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Toward Scheduling for Railway Systems Based on Max-Plus Linear Systems Extension of state equation for actual operation

Yumiko Kusunoki and Hirotaka Takahashi
Department of Information and Management Systems Engineering,
Nagaoka University of Technology,
1603-1 Kamitomioka, Nagaoka, Niigata 940-2188, Japan

Abstract*—We propose a scheduling method for railway systems based on Max-Plus Linear systems. Max-Plus Linear systems are suitable for representation of discrete event systems. In our research, we consider an application to the scheduling for actual railway systems. For the application to railway systems, we have to consider the number of trains that can exist between stations (the maximum number of trains), the number of trains that must stay between stations (the minimum number of trains) and the different passage route and stations depending on trains. Several models have been proposed in previous research. However, there are few models satisfying these three conditions. It is more problem that the state equation is complicated for models satisfying these conditions. The purpose of this research is to extend the existing model so that the different intermediate stations can be considered based on the model that state equation is simple. We also consider the maximum and minimum number of trains. Furthermore, in order to confirm the effectiveness, the numerical experiments of the normal operation and the occurrence of delay were conducted by using an imaginary route map and route diagram. As the result, the scheduling was performed correctly. Thus, we confirmed the effectiveness of our proposed method.

***Index Terms*—Max-Plus linear systems, railway systems, scheduling.**

I. INTRODUCTION

We propose a scheduling method for railway systems based on Max-Plus Linear (MPL) systems. MPL systems are the representation of the linear equations by using Max-Plus algebra that defines the “max” and “plus” operation as the addition and multiplication operations, respectively. This representation of MPL systems is similar to the state equation of modern control theory. MPL systems are suitable to represent a discrete event system and are known as a modeling method such as production system [1] and project management [2] etc.

This research focuses on the timetable scheduling of railway systems. Railway is widely used in various applications in our life such as the commutation and school attendance. Therefore, the scheduling to operate the train on time is essential and important. Moreover, if there is a delay in the train due to a train accident or breakdown, it is necessary to do again the scheduling. This is called rescheduling.

Railway systems are one of the discrete event systems. However, it is difficult to adapt the model used in the production systems to the railway systems. In the production systems and project management, all processes and works are done when the product and project are completed. However, the railway is different. For example, the starting station, terminal station, intermediate station are different. Therefore, it is conceivable that all trains must not pass through all stations. Moreover, the high-density railway networks are constructed in urban areas. In this network, we have to consider the number of trains that can exist between stations (the maximum number of trains) and the number of trains that must stay between stations (the minimum number of trains).

We briefly review the previous researches on timetable scheduling method of railway systems using MPL systems. Heidergott*et al.*[3] and Goverd [4] use the state equations of the model of the production system with a strong assumption that all trains stop at all stations. In Goto’s model [5], the maximum and minimum number of trains are taken into account and the state equation is simple. However, as the models of Heidergott*et al.*[3] and Goverd [4], there is an assumption that all trains stop at all stations. In addition, this model describes the general scheduling methods by using the MPL systems, not specialized in the railway systems. In the model ofGoto and Takahashi [6], the maximum number of trains and intermediate stations are taken into account, but the minimum number of trains is not considered. Maruyama’s model [7] based on the model of Goto and Takahashi [6] considers the maximum and minimum number of trains and the different intermediate stations. However, the state equation is very complex.

The purpose of this research is to extend the model so that different intermediate stations can be considered based on the method proposed by [5].

This paper is organized as follow. In section 2, we briefly review Max-Plus algebra and MPL systems. In section 3, we explain the model in the previous research, and we explain our extended model. In section 4, the results of the numerical experiments by using imaginary route map and diagram are described. Finally, in section 5, we summarize and conclude our work.

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II. MAX-PLUS LINEAR SYSTEMS

A. Max-Plus Algebra

In Max-Plus algebra, the "max" and "+" operations are define as addition and multiplication, respectively. Denote the real filed by . In $\mathbb{D} = \mathbb{R} \cup \{-\infty\}$, $x, y \in$,

$$\max(x, y) = x \oplus y, \; x + y = x \otimes y, \quad (1.)$$

and write the yth power of x by $x^{\otimes y}$.Operators for the multiple numbers are as follows.If $m \leq n$,

