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# Some new families of cnp graphs

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***Abstract:* *In this paper we investigate new families of closed neighborhood prime graphs (cnp).We show that $C_m$, One point union of k copies of $C_n$ given by $(Cn)^{(k)}$,Wheel graph Wn, Gear graph $G_m$,Shell graph Snare families of closed neighborhood prime (cnp) graphs.***

***Key words:*** **Closed, Neighborhood, prime, crowns, path unions, one point union, labeling etc.**

**Subject Classification: O5C78.**

## I.INTRODUCTION

The graphs considered in this paper are finite, simple and connected and not directed. Let G be a (p,q) graph. For definitions and terminology we depend on Gallian [4] and Harary [5]. Patel and Shrimali [7] has introduced neighborhood prime labeling of graph. We have introduced closed neighborhood prime labeling [3] In this labeling we have considered closed neighborhood of a vertex v given by $N_{[v]}$ .Define a bijective function f: V (G)→ {1,2, 3, …p} such that gcd of all labels of vertices incident with v including label of v is 1.i.e gcd $\{f(u)/u\epsilon N_{[v]}\}$ should be 1 . This is true for every vertex v in V(G) except the isolated vertices.The graph for which such a function f is defined then graph G is called as closed neighborhood Prime graph.(Cnp graph)And the function f is called as Closed Neighborhood Prime function. We show that the graphs $C_n^{(k)}$2)$C_n^{(k)}$3)$W_n$4)$G_n^{(k)}$5)$C_n^+$ and the path unions

1) Path union of $W_n$ given by G = $P_m(W_n)$.

2) Path union of G' = FL ($C_n$ ) given by G = $P_m$(G')

3)Pm($C_n^+$), the path union of $C_n^+$ are cnp graph families.

We also show that different nonisomorphic structures available on path union are cnp graphs. Following observations play important role in deciding the gcd of collection of positive numbers.

1) G.C.D. of any two consecutive integers is one.
2) If the set of numbers contains the number one the G.C.D. is equal to one.
3) If the set contain a prime and no multiple of it then G.C.D. is one.

## II. PRELIMINARIES

**Fusion of vertex**. Let G be a (p,q) graph. Let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges. [].If u$\epsilon G_1$ and v$\epsilon G_2$, where $G_1$ is $(p_1,q_1)$ and $G_2$ is $(p_2,q_2)$ graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has $p_1+p_2-1$ vertices and $q_1 + q_2$ edges. Sometimes this is referred as u is identified with v.

**Path union of G** i.e. $P_m$ (G) is obtained by taking a path $P_m$ and m copies of graph G. Fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq +m-1 edges, where G is a (p, q) graph. If we change the vertex on G that is fused with vertex of Pm then we generally get a path union non isomorphic to earlier structure. In this paper we define a e-cordial function f that does not depends on which vertex of given graph G is used to obtain path union. This allows us to obtain path union in which the same graph G is fused with vertices of Pm at different vertices of G, as our choice and the same function f is applicable to all such structures that are possible on $P_m$(G).

**$G^+$** is a crown graph which is same as G⊙$K_2$.Initially crown was defined for $C_n$. It is obtained by fusing an edge each to every vertex of G. If G is a (p,q) graph then $G^+$ has 2p vertices and q+p edges. In this paper we discuss crown of $C_n$.

**Fl (G)** is a graph obtained from (p,q) graph G by fusing an edge with any vertex of G .The resultant graph has p+1 vertices and q+1 edges.In this paper we discuss Fl($C_n$).

**$G^{(K)}$** it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If G is a (p, q) graph then $|V (G_{(k)}| = k(p-1)+1$and $|E(G)|= k.q$

## III. THEOREMS

**Theorem 1** $C_n$ is cnp graph.

Proof. Define $C_n$ with ordinary labeling as ($v_1$,$c_1$, $v_2$, $c_2$, ..$v_{n-1}$, $c_n$, $v_1$) where $C_i$=($v_iv_{i+1}$),i= 1, 2, ..n. and i+1 taken (modn). Define f:V($C_n$)→{1, 2, ..,n} as follows:

f($v_i$)=i; i= 1, 2, 3..,n.

