)]_\alpha$, $x\ [S(t)]_\alpha$ $and\ g(t) \in K(t)$ satisfying the generalized S-variation inequality (3.1).

Proof: From lemma 3.6, we know that the generalized S-variation inequality (3.1) is equivalent to the fuzzy fixed point

R(t) =
$t - g(t) +$
$P_{K(t)}[g(t) - \rho(w + h(x))] \ldots..(3.5)$

In order to prove the existence of a solution of it is sufficient to show that the problem (3.5) has a fixed point. Thus for $t_1, t_2 \in H\ and\ t_1 \neq t_2$, we have

$$\|R(t_1) - R(t_2)\|^2 = \left\| \begin{matrix} t_1 - g(t_1) + P_{K(t_1)}[g(t_1) - \rho(w_1 + h(x_1))] - \\ t_2 + g(t_2) - P_{K(t_2)}[g(t_2) - \rho(w_2 + h(x_2))] \end{matrix} \right\|^2$$

for $w_1 \in [T(t_1)]_\alpha\ and\ w_2 \in [T(t_2)]_\alpha$.

$$\leq \left\| t_1 - t_2 - (g(t_1) - g(t_2)) \right\|^2 +$$

$$\left\| P_{K(t_1)}[g(t_1) - \rho(w_1 + h(x_1))] - P_{K(t_2)}[g(t_2) - \rho(w_2 + h(x_2))] \right\|^2$$

Now,

$$\left\| P_{K(t_1)}[g(t_1) - \rho(w_1 + h(x_1))] - P_{K(t_2)}[g(t_2) - \rho(w_2 + h(x_2))] \right\|^2$$

$$\leq \gamma\|t_1 - t_2\|^2 + \|t_1 - t_2 - g(t_1) - g(t_2)\|^2$$

$$+ \|t_1 - t_2 - \rho(w_1 - w_2)\|^2 + \rho\|h(x_1) - h(x_2)\|^2$$

Hence,

$$\|R(t_1) - R(t_2)\|^2 \leq \gamma\|t_1 - t_2\|^2$$

$$+ \|t_1 - t_2 - g(t_1) - g(t_2)\|^2 +$$

$$\|t_1 - t_2 - \rho(w_1 - w_2)\|^2 + \rho\|h(x_1) - h(x_2)\|^2$$

Since, T is a fuzzy strong monotone and fuzzy D-Lipchitz continuous then,

$$\|t_1 - t_2 - \rho(w_1 - w_2)\|^2$$

$$\leq (1 - 2\rho\mu + \rho^2\xi^2)\|t_1 - t_2\|^2$$

Similarly,

$$\|t_1 - t_2 - g(t_1) - g(t_2)\|^2 \leq$$

$$(1 - 2\sigma + \zeta^2)\|t_1 - t_2\|^2$$

Since, h is Lipchitz continuous and S is fuzzy D-Lipchitz continuous, we have

$$\|h(x_1) - h(x_2)\|^2 \leq \eta^2\|x_1 - x_2\|^2 \leq \eta^2 D^2(S(t_1), S(t_2)) \leq \eta^2\varrho^2\|t_1 - t_2\|^2$$

Hence,

$$\|R(t_1) - R(t_2)\|^2 \leq \gamma\|t_1 - t_2\|^2$$

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$+2(1-2\sigma+\zeta^2)\|t_1-t_2\|^2+$

$(1-2\rho\mu+\rho^2\xi^2)\|t_1-t_2\|^2+\rho\eta^2\varrho^2\|t_1-t_2\|^2$

$\|R(t_1)-R(t_2)\|^2 \le \theta\|t_1-t_2\|^2$

Where,

$\theta=\kappa+t(\rho)+\rho\eta^2\varrho^2$

$\kappa=\gamma+2(1-2\sigma+\zeta^2)$

$t(\rho)=1-2\rho\mu+\rho^2\xi^2$

We have to show that $\theta<1$. It is clear that $t(\rho)$ assumes its minimum value for $\bar{\rho}=\frac{\mu}{\xi^2}$ with $t(\bar{\rho})=1-\frac{\mu^2}{\xi^2}$. For $\bar{\rho}=\rho$, $\kappa+t(\rho)+\rho\eta\varrho<1$ implies that $\rho\eta\varrho<1-\kappa$. Thus it follows that $\theta<1$, for ρ satisfying the conditions (3.2)-(3.4). Since, $\theta<1$ then the map R(t) define by (3.5) has a fuzzy fixed point t, w $\in H$ satisfying the generalized S-variation equation (3.1). This completes the proof. ∎