$$\bigoplus_{k=m}^{n} x_k = \max(x_m, x_{m+1}, \cdots, x_n). \quad (2.)$$

For matrix$X \in \mathbb{D}^{m\times n}$, $[X]_{ij}$express the(i, j)th element of X, and X^T is the transposed matrix of X. For $X, Y \in \mathbb{D}^{m\times n}$,

$$[X \oplus Y]_{ij} = \max([X]_{ij}, [Y]_{ij}). \quad (3.)$$

If $X \in \mathbb{D}^{m\times p}, Y \in \mathbb{D}^{p\times n}$,

$$[X \otimes Y]_{ij} = \bigoplus_{k=1}^{p}([X]_{ik} + [Y]_{kj}). \quad (4.)$$

The priority of operator$\otimes$ is higher than that of operator $\oplus$. For the scalar quantity, the zero and unit elements are given by $\varepsilon(= -\infty)$, $e(= 0)$, respectively. For the matrix operation, the unit elements are given by ϵ_{mn} and e_m . ϵ_{mn} is $\epsilon_{mn} \in \mathbb{D}^{m\times n}$ where all components are ε . e_m is $e_m \in \mathbb{D}^{m\times m}$ wherethe diagonal component e and off-diagonal component ε.

The following operator is known as the Kleene star. If $A \in \mathbb{D}^{n\times n}$,

$$A^* = e_n \oplus A \oplus A^{\otimes 2} \oplus \cdots \oplus A^{\ell-1}, \; A^{\otimes \ell} = \epsilon_{nn}, \quad (5.)$$

where an instance $\ell \; (1 \leq \ell \leq n)$ depends on the precedence-relationships of the system.

B. Max-Plus Linear Systems

Max-Plus Linear (MPL) systems are defined as the system described its behavior in a linear form by using Max-Plus algebra. MPL systems are similar to the state equation in the modern control theory. MPL system is expressed as follows :

$$\begin{aligned} x(k+1) &= A \otimes x(k) \oplus B \otimes u(k+1), \\ y(k) &= C \otimes x(k), \end{aligned} \quad (6.)$$

where is called an event counter and represents the number of occurrences of the events from the initial state. Moreover,

$x(k)$, $u($ and $y($ are vectors representing state, inputs and outputs variables, respectively. The number of their dimensions is , and , respectively. , and are referred to as the system, input and output matrices, respectively. More details of Max-Plus algebra and MPL systems are reviewed in [1].

III. EXPENDED STATE EQUATION

A. Previous Research

In this section, we briefly explain the model proposed by [5].As mention in introduction, this model describes the general scheduling method using the MPL systems, not specialized in railway system. We explain the assumption according to the railway system. The railway system in the previous research has the following constraints:

- The number of stations is n, α is the number of external inputs, and β is the number of external outputs.
- Constraints on in-process jobs are imposed only between stations and are not affected by external inputs or outputs.
- Other constraints such as no-concurrency, parallel processing, or synchronization are taken into account in analogous way to conventional MPL representation (e.g. [1]).

We explain the maximum and minimum number of trains. The maximum number of trains indicates the number of trains that can exist between stations. The section in which the maximum number of trains is set is called the maximum number of trains section. The reason why the maximum number of trains is taken into consideration is to prevent an increase in delay time when a delay occurs in the train. The minimum number of trains indicates the number of trains that must be present at minimum between stations. The section in which the minimum number of trains is set is called the minimum number of trains section. The reason why minimum number of trains is taken into consideration is that the delayed schedule can be returned quickly when a delay occurs in the train.

We define the matrices representing the state of the railway system as follows:

$$[P_0]_{ij} = \begin{cases} p_i : \text{if } i = j. \\ \varepsilon : \text{if } i \neq j. \end{cases} \quad (7.)$$

$$[F_0]_{ij} = \begin{cases} d_{ij}: \text{station } i \text{ has} \\ \text{a preceding station } j. \\ \varepsilon: \text{station } i \text{ does not} \\ \text{have a preceding} \\ \text{station } j. \end{cases} \quad (8.)$$

$$[B_0]_{ij} = \begin{cases} e: \text{station } i \text{ has} \\ \text{an external input } j. \\ \varepsilon: \text{station } i \text{ does not} \\ \text{have an external input } j. \end{cases} \quad (9.)$$

$$[H_0^{(h)}]_{ij} = \begin{cases} e: \text{the maximum number of train is } h \\ \text{between station } i \\ \text{and its downstream station } j. \\ \varepsilon: \textit{there are not any constraints} \\ \text{for the maximum number of train} \\ \text{between stations } i \text{ and } j. \end{cases} \quad (10.)$$

$$[L_0^{(l)}]_{ij} = \begin{cases} e: \text{the minimum number of train is } l \\ \text{between station } i \text{ and its upstream} \\ \text{staion } j. \\ \varepsilon: \text{there are not any constraints} \\ \text{for the minimum number of train} \\ \text{between stations } i \text{ and } j. \end{cases} \quad (11.)$$

Wherep_i and d_{ij} represents stop time of the train at the station i and time of movement of train from station i to station j.

