

# Path union of wheel related tail graphs as e-cordial families

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**Abstract:** In this paper we study path union of graphs  $P_m(G)$  obtained from attaching a path to wheel  $W_4$  graph.

**Key words:** E-cordial, path union, fusion, edge, vertex, wheel. Gear graph.

**Subject Classification:** 05C78.

## I. INTRODUCTION

The graphs we discuss are simple, connected and finite. For terminology and definitions we depend on Graph Theory by Harary [4], Dynamic survey of graph labeling [3].and West [6].

In 1997 Yilmaz and Cahit [5] introduced a weaker version of edge graceful labeling called E-cordial. The word cordial was used first time in this paper. Let  $G$  be a graph with vertex set  $V$  and edge set  $E$ . Let  $f$  be a function that maps  $E$  into  $\{0,1\}$ . Define  $f$  on  $V$  by  $f(v) = \sum_{\{f(uv)/(uv) \in E\}} \pmod{2}$ . The function  $f$  is called as E cordial labeling if  $|e_f(0) - e_f(1)| \leq 1$  and  $|v_f(0) - v_f(1)| \leq 1$ . Where  $e_f(i)$  is the number of edges labeled with  $i = 0,1$  and  $v_f(i)$  is the number of vertices labeled with  $i = 0,1$ . We also use  $v_f(0,1) = (a,b)$  to denote the number of vertices labeled with 0 are  $a$  in number and that with 1 are  $b$  in number. Similarly  $e_f(0,1) = (x,y)$  to denote number of edges labeled with 0 are  $x$  in number and that labeled with 1 are  $y$  in number. A lot of work has been done in this type of labeling and the above mentioned paper gave rise to number of papers on cordial labeling. A graph that admits E-cordial labeling is called as E-cordial graph. Yilmaz and Cahit has shown that Trees  $T_n$  with  $n$  vertices and Complete graphs  $K_n$  on  $n$  vertices are E-cordial iff  $n$  is not congruent to 2 (modulo 4). Friendship graph  $C_3^{(n)}$  for all  $n$  and fans  $F_n$  for  $n$  not congruent to 1 (mod 4). They observe that a graph with  $n$  vertices is not e-cordial if  $n \equiv 2 \pmod{4}$ . One may refer A Dynamic survey of graph labeling for more details on completed work.

In this paper we discuss path unions of graphs obtained from  $W_4$ . WE discuss  $\text{tail}W_4(P_m)$ . The e-cordial function  $f$  is independent of the vertex on  $G$  used to fuse on the vertex of path  $P_m$ .

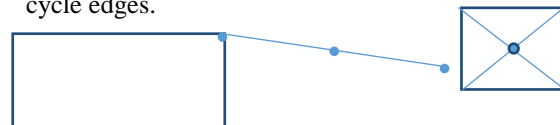
## II. PRELIMINARIES

**Fusion of vertex.** [6] Let  $G$  be a  $(p,q)$  graph. Let  $u \neq v$  be two vertices of  $G$ . We replace them with single vertex  $w$  and all edges incident with  $u$  and that with  $v$  are made incident with  $w$ . If a loop is formed is deleted. The new graph has  $p-1$  vertices and at least  $q-1$  edges.[6]. If  $u \in G_1$  and  $v \in G_2$ , where  $G_1$  is  $(p_1, q_1)$  and  $G_2$  is  $(p_2, q_2)$  graph. Take a new vertex  $w$  and all the edges incident to  $u$  and  $v$  are joined to  $w$  and vertices  $u$  and  $v$  are deleted. The new

graph has  $p_1+p_2-1$  vertices and  $q_1 + q_2$  edges. Sometimes this is referred as  $u$  is identified with  $v$ .

**Path union of  $G$**  i.e.  $P_m(G)$  [1] is obtained by taking a path  $P_m$  and  $m$  copies of graph  $G$ . Fuse a copy each of  $G$  at every vertex of path at given fixed point on  $G$ . It has  $mp$  vertices and  $mq + m-1$  edges, where  $G$  is a  $(p,q)$  graph. If we change the vertex on  $G$  that is fused with vertex of  $P_m$  then we generally get a path union non isomorphic to earlier structure. In this paper we define a e-cordial function  $f$  that does not depends on which vertex of given graph  $G$  is fused to obtain path union. This allows us to obtain path union in which the same graph  $G$  is fused with vertices of  $P_m$  at different vertices of  $G$ , as our choice and the same function  $f$  is applicable to all such structures that are possible on  $P_m(G)$ . 3) Tail graph  $\text{tail}(G, P_m)$  is obtained from  $G$  by fusing an end point of  $P_m$  with a vertex of  $G$ . If  $G$  is a  $(p,q)$  graph then  $\text{tail}(G, P_m)$  has  $p+m-1$  vertices and  $q+m-1$  edges. Below is copy of  $\text{tail}C_4, p_3$