Definition 3.8: Let H be a Hilbert space, T,S: H$\to W(H)$ be a fuzzy mappings , $\{\lambda_n\},\{\beta_n\}$ be a real sequence in [0 , 1] and $h,g:H\to H$ be a single valued operators. Let K(x_n), $K(b_n)$ and $K(z_n)$ be a closed convex sets in H. For given $x_0,y_0,,w_0\in H$,let $w_0\in[T(x)]_\alpha,y_0\in[S(x)]_\alpha and$ for n $\in N$ and $\rho>0$, compute the sequence $\{x_n\},\{b_n\}$ and $\{z_n\}$ from

$$x_{n+1}=b_n-g(b_n)+P_{K(b_n)}\begin{bmatrix} g(b_n)- \\ \rho(w_n+h(y_n)) \end{bmatrix}$$

$$b_n=(1-\lambda_n)z_n+$$

$$\lambda_n\begin{Bmatrix} z_n-g(z_n)+ \\ P_{K(z_n)}[g(z_n)-\rho(w_n+h(y_n))] \end{Bmatrix}$$

$$z_n=(1-\beta_n)x_n+$$

$$\beta_n\begin{Bmatrix} x_n-g(x_n)+ \\ P_{K(x_n)}[g(x_n)-\rho(w_n+h(y_n))] \end{Bmatrix}\dots(3.6)$$

Then the equation (3.6) is called Fuzzy SM- Iteration Scheme.

Theorem 3.9: Let K(x) , K(z) and K(b) be a closed convex sets in H. Let T: H$\to W(H)$be fuzzy strong monotone with constant $\mu\in(0,1)$ and fuzzy D-Lipchitz continuous. Let the operator $g:H\to H$ be a strongly monotone with constant $\sigma>0$ and Lipchitz continuous with constant $\zeta>0$. Assume that the operator $h:H\to H$ is Lipchitz continuous with constant $\eta>0$ and S: H$\to W(H)$ is fuzzy D-Lipchitz continuous with constant$\varrho>0$. If

$$0<h(\varsigma\varepsilon+\rho^2\xi^2)+\rho^2(\xi^2+\eta^2\varrho^2)<1\dots\dots(3.7)$$

Where

$$\varepsilon=1-\lambda_n+2\lambda_n\{(1-2\rho\sigma+\zeta^2)\}+\lambda_n\gamma+\lambda_n(1-2\rho\mu)$$

$$\varsigma=(1-\beta_n+2\beta_n\{(1-2\sigma+\zeta^2)\}+\beta_n+\beta_n(1-2\rho\mu+\rho^2(\xi^2+\eta^2\varrho^2))$$

$$h=3+2\zeta^2-2(\sigma+\rho\mu)+\gamma$$

Then, there exists a solution , x ,y ,w $\in H$ such that $w\in[T(x)]_\alpha, y\in[S(x)]_\alpha$ and $g(x)\in K(x)$ satisfying the generalized S-variation inequality (3.1), and the sequences $\{x_n\},\{y_n\}and$ $\{w_n\}$ generated by (3.6) converge to x,y and w strongly in H, respectively.

Proof: From fuzzy SM iteration Scheme, we have

$$\|x_{n+1}-x_n\|^2=\left\|\begin{matrix} b_n-g(b_n)+P_{K(b_n)}[g(b_n)-\rho(w_n+h(y_n))]- \\ b_{n-1}-g(b_{n-1})+P_{K(b_{n-1})}[g(b_{n-1})-\rho(w_{n-1}+h(y_{n-1}))] \end{matrix}\right\|^2$$

for

$w_n\in[T(x_{n-1})]_\alpha, w_{n-1}\in[T(x_n)]_\alpha, y_n\in[S(x_{n-1})]_\alpha$ and $y_{n-1}\in[S(x_n)]_\alpha$

.

$$\le\|b_n-b_{n-1}\|^2+\|g(b_n)-g(b_{n-1})\|^2+$$

$$\|P_{K(b_{n-1})}[g(b_{n-1})-\rho(w_{n-1}+h(y_{n-1}))]-P_{K(b_n)}[g(b_n)-\rho(w_n+h(y_n))]\|^2$$