If train number is k, the number of stations is n, the maximum number of train is H, the minimum number of train is L, the earliest departure time $x_E(k)$,then the state equation of railway system can be written as follows:

$$X_E(k) = \Gamma^* \otimes (\Delta_k \oplus X_k^- \oplus X_k^+), \quad (12.)$$

Where

$$\Gamma = \begin{bmatrix} P_0 & \epsilon_{nn} & \cdots & \epsilon_{nn} \\ \epsilon_{nn} & P_0 & \cdots & \epsilon_{nn} \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon_{nn} & \epsilon_{nn} & \cdots & P_0 \end{bmatrix} \otimes \begin{bmatrix} F_0 & L_0^{(1)} & \cdots & \epsilon_{nn} \\ H_0^{(1)} & F_0 & \cdots & \vdots \\ \vdots & \vdots & \ddots & L_0^{(1)} \\ \epsilon_{nn} & \cdots & H_0^{(1)} & F_0 \end{bmatrix} \quad (13.)$$

$$X_E(k) = \begin{bmatrix} x_E(k) \\ x_E(k+1) \\ \vdots \\ x_E(k+n-1) \end{bmatrix}, \quad (14.)$$

$$\Delta_k = \begin{bmatrix} B_0 u(k) \\ B_0 u(k+1) \\ \vdots \\ B_0 u(k+n-1) \end{bmatrix}, \quad (15.)$$

$$X_k^- = \begin{bmatrix} H_0^{(H)} & H_0^{(H-1)} & \cdots & H_0^{(1)} \\ \epsilon_{nn} & H_0^{(H-2)} & \cdots & H_0^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon_{nn} & \epsilon_{nn} & \cdots & \epsilon_{nn} \end{bmatrix} \otimes \begin{bmatrix} x_E(k-H) \\ x_E(k-H- \\ \vdots \\ x_E(k-1) \end{bmatrix} \quad (16.)$$

,

$$X_k^+ = \begin{bmatrix} \epsilon_{nn} & \epsilon_{nn} & \cdots & \epsilon_{nn} \\ \vdots & \ddots & \vdots & \vdots \\ L_0^{(2)} & \cdots & L_0^{(L)} & \epsilon_{nn} \\ L_0^{(1)} & \cdots & L_0^{(L-1)} & L_0^{(L)} \end{bmatrix} \otimes \begin{bmatrix} x_E(k+1) \\ x_E(k+2) \\ \vdots \\ x_E(k+L-1 \end{bmatrix} \quad (17.)$$

We describe the method of rescheduling if the delay occurs. Suppose that one or more trains are delayed. At that time, the value of state vector $X_E(k)$ representing the earliest departure time is changed to the up dated value $\tilde{X}_E(k)$. On the assumption is $\tilde{X}_E(k) \gg X_E(k)$, the state equation which is rescheduled can be written as follows:

$$\tilde{X}_E(k) = \Gamma^* \otimes \tilde{X}_E(k). \quad (18.)$$

From Eq.(18) , it is possible to obtain the earliest departure time after rescheduling by using updated state vectors $\tilde{X}_E(k)$ and Γ^*

B. *Expended State Equation*

In model of [5], all trains stop at all stations. However, in reality, there are few routes that all trains stop at all stations. It is necessary to extend the model so that it can accommodate with more routes (e.g. considering express lines and trains with different stops for each train). In this research, we extended the train so that it can deal with routes with different stops.

We describe the following assumptions to extend the model:

- All trains go through two or more stations.
- Every train goes through all midway stations sequentially from upstream to downstream. However, train does not necessarily stop in all midway stations.
- Any succeeding trains never overpass the preceding ones.
- The route may have a “join” structure. If the route has a “join” structure, trains may have single or multiple starting stations. In this case, the branched trains with the same train number must be merged in the joining station.
- The route may have a “fork” structure. If the line has a “fork” structure, trains may have single or multiple terminal stations. In this case, the trains must be separated in the branching station.