**Wheel graph  $W_n$ .** Take a cycle  $C_n$  and a vertex  $w$  not on  $C_n$ . Join every vertex of  $C_n$  by an edge each to  $w$ . The resultant graph is wheel  $W_n$ . Sometimes the same graph is shown by  $W_{n+1}$ . It has  $n+1$  vertices and  $2n$  edges. The edges incident with  $w$  are called as pokes and vertex as hub. The other edges that lie on cycle  $C_n$  are called as cycle edges.



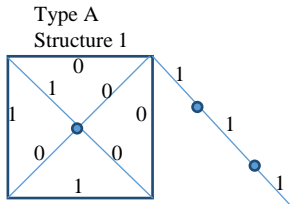
In all the diagrams given below the number indicates edge label numbers.

## III. RESULTS

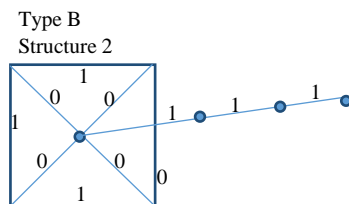
**Theorem 1.**  $P_m(G)$  is e-cordial where  $G = \text{tail}(W_4, P_4)$  is obtained by fusing  $P_4$  at it's pendent vertex with any of  $W_4$  vertex.

**Proof.** We take a path  $P_m = (v_1, v_2, \dots, v_m)$  and at each of it's vertex fuse a copy of  $G$  at a fixed vertex of  $G$ . Define a function  $f: E(G) \rightarrow \{0,1\}$  as follows: Label of every edge on path  $P_m$  is '0'. Atail can be fused with  $W_4$  at any  $C_4$  vertex of  $W_4$  or at hub. Using  $f$  we get following labeled copies of  $\text{tail}(W_4, P_4)$  which is E-cordial. We fuse Type A label with every vertex of  $P_m$  resulting in structure 1 and Type B in resulting structure 2. In any case it gives us E-cordial labeled copy of  $G$ . Moreover it is important to note that we get a mixed path union if we use type A label and Type B label alternately on  $P_m$ . Also we can fuse  $G$  at any of it's vertex with  $P_m$  and get some different

(non isomorphic to other structures) structure. The same function  $f$  will work for all structures as e-cordial labeling



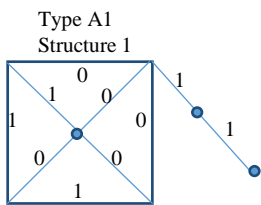
**Fig 1: tail( $W_4, P_4$ )**  
 $v_f(0,1) = (4,4)$ ,  
 $e_f(0,1) = (5,6)$



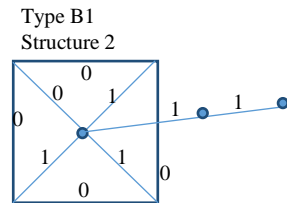
**Fig 2: tail( $W_4, P_4$ )**  
 $v_f(0,1) = (4,4)$ ,  
 $e_f(0,1) = (5,6)$

Fuse the above labeled copy with each vertex at given fixed point. The resultant graph has label distribution given by  $v_f(0,1) = (4m, 4m)$ ,  $e_f(0,1) = (5+6(m-1), 6+6(m-1))$  where  $m = 1, 2, 3$ . Thus the graph is e-cordial for all  $m$ . #

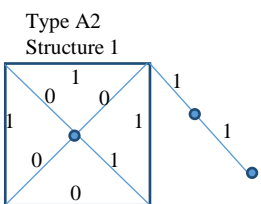
Theorem 2  $P_m(G)$  is e-cordial iff  $m$  is not congruent to 2 (mod 4) where  $G = \text{tail}(W_4, P_3)$  is obtained by fusing  $P_3$  at its pendent vertex with any of  $W_4$  vertex.