Now,

$$\|P_{K(b_{n-1})}[g(b_{n-1})-\rho(w_{n-1}+h(y_{n-1}))]-P_{K(b_n)}[g(b_n)-\rho(w_n+h(y_n))]\|^2$$

$$\le\gamma\|b_{n-1}-b_n\|^2+$$

$$\|b_{n-1}-b_n-g(b_{n-1})-g(b_n)\|^2+$$

$$\|b_{n-1}-b_n-\rho(w_{n-1}-w_n)\|^2+\rho^2\|h(y_n)-h(y_{n-1})\|^2$$

Since, T is a fuzzy strong monotone and fuzzy D-Lipchitz continuous then,

$$\|b_{n-1} - b_n - \rho(w_{n-1} - w_n)\|^2$$

$$\le (1 - 2\rho\mu)\|b_{n-1} - b_n\|^2 + \rho^2\xi^2\|x_{n-1} - x_n\|^2$$

Similarly,

$$\|b_{n-1} - b_n - g(b_{n-1}) - g(b_n)\|^2 \le$$

$$(1 - 2\sigma + \zeta^2)\|b_{n-1} - b_n\|^2$$

Since, S is a fuzzy strong monotone and fuzzy D-Lipchitz continuous with constant $\varrho > 0$ and h is Lipchitz continuous with constant $\eta > 0$, then

$$\rho^2\|h(y_n) - h(y_{n-1})\|^2 \le \rho^2\eta\|y_n - y_{n-1}\|^2 \le \rho^2\eta D^2(S(x_{n-1}), S(x_n)) \le \rho^2\eta^2\varrho^2\|x_{n-1} - x_n\|^2$$

Hence,

$$\|x_{n+1} - x_n\|^2 \le \|b_n - b_{n-1}\|^2 + \left\|\left(g(b_n) - g(b_{n-1})\right)\right\|^2 +$$

$$\gamma\|b_{n-1} - b_n\|^2 + \|b_{n-1} - b_n - g(b_{n-1}) - g(b_n)\|^2 +$$

$$\|b_{n-1} - b_n - \rho(w_{n-1} - w_n)\|^2 + \rho^2\|h(y_n) - h(y_{n-1})\|^2$$

$$\|x_{n+1} - x_n\|^2 \le (1 + \zeta^2 + \gamma)\|b_n - b_{n-1}\|^2$$

$$+(1 - 2\sigma + \zeta^2)\|b_{n-1} - b_n\|^2 +$$

$$(1 - 2\rho\mu)\|b_{n-1} - b_n\|^2 + \rho^2\xi^2\|x_{n-1} - x_n\|^2 + \rho\eta^2\varrho^2\|x_{n-1} - x_n\|^2$$

$$\|x_{n+1} - x_n\|^2 \le (3 + 2\zeta^2 - 2(\rho\mu + \sigma) + \gamma)$$

$$\|b_n - b_{n-1}\|^2 + \rho^2(\xi^2 + \eta^2\varrho^2)$$

$$\|x_{n-1} - x_n\|^2$$

Since,

$$b_n = (1 - \lambda_n)z_n + \lambda_n\{z_n - g(z_n) +$$

$$P_{K(z_n)}[g(z_n) - \rho(w_n + h(y_n))]\}$$

then,

$$\|b_{n-1} - b_n\|^2 = \left\| \begin{matrix} (1 - \lambda_n)z_{n-1} + \lambda_n\{z_{n-1} - g(z_{n-1}) + \\ P_{K(z_{n-1})}[g(z_{n-1}) - \rho(w_{n-1} + h(y_{n-1}))]\} - \\ (1 - \lambda_n)z_n + \lambda_n\{z_n - g(z_n) + \\ P_{K(z_n)}[g(z_n) - \rho(w_n + h(y_n))]\} \end{matrix} \right\|^2$$

Thus,

$$\|b_{n-1} - b_n\|^2 \le (1 - \lambda_n)\|z_{n-1} - z_n\|^2 + \lambda_n$$

$$\|z_{n-1} - z_n - \left(g(z_{n-1}) - g(z_n)\right)\|^2 + \lambda_n$$

$$\left\{\left\| \begin{matrix} P_{K(z_{n-1})}[g(z_{n-1}) - \rho(w_{n-1} + h(y_{n-1}))] - \\ P_{K(z_n)}[g(z_n) - \rho(w_n + h(y_n))] \end{matrix} \right\|^2\right\}$$