Moreover, we define the following system matrices:

$$[F_k]_{ij} = \begin{cases} d_{ij}(k)\text{: station } i \text{ has} \\ \text{a preceding station } j \text{ in train } k. \\ \epsilon\text{: station } i \text{ does not} \\ \text{have a preceding} \\ \text{station } j \text{ in train } k. \end{cases} \quad (19.)$$

$$[B_k]_{ij} = \begin{cases} p_i(k)\text{: station } i \text{ has} \\ \text{an external input } j \text{ in train } k. \\ \epsilon\text{: station } i \text{ does not} \\ \text{have an external input } j \text{ in train } k. \end{cases} \quad (20.)$$

$$[R_k]_{ij} = \begin{cases} e\text{: if } i = j \text{ and} \\ \text{the train stops station } i. \\ \epsilon\text{: if } i \neq j \text{ or the train does not} \\ \text{stop station } i. \end{cases} \quad (21.)$$

$$[L_a^{(s)}]_{ij} = \begin{cases} e\text{: the minimum number of trains} \\ \text{between station } i \text{ and} \\ \text{its upstream station } j \text{ is } s. \\ \epsilon\text{: there is no constraint regarding} \\ \text{the minimum number of trains} \\ \text{between stations } i \text{ and } j. \end{cases} \quad (22.)$$

$$[L_k^{(s)}]_{jf} = \begin{cases} e\text{: the minimum number} \\ \text{of trains between station } i \\ \text{and its upstream} \\ \text{station } j \text{ is } s \text{ and} \\ \text{the terminal station } f \\ \text{is inside of the section} \\ \text{i.e. } (j \succeq f \succeq i) \text{ in the train } k. \\ \epsilon\text{: there is no constraint} \\ \text{regarding the minimum number} \\ \text{of trains between} \\ \text{stations } i \text{ and } j \text{ in the train } k. \end{cases} \quad (23.)$$

Where $p_i(k)$ and $d_{ij}(k)$ represents stop time of the train k at the station i and time of movement of train k from station i to station j, respectively. $x \succeq y$ Expresses that station y is located downstream of x or is equivalent to station x itself. The added system matrix represents the stopping station of the train number k and it is set for each train. We also need to extend the existing matrix. The matrix to be extended is Γ_k, X_k^- and X_k^+:

$$\Gamma_k = \begin{bmatrix} P_k & \epsilon_{nn} & \cdots & \epsilon_{nn} \\ \epsilon_{nn} & P_k & \cdots & \epsilon_{nn} \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon_{nn} & \epsilon_{nn} & \cdots & P_k \end{bmatrix} \otimes \begin{bmatrix} F_k & L^{(1)} & \cdots \\ H_0^{(1)} & F_{k+1} & \cdots \\ \vdots & \vdots & \ddots \\ \epsilon_{nn} & \cdots & H_0^{(1)} \end{bmatrix} \quad (24.)$$

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$$X_k^- = \begin{bmatrix} H_0^{(H)} & H_0^{(H-1)} & \cdots & H_0^{(1)} \\ \epsilon_{nn} & H_0^{(H-2)} & \cdots & H_0^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon_{nn} & \epsilon_{nn} & \cdots & \epsilon_{nn} \end{bmatrix} \otimes \begin{bmatrix} x_E\{k| - H \\ x_E\{k| - H + \\ \vdots \\ x_E\{k| - 1\} \end{bmatrix} \quad (25.)$$

,

$$X_k^+ = \begin{bmatrix} L^{(L)} & L^{(L-1)} & \cdots & L^{(1)} \\ \vdots & L^{(L)} & \cdots & L^{(2)} \\ \epsilon_{nn} & \cdots & \ddots & \vdots \\ \epsilon_{nn} & \cdots & \epsilon_{nn} & \epsilon_{nn} \end{bmatrix} \otimes \begin{bmatrix} x\{k|+L\} \\ x\{k|+L-1\} \\ \vdots \\ x\{k|+1\} \end{bmatrix}, \quad (26.)$$

where $L^{(s)}$ means $L_a^{(s)}$ or $L_k^{(s)}$, which satisfies the condition in Eq.(22) or Eq.(23). $x_E\{k| - h\}$ is vector indicating the earliest departure time before hth trains of kth train, $x_E\{k| + l\}$ is vector indicating the departure time after lth trains in the minimum number section of kth train.