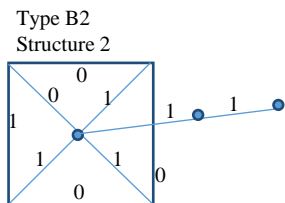
**Fig 3: tail( $W_4, P_3$ )**  
 $v_f(0,1) = (3,4)$ ,  
 $e_f(0,1) = (5,5)$



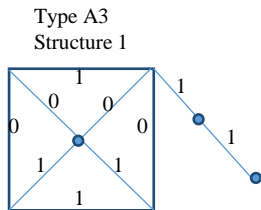
**Fig 4: tail( $W_4, P_3$ )**  
 $v_f(0,1) = (3,4)$ ,  
 $e_f(0,1) = (5,5)$



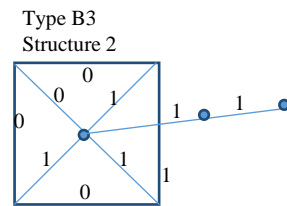
**Fig 5: tail( $W_4, P_3$ )**  
 $v_f(0,1) = (3,4)$ ,  
 $e_f(0,1) = (4,6)$



**Fig 6: tail( $W_4, P_3$ )**  
 $v_f(0,1) = (3,4)$ ,  
 $e_f(0,1) = (4,6)$



**Fig 7 tail( $W_4, P_3$ )** :  
 $v_f(0,1) = (5,2)$ ,  
 $e_f(0,1) = (4,6)$



**Fig 8 :**  
**tail( $W_4, P_3$ )** :  
 $v_f(0,1) = (5,2)$ ,

Proof. We take a path  $P_m = (v_1, v_2, \dots, v_m)$  and at each of its vertex fuse a copy of  $G$  at a fixed vertex of  $G$ . Define a function  $f: E(G) \rightarrow \{0,1\}$  as follows: Label of every edge on path  $P_m$  is '0'. A tail can be fused with  $W_4$  at any  $C_4$

vertex of  $W_4$  resulting in structure 1 or if tail is fused at hub resulting in structure 2. Using  $f$  we get labeled copies of  $\text{tail}(W_4, P_3)$  as Type A1, type A2 and Type A3 in structure 1 and Type B1, type B2 and Type B3 in structure 2. Of these only Type A1 and Type B1 are E-cordial.

To obtain structure 1, we fuse Type A1 at vertex  $v_i$  of  $P_m$  when  $i=1$  and  $i \equiv 3, 0 \pmod{4}$  and Type A3 when  $i=2$  and  $(i>1) i \equiv 2 \pmod{4}$ , Type A2 label when  $(i>1) i \equiv 1 \pmod{4}$  with vertex  $v_i$  of  $P_m$  resulting in structure 1. Similarly to obtain structure 2 we fuse Type B1 when  $i=1$  and  $i \equiv 3, 0 \pmod{4}$  and Type B3 when  $i=2$  and  $(i>1) i \equiv 2 \pmod{4}$ , Type B2 label when  $(i>1) i \equiv 1 \pmod{4}$  with vertex  $v_i$  of  $P_m$  resulting in structure 2. In both cases it gives us E-cordial labeled copy of  $G$ . Moreover it is important to note that we get a mixed path union if we use type  $A_i$  label in place of Type  $B_i$  label alternately on vertices  $v_i$  of  $P_m$  resulting in e-cordial labeling. This is valid in both structures, in structure 1 and in structure 2. The same function  $f$  will work for all structures and produce e-cordial labeling.

The resultant graph has label distribution given by (for both structures)

$v_f(0,1) = (3+14x, 4+14x)$ ,  $e_f(0,1) = (5+22x, 5+22x)$  where  $m = 2x+1$  and  $x = 0, 1, 2, 3..$

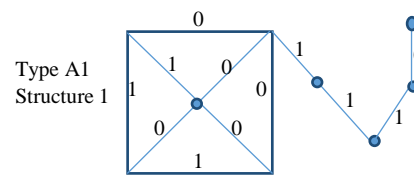
$v_f(0,1) = (8+14x, 6+14x)$ ,  $e_f(0,1) = (10+22x, 11+22x)$  where  $m = 2x+2$  and  $x = 0, 1, 2, 3..$

$v_f(0,1) = (11+14x, 10+14x)$ ,  $e_f(0,1) = (16+22x, 16+22x)$  where  $m = 2x+3$  and  $x = 0, 1, 2, 3..$

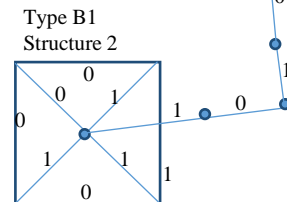
$v_f(0,1) = (14x, 14x)$ ,  $e_f(0,1) = (22x, 22x-1)$  where  $m = 4x$  and  $x = 1, 2, 3$ . Thus the graph is e-cordial for all  $m$  not congruent to 2 (mod 4). #

Theorem.3  $P_m(G)$  is e-cordial iff  $m$  is not congruent to 2 (mod 4) where  $G = \text{tail}(W_4, P_5)$  is obtained by fusing  $P_5$  at its pendent vertex with any of  $W_4$  vertex.