For any vertex of $C_n$ the label of itself and one of the neighborhood vertex label are consecutive integers. Therefore the collection of labels of vertex itself and label of it's neighbor have gcd equal to 1. Thus the graph is cnp graph.

**Theorem 2** G = $C_n^{(k)}$ is cnp graph.

Proof. The vertices of $t^{th}$ copy of $C_n$ are given by ($v_{t,1}$, $v_{t,2}$, $v_{t,3}$, .. $v_{t,n}$) ; t = 1, 2, ..k. The vertex common to all copies be $v_{t,1}$ for t= 1, 2, ..k.

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Define a function $f:V(G) \rightarrow \{1,2,..|V(G)|\}$ by :

$f(v_{t,1}) = 1$; $f(v_{1,i}) = i$, i = 2, 3, ..n.

$f(v_{i,j}) = n+(i-2)(n-1)+j-1$, i = 2, 3, ..k and j = 2, 3, ..n.

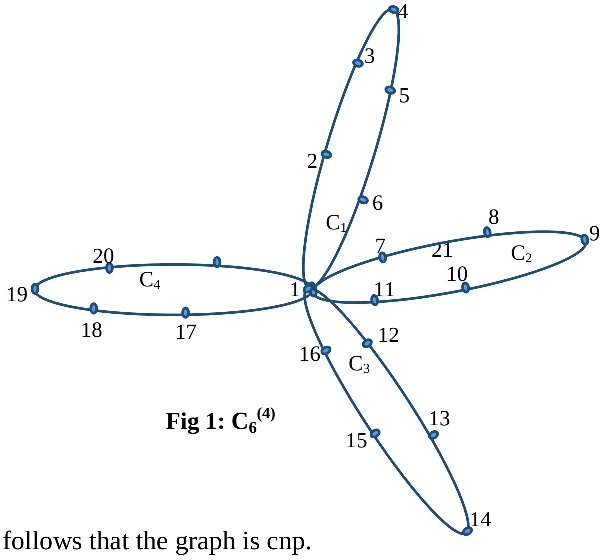


**Fig 1: $C_6^{(4)}$**

It follows that the graph is cnp.

**Theorem 3**.G = $W_n$ is cnp graph.

Proof: The $W_n$ be given ordinary names as: the cycle $C_n$ vertices as $v_1$, $v_2$, ,, $v_n$. and the hub be given by w.Define a function $f : V(G) \rightarrow \{1,2,..,(n+1)\}$ by :

f(w) =1;

$f(v_i) = i+1$, i = 1, 2, ,n.

**Theorem 4**.G = $W_n^{(k)}$ is cnp graph.

Proof: The $t^{th}$ copy of $W_n$ be given ordinary labels as follows: The hub which is common to all copies is w.The cycle vertices be given by $v_{t,1}$, $v_{t,2}$, ..$v_{t,n}$; t = 1, 2, ..k. Note w is adjacent to $v_{t,j}$ for all j= 1, 2, ..n. for all t= 1, 2, .. k.

Define a function $f:V(G) \rightarrow \{1,2,..|V(G)|\}$ by :

f(w) = 1;At this point the union is taken.

$f(v_{t,j}) = (t-1)(n)+1 + j$; j=1, 2, 3, ..n; t = 1, 2, ..k

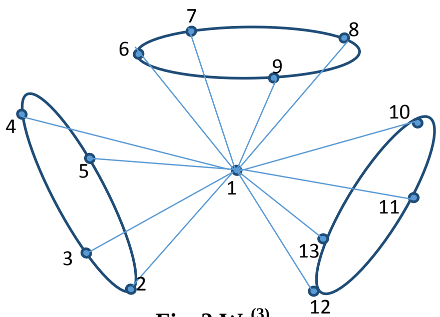


**Fig. 2 $W_4^{(3)}$**

Thus $W_n^{(k)}$ is cnp graph.

Suppose we are interested in changing point common to all copies in $(W_n)^{(k)}$ which is hub in this case to a cycle vertex say $v_{t,j}$ ,then following change in f is required.