The extended state equation is written as follows:

$$X(k) = \Gamma_k^* \otimes (\Delta_k \oplus X_k^- \oplus X_k^+), \quad x_E(k) = R_k \otimes x(k). \quad (27.)$$

We describe a method of rescheduling in case of delay. The train performing rescheduling substitutes ϵ for the arrival time of the train Δ_k for station i that needs rescheduling, and again calculates Eq.(27).

IV. NUMERICAL SIMULATIONS

Numerical simulations are conducted to confirm the extended scheduling method.

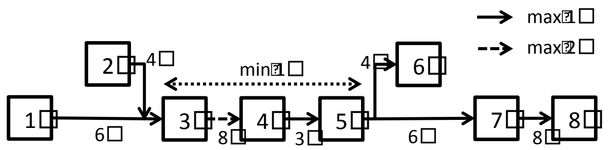

Fig. 1 : Route map with eight stations

Figure 1 shows a route map of an imaginary line with eight stations. Suppose that the trains stop for 1 time unit in each station. The traveling/moving time between each station is set as shown in Fig. 1. We set the maximum number of trains 2 between station 3 and station 4, and set the maximum number of trains 1 between other stations. Also, the minimum number of trains 1 is set in the section between station 3 and station 5.

Sta. \ No.	1	2	3	4	5	6	7	8	9	10
1	–		–	–					–	
2			–		–	–				–
3			–							
4			–				–	–		
5						–	–	–		
6	–		–		–	–	–	–	–	
7	–			–	–	–	–	–	–	
8	–			–	–	–	–	–	–	–

Fig. 2 : Route diagram of each train

Figure 2 shows a route diagram for each train. The station number is taken on the vertical axis and the train number is taken on the horizontal axis. In this case, the number of trains is 10. In the route diagram, the train stops at the filled station and does not stop at the station where "-" is written. For example, train number 2 departs from the station 1 and station 2, and joins at station 3. After that, the train is separated at the station 5, branching station number 6 and station number 7, each goes terminal stations. Using the route map of Fig.1 and route diagram of Fig.2, we perform the simulation. The matrices representing this system of route map and diagram are represented as follows:

$$P_k = \mathrm{diag}(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1), \quad (28.)$$

$$F_1 = \begin{pmatrix} \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & 4 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & 8 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & 3 & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \end{pmatrix}, \quad (29.)$$

$$F_2 = \begin{pmatrix} \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ 6 & 4 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & 8 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & 3 & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & 4 & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & 6 & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 8 & \epsilon \end{pmatrix}, \quad (30.)$$

$$H_0^{(1)} = \begin{pmatrix} e & \epsilon & e & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & e & e & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & e & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & e & e & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & e & e & e & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & e & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & e & e \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & e \end{pmatrix}, \quad (31.)$$

$$H_0^{(2)} = \begin{pmatrix} \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & e & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \end{pmatrix}, \quad (32.)$$

$$L_a^{(1)} = \begin{pmatrix} \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & e & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \end{pmatrix}, \quad (33.)$$

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$$L_6^{(1)} = \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & e & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{pmatrix}, \quad (34.)$$

$$R_1 = \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & e & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & e & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & e & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{pmatrix}, \quad (35.)$$

$$R_2 = \begin{pmatrix} e & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & e & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & e & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & e & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e \end{pmatrix}. \quad (36.)$$

Note that regarding $\boldsymbol{F}_k$ and $\boldsymbol{R}_k$, due to space reasons, only train number 1 and 2 are shown here.

The time when the operation of the train started was set to 0 i.e. the train number 1 starts at the time 0 from station 2.Figure 3 shows the calculation result of the earliest departure time during normal operation. In Fig. 3, the vertical axis indicates the station number and the horizontal axis indicates the train number.

First, the train number 1 arrives at the station 2 and departs at time 1. Since there is no delay in the train with train number 1, it will be operated according to the required time set between the stations. The train number 2 departs from station 1 and station 2 when the train number 1 departs at station number 3 at time 6. From this, it can be confirmed that it is scheduled according to the precedence constraint. Since the maximum number of trains is two between the station 3 and station 4, the departure of the station 3 at 13 is earlier than 15 that is the departure time of station 3 of the preceding train 1. Next, we confirm the train number 4. The train number 4 can arrive if the train number 2 departs from station 3. This is because that the maximum number of trains between the station 2 and station 3 is one and that the train number 3 does not operate the station 3. The train number 4 then departs station 4 at time 33. The reason is that the maximum number of trains is one between the station 4 and the station 5, and the train of the preceding train number 3 departs from the station 5 is the time 33. The departure time of train number 6 at station 3 is 33, but the departure time at station 4 is 52. Since the movement time between station 3 and station 4 is 8, a delay of departure of has occurred. This is because the minimum number of trains section is set from station 3 to station 5, trains number 7 and train number 8 does not pass through the minimum number of trains section. Since the train number 7 and train number 8 does not pass through the minimum number of trains section, the departure time at station 3 of train number 9 is the departure time at station 4 of train number 6. It can be confirmed that the following trains are also scheduled according to the precedence constraint.