Proof: We take a path  $P_m = (v_1, v_2, \dots, v_m)$  and at each of its vertex fuse a copy of  $G$  at a fixed vertex of  $G$ . Define a function  $f: E(G) \rightarrow \{0,1\}$  as follows: Label of every edge on path  $P_m$  is '0'. A tail can be fused with  $W_4$  at any  $C_4$  vertex of  $W_4$  resulting in structure 1 or if tail is fused at hub resulting in structure 2.



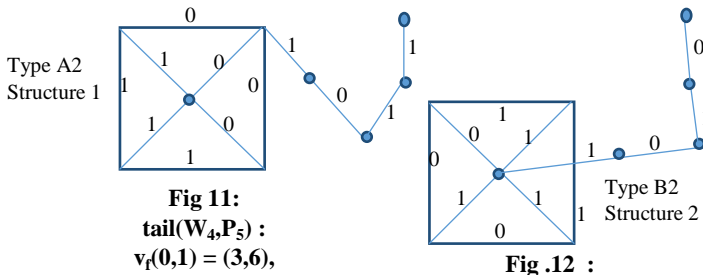
**Fig 9: tail( $W_4, P_5$ )**  
 $v_f(0,1) = (5,4)$ ,  
 $e_f(0,1) = (6,6)$



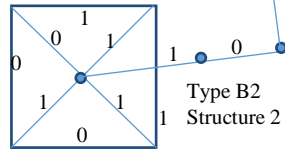
**Fig 10 :**  
**tail( $W_4, P_5$ )** :  
 $v_f(0,1) = (5,4)$ ,

Proof: We take a path  $P_m=(v_1, v_2, \dots, v_m)$  and at each of its vertex fuse a copy of  $G$  at a fixed vertex of  $G$ . Define a function  $f:E(G)\rightarrow \{0,1\}$  as follows: Label of every edge on path  $P_m$  is '0'. Two tails can be fused with  $W_4$  at any  $C_4$  vertex of  $W_4$  resulting in structure 1 or at hub resulting in structure 2. Using  $f$  we get labeled copies of  $\text{tail}(W_4, 2P_2)$  as Type A1, type A2 and Type A3 in structure 1 and Type B1, type B2 and Type B3 in structure 2. Of these only Type A1 and Type B1 are E-cordial. In both cases it gives us E-cordial labeled copy of  $G$ .

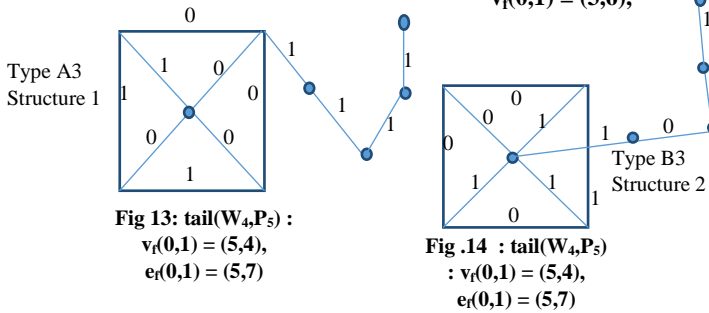
Moreover it is important to note that we get a mixed path union if we use type  $A_i$  label in place of Type  $B_i$  label alternately on vertices  $v_i$  of  $P_m$  resulting in e-cordial labeling. This is valid in both structures, in structure 1 and in structure 2. Also if we obtain a path union of  $G$  in which  $G$  is fused with vertex of  $P_m$  at random vertex of  $G$ , the same function  $f$  will work for all structures and produce e-cordial labeling. The resultant graph has label distribution given by (for both structures)



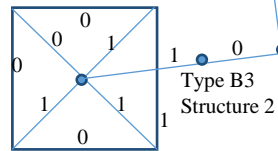
**Fig 11:**  
 $\text{tail}(W_4, P_5)$   
 $v_f(0,1) = (3,6)$



**Fig.12 :**  
 $\text{tail}(W_4, P_5)$   
 $v_f(0,1) = (3,6)$



**Fig 13:**  $\text{tail}(W_4, P_5)$   
 $v_f(0,1) = (5,4)$   
 $e_f(0,1) = (5,7)$



**Fig.14 :**  $\text{tail}(W_4, P_5)$   
 $v_f(0,1) = (5,4)$   
 $e_f(0,1) = (5,7)$

To obtain structure 1, we fuse Type A1 at vertex  $v_i$  of  $P_m$  when  $i=1$  and  $i \equiv 3,0 \pmod{4}$  and Type A2 when  $i=2$  and  $i \equiv 2 \pmod{4}$ , Type A3 label when  $(i>1) i \equiv 1 \pmod{4}$  with vertex  $v_i$  of  $P_m$  resulting in structure 1. Similarly to obtain structure 2 we fuse Type B1 at vertex  $v_i$  of  $P_m$  when  $i=1$  and  $i \equiv 3,0 \pmod{4}$  and Type B2 when  $i=2$  and  $i \equiv 2 \pmod{4}$ , Type B3 label when  $(i>1) i \equiv 1 \pmod{4}$  with vertex  $v_i$  of  $P_m$ .