$f(v_{t,j}) = 1$ for all k and fixed t and label of $w_i$ is redefined as label of $v_{t,j}$ which is given by (t-1)(n)+1 + j; j=1, 2, 3, ..n; t = 1, 2, ..k. The resultant graph is cnp.

**Theorem 5** Gear graph G = $G_n$ is cnp graph.

Proof: We define gear graph with ordinary labeling as: The $C_{2n}$ cycle vertices are ($v_1$, $v_2$,$v_3$, ..$v_{2n}$). The hub vertex is w.Define a function $f:V(G) \rightarrow \{1,2,.., (2n+1)\}$ by :

f(w) = 1;

$f(v_i) = i+1$.The resultant function is Cnp function.

**Theorem 6** G=$G_n^{(k)}$ is cnp graph.

Proof : Let the j th copy of $G_n$ in $G_n^{(k)}$ be given by : The cycle vertices are ($v_{j,1}$ $v_{j,2}$, .., $v_{j,2n}$) .The hub be w. It is common for all copies of $G_n$.

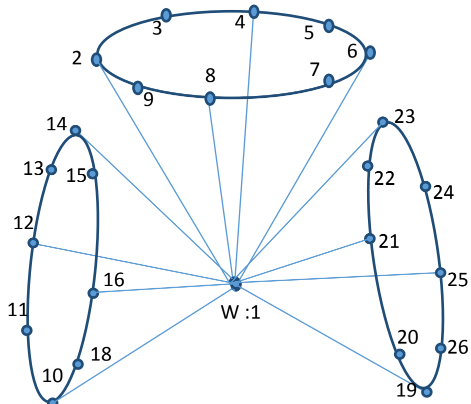


**Fig. 3 $G_4^{(3)}$**

Define a function $f: V(G) \rightarrow \{1,2,..|V(G)|\}$ by:

f(w) =1 ;

$f(v_{j,m}) = 1+ 2(j-1)n+m$.i = 1,2, …,k; m = 1, 2, ..2n

Shift the point common on $G^{(K)}$ which is hub in present example to any cycle point say $v_{j,m}$ one change in f defined above is required. $f(v_{j,m}) = 1$ for all j and fixed m and label of $w_i$ is redefined as label of $v_{j,m}$ which is given by 1 + 2(j-1)n + m. i = 1,2, …,k; m = is fixed vertex. The resultant graph is cnp.

**Theorem 7** Path union of$C_n$ given by G = $P_m(C_n)$ is cnp prime.

Proof: We start with a path $P_m$=( $v_1$, $e_1$, $v_2$, $e_2$,…$v_m$). At each vertex of $P_m$ a copy of $C_n$ is fused. The consecutive vertices of copy of $C_n$ fused at vertex $v_t$ of $P_m$ is given by ($v_{t,1}$ $v_{t,2}$ ..$v_{t,n}$). The point of fusion is vertex $v_{t,1}$= $v_t$.. Define a function

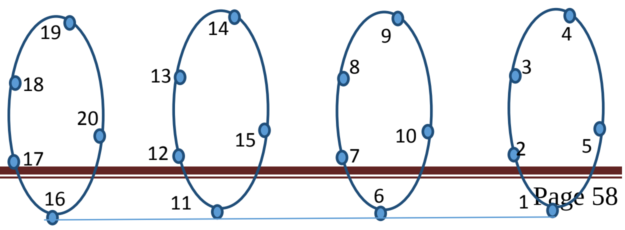


**Fig. 4 $P_4$ ($C_5$)**

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f:V(G)→{ 1,2,..|V(G)|} by :
$f(v_{t,j}) = (t-1)n+j$; j = 1, 2, ..n.

Each copy of cycle is independently cnp. Therefore which vertex on $C_n$ is fused with the vertex of Pm ios not important in deciding cnp property? Thus the graph is cnp.