Now,

$$\left\| \begin{matrix} P_{K(z_{n-1})}[g(z_{n-1}) - \rho(w_{n-1} + h(y_{n-1}))] - \\ P_{K(z_n)}[g(z_n) - \rho(w_n + h(y_n))] \end{matrix} \right\|^2$$

$$\le \left\| \begin{matrix} P_{K(z_{n-1})}[g(z_{n-1}) - \rho(w_{n-1} + h(y_{n-1}))] - \\ P_{K(z_{n-1})}[g(z_n) - \rho(w_n + h(y_n))] \end{matrix} \right\|^2 +$$

$$\left\| \begin{matrix} P_{K(z_{n-1})}[g(z_n) - \rho(w_n + h(y_n))] - \\ P_{K(z_n)}[g(z_n) - \rho(w_n + h(y_n))] \end{matrix} \right\|^2$$

$$\le \gamma\|z_{n-1} - z_n\|^2 + \|g(z_{n-1}) - g(z_n) - \rho(w_{n-1} - w_n)\|^2$$

$$\le \gamma\|z_{n-1} - z_n\|^2 + \|z_{n-1} - z_n - g(z_{n-1}) - g(z_n)\|^2 +$$

$$\|z_{n-1} - z_n - \rho(w_{n-1} - w_n)\|^2 + \rho^2\|h(y_n) - h(y_{n-1})\|^2$$

Hence,

$$\|b_{n-1} - b_n\|^2 \le (1 - \lambda_n)\|z_{n-1} - z_n\|^2 + 2\lambda_n$$

$$\|z_{n-1} - z_n - \left(g(z_{n-1}) - g(z_n)\right)\|^2 +$$

$$\lambda_n\gamma\|z_{n-1} - z_n\|^2 +$$

$$\lambda_n\|z_{n-1} - z_n - \rho(w_{n-1} - w_n)\|^2 +$$

$$\lambda_n\rho^2\|h(y_n) - h(y_{n-1})\|^2$$

Since, T is a fuzzy strong monotone and fuzzy D-Lipchitz continuous

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then,

$\|z_{n-1} - z_n - \rho(w_{n-1} - w_n)\|^2 \leq (1 - 2\rho\mu)$

$\|z_{n-1} - z_n\|^2 + \rho^2\xi^2\|x_{n-1} - x_n\|^2$

Similarly,

$\|z_{n-1} - z_n - g(z_{n-1}) - g(z_n)\|^2 \leq$

$(1 - 2\sigma + \zeta^2)\|z_{n-1} - z_n\|^2$

Now,

$\|b_{n-1} - b_n\|^2 \leq \begin{pmatrix} 1 - \lambda_n + 2\lambda_n\{(1 - 2\sigma + \zeta^2)\} + \lambda_n\gamma \\ +\lambda_n\{(1 - 2\rho\mu) \end{pmatrix}$

$\|z_{n-1} - z_n\|^2 +$

$\lambda_n\rho^2(\xi^2 + \eta^2\varrho^2)\|x_{n-1} - x_n\|^2$

In the same way, we compute $\|z_{n-1} - z_n\|^2$

$$\|z_{n-1} - z_n\|^2 = \left\| \begin{matrix} (1 - \beta_n)x_{n-1} + \beta_n\{x_{n-1} - g(x_{n-1}) + \\ P_{K(x_{n-1})}[g(x_{n-1}) - \rho(w_{n-1} + h(y_{n-1}))]\} - \\ (1 - \beta_n)x_n + \beta_n\{x_n - g(x_n) + \\ P_{K(x_n)}[g(x_n) - \rho(w_n + h(y_n))]\} \end{matrix} \right\|^2$$

Thus,

$\|z_{n-1} - z_n\|^2 \leq (1 - \beta_n)\,\|x_{n-1} - x_n\|^2 +$

$\beta_n\left\|x_{n-1} - x_n - \left(g(x_{n-1}) - g(x_n)\right)\right\|^2 +$

$$\beta_n\left\{\left\| \begin{matrix} P_{K(x_{n-1})}[g(x_{n-1}) - \rho(w_{n-1} + h(y_{n-1}))] - \\ P_{K(x_n)}[g(x_n) - \rho(w_n + h(y_n))] \end{matrix} \right\|^2\right\}$$

Now,

$$\left\| \begin{matrix} P_{K(x_{n-1})}[g(x_{n-1}) - \rho(w_{n-1} + h(y_{n-1}))] - \\ P_{K(x_n)}[g(x_n) - \rho(w_n + h(y_n))] \end{matrix} \right\|^2$$