In both cases it gives us E-cordial labeled copy of  $G$ . Moreover it is important to note that we get a mixed path union if we use type  $A_i$  label in place of Type  $B_i$  label alternately on vertices  $v_i$  of  $P_m$  resulting in e-cordial labeling. This is valid in both structures, in structure 1 and in structure 2. The same function  $f$  will work for all structures and produce e-cordial labeling. The resultant graph has label distribution given by (for both structures).

$$v_f(0,1) = (5+18x, 4+18x), e_f(0,1) = (6+26x, 6+26x) \text{ where } m = 2x+1 \text{ and } x=0, 1, 2, 3..$$

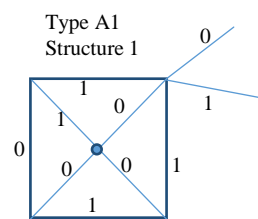
$$v_f(0,1) = (8+18x, 10+18x), e_f(0,1) = (12+26x, 13+26x) \text{ where } m = 2x+2 \text{ and } x=0, 1, 2, 3..$$

$$v_f(0,1) = (13+18x, 14+18x), e_f(0,1) = (19+26x, 19+26x) \text{ where } m = 2x+3 \text{ and } x=0, 1, 2, 3..$$

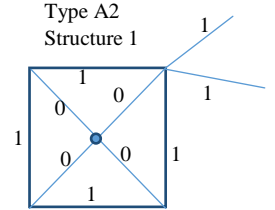
$$v_f(0,1) = (18x, 18x), e_f(0,1) = (26x, 26x-1) \text{ where } m = 4x \text{ and } x=1, 2, 3..$$

Thus the graph is e-cordial for all  $m$  not congruent to 2 (mod 4).#

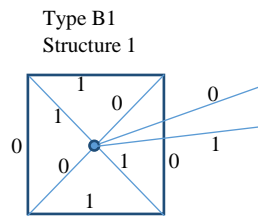
Theorem.4  $P_m(G)$  is e-cordial iff  $m$  is not congruent to 2 (mod 4) where  $G$  is  $\text{tail}(W_4, 2P_2)$



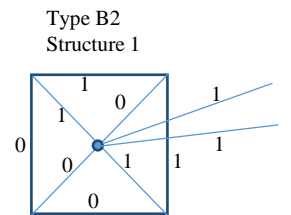
**Fig 15:**  $\text{tail}(W_4, 2P_2)$   
 $v_f(0,1) = (3,4)$   
 $e_f(0,1) = (5,5)$



**Fig 16:**  $\text{tail}(W_4, 2P_2)$   
 $v_f(0,1) = (5,2)$   
 $e_f(0,1) = (4,6)$



**Fig.17:**  $\text{tail}(W_4, 2P_2)$   
 $v_f(0,1) = (3,4)$   
 $e_f(0,1) = (5,5)$



**Fig 18:**  $\text{tail}(W_4, 2P_2)$   
 $v_f(0,1) = (5,2)$   
 $e_f(0,1) = (4,6)$

To obtain structure 1, we fuse Type A1 at vertex  $v_i$  of  $P_m$  when  $i=1$  and  $i \equiv 3,0 \pmod{4}$  and Type A2 when  $i=2$  and  $i \equiv 2 \pmod{4}$ , Type A3 label when  $(i>1) i \equiv 1 \pmod{4}$  with vertex  $v_i$  of  $P_m$  resulting in structure 1. Similarly to obtain structure 2 we fuse Type B1 at vertex  $v_i$  of  $P_m$  when  $i=1$  and  $i \equiv 3,0 \pmod{4}$  and Type B2 when  $i=2$  and  $i \equiv 2 \pmod{4}$ , Type B3 label when  $(i>1) i \equiv 1 \pmod{4}$  with vertex  $v_i$  of  $P_m$ .