**Theorem 8** Path union of $W_n$ given by G = $P_m(W_n)$ is cnp graph. `
Proof: We start with a path $P_m$ =( $v_1$, $e_1$, $v_2$, $e_2$,…$v_m$). At each vertex of $P_m$ a copy of $W_n$ is fused. The consecutive vertices of copy of $W_n$ fused at vertex $v_t$ of $P_m$ is given by ($v_{t,1}$ $v_{t,2}$ ..$v_{t,n}$ and hub $w_t$.). The point of fusion is vertex $v_{t,1}$= $w_t$..
Define a function f:V(G)→{ 1,2,..,|V(G)|} by :
$f(v_{t,j}) = (t-1)(n+1)+j$; j = 1, 2, ..n.

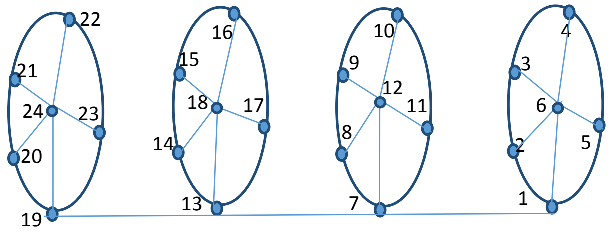


**Fig. 5 $P_4(W_5)$**

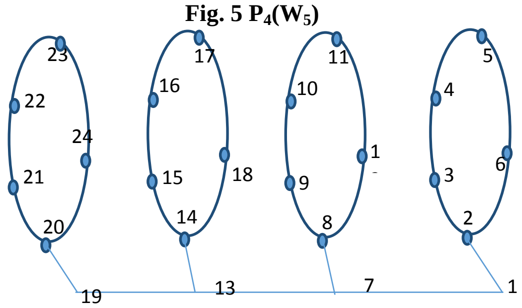


**Fig. 6 $P_4$ (FL ($C_5$): structure 1**

There are two structures possible on $P_m$ ($W_n$) depending on if we use hub or a cycle vertex to fuse with vertex of $P_m$. Each copy of wheel $W_n$ is independently cnp. Therefore hub or the vertex on $C_n$ is fused with the vertex of $P_m$ is not important in deciding cnp property. If we do away the restriction that the same fixed vertex on $W_n$ be fused with vertices of $p_m$,we will get irregular mixed $P_m(W_n)$.The same function f will be applicable and gives the resultant graph as cnp.

Thus the graph is cnp.

**Theorem 9**.Path union of G' = FL ($C_n$) given by G = $P_m$(G') is cnp graph.
Proof: We start with a path $P_m$= ( $v_1$, $e_1$, $v_2$, $e_2$,…$v_m$). At each vertex of $P_m$ a copy of FL ($C_n$) is fused. The consecutive vertices of copy of FL ($C_n$) fused at vertex $v_t$ of $P_m$ is given by ($v_{t,1}$ $v_{t,2}$ ..$v_{t,n}$ and $v_{t,n+1}$).Where $v_{t,n+1}$is the pendent vertex attached to $v_{t,1}$ at $t^{th}$ copy and The point of fusion is vertex $v_{t,1}$= $v_t$..

Define a function f:V(G)→{ 1,2,..|V(G)|} by :
$f(v_{t,n+1}) = (t-1)(n+1)+1$;

$f(v_{t,j}) = (t-1)(n+1)+j+1$; j = 1, 2, ..n.

In regular path union we fuse a given fix point from each copy of given graph with vertices of $P_m$ to obtain path union. But in case at different points of $P_m$ we can fuse different points of FL ($C_n$) which results in irregular path union is also cnp graph. This is because each copy of FL ($C_n$) is independently cnp. (The vertices in closed neighborhood includes labels which are consecutive integers).