$\leq \gamma\|x_{n-1} - x_n\|^2 +$

$\|x_{n-1} - x_n - g(x_{n-1}) - g(x_n)\|^2 +$

$\|x_{n-1} - x_n - \rho(w_{n-1} - w_n)\|^2 +$

$\rho^2\|h(y_n) - h(y_{n-1})\|^2$

Hence,

$\|z_{n-1} - z_n\|^2 \leq (1 - \beta_n)\,\|x_{n-1} - x_n\|^2 +$

$2\beta_n\left\|x_{n-1} - x_n - \left(g(x_{n-1}) - g(x_n)\right)\right\|^2 +$

$\beta_n\gamma\|x_{n-1} - x_n\|^2 + \beta_n\left\| \begin{matrix} x_{n-1} - x_n - \\ \rho(w_{n-1} - w_n) \end{matrix} \right\|^2 +$

$\beta_n\rho^2\|h(y_n) - h(y_{n-1})\|^2$

Since, T is a fuzzy strong monotone and fuzzy D-Lipchitz continuous

Then,

$\|x_{n-1} - x_n - \rho(w_{n-1} - w_n)\|^2 \leq$

$(1 - 2\rho\mu + \rho^2\xi^2)\|x_{n-1} - x_n\|^2$

Similarly,

$\|x_{n-1} - x_n - g(x_{n-1}) - g(x_n)\|^2 \leq$

$(1 - 2\sigma + \zeta^2)\|x_{n-1} - x_n\|^2$

Now,

$\|z_{n-1} - z_n\|^2 \leq (1 - \beta_n + 2\beta_n\{(1 - 2\sigma + \zeta^2)\} + \beta_n + \beta_n(1 - 2\rho\mu + \rho^2(\xi^2 + \eta^2\varrho^2))\|x_{n-1} - x_n\|^2$

Since,

$\|b_{n-1} - b_n\|^2 \leq \{[\begin{pmatrix} 1 - \lambda_n + 2\lambda_n\{(1 - 2\sigma + \zeta^2)\} + \lambda_n\gamma \\ +\lambda_n\{(1 - 2\rho\mu) \end{pmatrix}$

$(1 - \beta_n + 2\beta_n\{(1 - 2\sigma + \zeta^2)\} + \beta_n + \beta_n(1 - 2\rho\mu + \rho^2(\xi^2 + \eta^2\varrho^2))] +$

$\lambda_n\rho^2(\xi^2 + \eta^2\varrho^2)\}\|x_{n-1} - x_n\|^2$

Let

$\varepsilon = 1 - \lambda_n + 2\lambda_n\{(1 - 2\rho\sigma + \zeta^2)\} + \lambda_n\gamma +$
$\lambda_n(1 - 2\rho\mu)$
$\varsigma = (1 - \beta_n + 2\beta_n\{(1 - 2\sigma + \zeta^2)\} + \beta_n + \beta_n(1 - 2\rho\mu + \rho^2(\xi^2 + \eta^2\varrho^2))$

Then,

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$\|b_{n-1} - b_n\|^2 \leq (\varsigma\varepsilon + \lambda_n \rho^2 (\xi^2 +$
$\eta^2 \varrho^2))\|x_{n-1} - x_n\|^2$

Hence,

$\|x_{n+1} - x_n\|^2 \leq h\left(\varsigma\varepsilon + \lambda_n \rho^2 (\xi^2 + \eta^2 \varrho^2)\right)$

$\|x_{n-1} - x_n\|^2 +$

$\rho^2 (\xi^2 + \eta^2 \varrho^2)\|x_{n-1} - x_n\|^2$

Such that, h=$3 + 2\zeta^2 - 2(\sigma + \rho\mu) + \gamma$

Hence, $\|x_{n+1} - x_n\|^2 \leq \theta\|x_{n-1} - x_n\|^2$, where
$\theta = h(\varsigma\varepsilon + \rho^2 \xi^2) + \rho^2 (\xi^2 + \eta^2 \varrho^2)$

Now, from (4.3.7) , we have $0 < \theta < 1$.

Then $\|x_{n-1} - x_n\|^2 \to 0 \quad as \quad n \to \infty$

Hence, $\{x_n\}$ is a Cauchy sequence in H so that there exists $x, w \in H$ with

$x_{n+1} \to x, y_{n+1} \to y$ and $w_{n+1} \to$
w strongly in H

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