$$v_f(0,1) = (3+14x, 4+14x), e_f(0,1) = (5+22x, 5+22x) \text{ where } m = 2x+1 \text{ and } x=0, 1, 2, 3..$$

$$v_f(0,1) = (8+14x, 6+14x), e_f(0,1) = (10+22x, 11+22x) \text{ where } m = 2x+2 \text{ and } x=0, 1, 2, 3..$$

$v_f(0,1) = (11+18x,10+18x)$ ,  $e_f(0,1) = (16+22x,16+22x)$  where  $m = 2x+3$  and  $x = 0, 1, 2, 3..$

$v_f(0,1) = (14x,14x)$ ,  $e_f(0,1) = (22x,22x-1)$  where  $m = 4x$  and  $x = 1, 2, 3..$

Thus the graph is e-cordial for all  $m$  not congruent to 2 (mod 4).#

Theorem4.5  $P_m(G)$  is e-cordial iff  $m$  not congruent to 2 (mod 4) where  $G = \text{tail}(W_4, 2P_4)$  is obtained by fusing two copies of  $P_4$  at its pendent vertex with any of  $W_4$  vertex

Proof: We

take a path  $P_m = (v_1, v_2, \dots, v_m)$  and at each of its vertex fuse a copy of  $G$  at a fixed vertex of  $G$ . Define a function  $f: E(G) \rightarrow \{0,1\}$  as follows: Label of every edge on path  $P_m$  is '0'. Two tails can be fused with  $W_4$  at any  $C_4$  vertex of  $W_4$  resulting in structure 1 or at hub resulting in structure 2. Using  $f$  we get labeled copies of  $\text{tail}(W_4, 2P_2)$  as Type A1, type A2 and Type A3 in structure 1 and Type B1, type B2 and Type B3 in structure 2. Of these only Type A1 and Type B1 are E-cordial. In both cases it gives us E-cordial labeled copy of  $G$ .

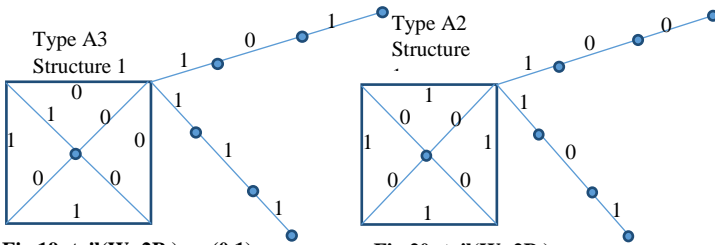


Fig 19:  $\text{tail}(W_4, 2P_4) : v_f(0,1) = (5,6), e_f(0,1) = (6,8)$

Fig 20:  $\text{tail}(W_4, 2P_4) : v_f(0,1) = (7,4), e_f(0,1) = (7,7)$

Type A3 label when  $i \equiv 3 \pmod{4}$  with vertex  $v_i$  of  $P_m$  resulting in structure 1. Similarly to obtain structure 2 we fuse Type B1 at vertex  $v_i$  of  $P_m$  when  $i \equiv 1, 0 \pmod{4}$  and Type B2 when  $i \equiv 2 \pmod{4}$ , Type B3 label when  $i \equiv 3 \pmod{4}$  with vertex  $v_i$  of  $P_m$ .

Moreover it is important to note that we get a mixed path union if we use type  $A_i$  label in place of Type  $B_i$  label alternately on vertices  $v_i$  of  $P_m$  resulting in e-cordial labeling ( $i = 1, 2, 3$ ). This is valid in both structures, in structure 1 and in structure 2. The same function  $f$  will work for all structures and produce e-cordial labeling. The same function  $f$  allows us to design a path union in which at each  $v_i$  ( $i = 1, 2, \dots, m$ ) on  $P_m$ ,  $G$  may be fused at different vertex on  $G$  with  $v_i$  of  $P_m$  then also function  $f$  works as e-cordial function. The resultant graph has label distribution given by (for both structures) For both structures the label number distribution is given by :

$v_f(0,1) = (5+22x, 6+22x)$ ,  $e_f(0,1) = (7+30x, 7+30x)$  where  $m = 2x+1$  and  $x = 0, 1, 2, 3..$

$v_f(0,1) = (12+22x, 10+22x)$ ,  $e_f(0,1) = (15+30x, 14+30x)$  where  $m = 2x+2$  and  $x = 0, 1, 2, 3..$

$v_f(0,1) = (17+22x, 16+22x)$ ,  $e_f(0,1) = (22+22x, 22+30x)$  where  $m = 2x+3$  and  $x = 0, 1, 2, 3..$

$v_f(0,1) = (22x, 22x)$ ,  $e_f(0,1) = (30x, 30x-1)$  where  $m = 4x$  and  $x = 1, 2, 3..$