Sta. \ No.	1	2	3	4	5	6	7	8	9	10
1	–	6	–	–	18	25	33	40	–	52
2	1	6	–	13	–	–	33	40	47	–
3	6	13	–	18	25	33	40	47	52	59
4	15	22	–	33	37	52	–	–	61	68
5	19	26	33	37	41	–	–	–	65	72
6	–	31	–	42	–	–	–	–	–	77
7	–	33	42	–	–	–	–	–	–	79
8	–	42	51	–	–	–	–	–	–	–

Fig. 3: Earliest departure times for normal operation

Next, let us consider a case when train number 2 causes a delay of 10 time units at the station 3. A delay occurs and the result of rescheduling is shown in Fig. 4. The train number 1 departs the station 2 at time 1. However, though train number 1 does not have preceding trains, the delay of 6 time units has occurred at station 5 before the rescheduling. The reason for this is that the minimum number of trains is set between station 3 and station 5. Since the train number 2 is delayed at station 3, there is delay in the time when the train number 3 operates the minimum number of trains section. Therefore, a train number 1 also has a delay, and it is necessary to reschedule the train before the delay. Next, the train number 4 will arrive at station 3 if the train number 2 departs from station 3 even if delay occurs. This is because the maximum number of trains between stations is 2 as in the case where there is no delay. As the delay occurred, it is understood that the operation is operated considering the departure time of the preceding train. It can be confirmed that the following trains are also rescheduled according to the precedence constraint.

Sta. \ No.	1	2	3	4	5	6	7	8	9	10
1	–	6	–	–	28	35	43	50	–	62
2	1	6	–	23	–	–	43	50	57	–
3	6	23	–	28	35	43	50	57	62	69
4	15	32	–	43	47	62	–	–	71	78
5	23	36	43	47	51	–	–	–	75	82
6	–	41	–	52	–	–	–	–	–	87
7	–	43	52	–	–	–	–	–	–	89
8	–	52	61	–	–	–	–	–	–	–

Fig. 4: Earliest departure times when the delay occurs

V. SUMMARY

In our research, we have developed a scheduling method for actual railway systems based on Max-Plus Linear systems. In this paper, as the first step, we extended the existing model so that the different intermediate stations could be considered based on the model that state equation was simple. We also considered the maximum and minimum number of trains. In addition, numerical experiments with a simple route map and diagram were carried out. Then, it was confirmed that the scheduling and rescheduling was performed correctly.

However, the route and conditions were ideal. Therefore, in future, we will have to conduct the scheduling using the actual route map and diagram. Then, we will be able to compare the results with the actual timetable. In actual railway systems, there are cases when train overtakes the preceding trains. However, since our expanded model had the assumption “Any succeeding trains never overpass the preceding ones”, of course, it is impossible to consider overtaking. If we can consider the overtaking, we can approach the actual railway systems. It will discuss elsewhere.

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AUTHOR BIOGRAPHY

Yumiko Kusunoki is a currently graduate student in the Department of Information and Management Systems Engineering, Nagaoka University of Technology (NUT), Japan. Her research interests include operations research and Controller Design for Discrete Event Systems. She is a student member of the Operations Research Society of Japan.

Hirotaka Takahashi is currently an Associate Professor of the Department of Information and Management Systems Engineering, Nagaoka University of Technology (NUT), Japan. He received his D.Sci. From the Graduate School of Science and Technology at Niigata University, Japan in 2005. He was a PD at Max Planck Institute for Gravitational Physics (Albert Einstein Institute) in Germany from 2005 to 2007. His research interests include Controller Design for Discrete Event Systems, Data Science and Physics (General Relativity), Gravitational Astronomy. He is a member of the Operations Research Society of Japan (ORSJ) and the Physical Society of Japan (JPS) etc.