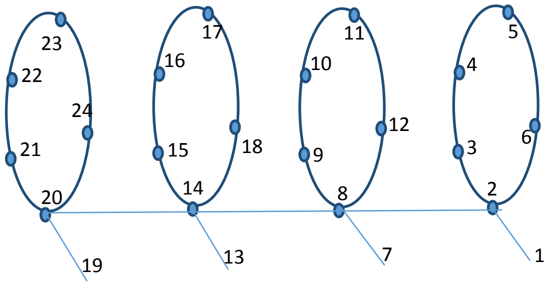


**Fig. 7: $P_4$ (FL ($C_n$) $P_4$ ($C_5$): structure 2**

In the above diagram only two structures are shown. The other two structures are not shown. These structures are possible by taking path union on vertex with label 3+6x, or vertex 4+6x;x= 0,1, 2,3.The irregular path union has no restriction of vertex on FL($C_n$) to fuse with vertex of path Pm.In all these cases the graph is cnp.

**Theorem 10.**7$C_n^+$ is cnp graph.

Proof: In ordinary labeling of $C_n^+$ we have $V(C_n^+)$ =($v_1$, $v_2$, ..$v_n$, $v_{i,1}$), i = 1, 2, ..n. $E(C_n^+)$= {$e_j$= ($v_jv_{j+1}$), j = 1,2,..n} U { $C_i$=($v_iv_{i,1}$).

Define a function f:V(G)→{ 1,2,..|V(G)|} by
$f(v_i) = 2(i-1)+ 1$, i = 1,2,..n.
$f(v_{i,1}) = f(v_i) +1$, i = 1, 2, ..n

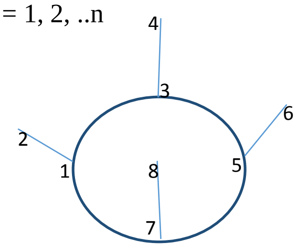


**Fig 8 $C_n^+$ : labeled copy.**

Thus the graph is cnp.

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**Theorem 11** G = Pm($C_n^+$), the path union of $C_n^+$ is cnp graph.

Proof: We define the path $P_m$as ( $v_1$, $e_1$, $v_2$, $e_2$,…$v_m$). At each vertex of $P_m$ a copy of $C_n^+$ is fused. The vertices of copy of $C_n^+$ fused at vertex $v_t$ of $P_m$ is given by {$v_{t,1}$ $v_{t,2}$ ..$v_{t,n}$}U { pendent vertices $u_{t,i}$ at the vertex $v_{t,i}$ of $C_n$;i = 1, 2, ..m) }.The $C_n^+$ is fused at its vertex $v_{t,1}$ with vertex $v_t$ of $P_m$. ( t = 1,2,..m).

Define a function f:V(G)→{ 1,2,..|V(G)|} by :

$f(v_{t,i}) = (t-1)2n+(2i-1)$;

$f(u_{t,i}) = f(v_{t,i})+1$.

The function f defined above is independent of the choice of vertex on $C_n^+$ used to fuse with vertex of Pm is designing the path union.

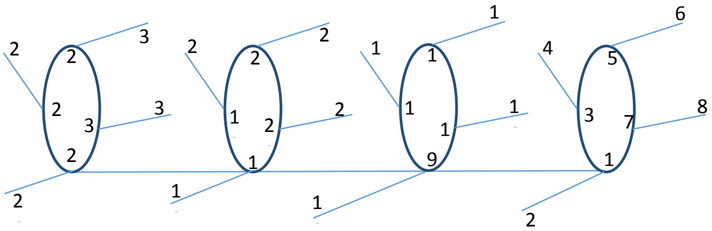

Fig 9 : $P_4(C_4^+)$ : labeled copy.

## IV. CONCLUSION

We have taken cycle related graphs and obtain their one point union and path union. we have shown that all these are cnp.To obtain irregular path union $P_m(G)$ we first obtain cnp labeling of path union and then change the point of contact at different nodes of Pm. This is because the individual labeling of G at any node is independent of point of contact with $P_m$.

The results obtained are: The cnp graphs 1)$C_n^{(k)}$ 2)$C_n^{(k)}$ 3)$W_n$ 4)$G_n^{(k)}$ 5)$C_n^+$ These are the individual graphs.The path unions proved to be cnp graphs are

1) Path union of $W_n$ given by G = $P_m(W_n)$
2) Path union of G' = FL($C_n$ ) given by G = $P_m$(G')
3) Pm($C_n^+$), the path union of $C_n^+$ is cnp graph.

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