Thus the graph is e-cordial for all  $m$  not congruent to 2 (mod 4).#

Theorem4.6  $P_m(G)$  is e-cordial iff  $m$  not congruent to 2 (mod 4) where  $G = \text{tail}(W_4, 2P_3)$  is obtained by fusing two copies of  $P_4$  at its pendent vertex with any of  $W_4$  vertex

Proof: We take a path  $P_m = (v_1, v_2, \dots, v_m)$  and at each of its vertex fuse a copy of  $G$  at a fixed vertex of  $G$ . Define a function  $f: E(G) \rightarrow \{0,1\}$  as follows: Label of every edge on path  $P_m$  is '0'. Two tails can be fused with  $W_4$  at any  $C_4$  vertex of  $W_4$  resulting in structure 1 or at hub resulting in structure 2. Using  $f$  we get labeled copies of  $\text{tail}(W_4, 2P_2)$  as Type A1, type A2 and Type A3 in structure 1 and Type B1, type B2 and Type B3 in structure 2. Of these only Type A1 and Type B1 are E-cordial. In both cases it gives us E-cordial labeled copy of  $G$ .

To obtain structure 1, we fuse Type A1 at vertex  $v_i$  of  $P_m$  when  $i = 1$  and  $i \equiv 3, 0 \pmod{4}$  and Type A2 when  $i \equiv 2 \pmod{4}$ , Type A3 label when  $(i > 1) i \equiv 1 \pmod{4}$  with vertex  $v_i$  of  $P_m$  resulting in structure 1. Similarly to obtain structure 2 we fuse Type B1 at vertex  $v_i$  of  $P_m$  when  $i = 1$  and  $i \equiv 3, 0 \pmod{4}$  and Type B2 when  $i = 2$

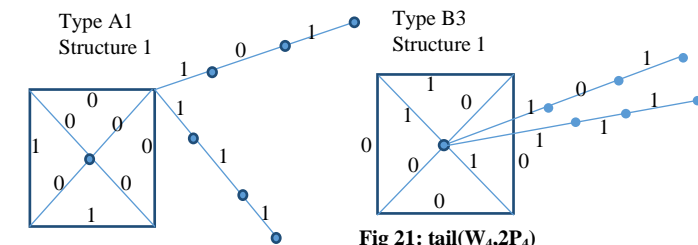


Fig 20  $\text{tail}(W_4, 2P_4) : v_f(0,1) = (5,6), e_f(0,1) = (7,7)$

Fig 21:  $\text{tail}(W_4, 2P_4) : v_f(0,1) = (5,6), e_f(0,1) = (6,8)$

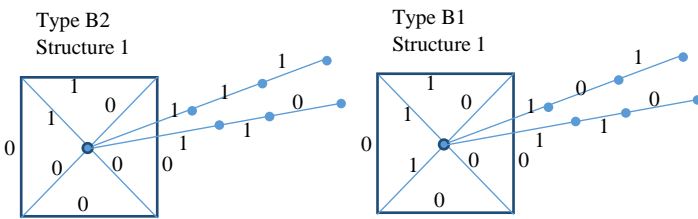


Fig 22:  $\text{tail}(W_4, 2P_4) : v_f(0,1) = (7,4), e_f(0,1) = (7,7)$

Fig 23:  $\text{tail}(W_4, 2P_4) : v_f(0,1) = (5,6), e_f(0,1) = (7,7)$

To obtain structure 1, we fuse Type A1 at vertex  $v_i$  of  $P_m$  when  $i \equiv 1, 0 \pmod{4}$  and Type A2 when  $i \equiv 2 \pmod{4}$ ,

and  $i \equiv 2 \pmod{4}$ , Type B3 label when  $(i > 1) i \equiv 1 \pmod{4}$  with vertex  $v_i$  of  $P_m$ .

Moreover it is important to note that we get a mixed path union if we use type  $A_i$  label in place of Type  $B_i$  label alternately on vertices  $v_i$  of  $P_m$  resulting in e-cordial labeling ( $i = 1, 2, 3$ ). This is valid in both structures, in structure 1 and in structure 2. The same function  $f$  will work for all structures and produce e-cordial labeling. The same function  $f$  allows us to design a path union in which at each  $v_i$  ( $i = 1, 2, \dots, m$ ) on  $P_m$ ,  $G$  may be fused at different vertex on  $G$ . The resultant graph has label distribution given by (for both structures).

For both structures the label number distribution is given by :

$v_f(0,1) = (5+18x, 4+18x)$ ,  $e_f(0,1) = (6+26x, 6+26x)$  where  $m = 2x+1$  and  $x = 0, 1, 2, 3..$

$v_f(0,1) = (8+18x, 10+18x)$ ,  $e_f(0,1) = (12+26x, 13+26x)$  where  $m = 2x+2$  and  $x = 0, 1, 2, 3..$

$v_f(0,1) = (13+18x, 14+18x)$ ,  $e_f(0,1) = (19+26x, 19+26x)$  where  $m = 2x+3$  and  $x = 0, 1, 2, 3..$

$v_f(0,1) = (18x, 18x)$ ,  $e_f(0,1) = (26x, 26x-1)$  where  $m = 4x$  and  $x = 1, 2, 3..$

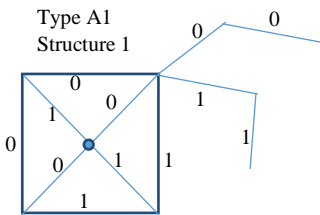


Fig 24:  $\text{tail}(W_4, 2P_3)$   
:  $v_f(0,1) = (5,4)$ ,  
 $e_f(0,1) = (6,6)$

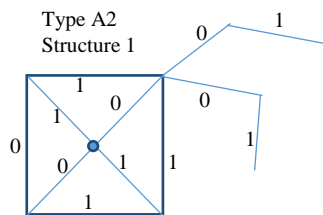


Fig 25:  $\text{tail}(W_4, 2P_3)$   
:  $v_f(0,1) = (3,6)$ ,  
 $e_f(0,1) = (5,7)$

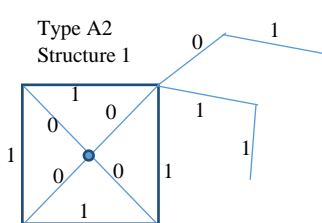


Fig 26:  $\text{tail}(W_4, 2P_3)$   
:  $v_f(0,1) = (5,4)$ ,  
 $e_f(0,1) = (5,7)$

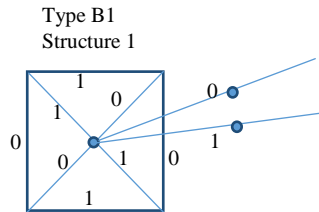


Fig 27:  $\text{tail}(W_4, 2P_2)$   
:  $v_f(0,1) = (5,4)$ ,  
 $e_f(0,1) = (6,6)$

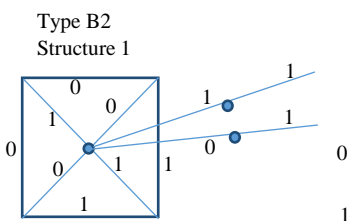


Fig 28:  $\text{tail}(W_4, 2P_2)$   
:  $v_f(0,1) = (5,6)$ ,  
 $e_f(0,1) = (5,7)$

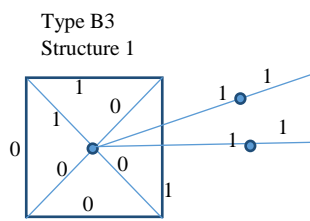


Fig 29:  $\text{tail}(W_4, 2P_2)$   
:  $v_f(0,1) = (5,4)$ ,  
 $e_f(0,1) = (5,7)$

Thus the graph is e-cordial for all  $m$  not congruent to  $2 \pmod{4}$ .#

#### IV. CONCLUSION

In this paper we construct path union different families of graphs from  $W_4$  and paths attached to it. We have arrived at following results.

1)  $P_m(G)$  is e-cordial where  $G = \text{tail}(W_4, P_4)$  is obtained by fusing  $P_4$  at its pendent vertex with any of  $W_4$  vertex

2)  $P_m(G)$  is e-cordial iff  $m$  is not congruent to  $2 \pmod{4}$  where  $G = \text{tail}(W_4, P_3)$  is obtained by fusing  $P_3$  at its pendent vertex with any of  $W_4$  vertex

3)  $P_m(G)$  is e-cordial iff  $m$  is not congruent to  $2 \pmod{4}$  where  $G = \text{tail}(W_4, P_5)$  is obtained by fusing  $P_5$  at its pendent vertex with any of  $W_4$  vertex.

4)  $P_m(G)$  is e-cordial iff  $m$  is not congruent to  $2 \pmod{4}$  where  $G$  is  $\text{tail}(W_4, 2P_2)$

5)  $P_m(G)$  is e-cordial iff  $m$  not congruent to  $2 \pmod{4}$  where  $G = \text{tail}(W_4, 2P_4)$  is obtained by fusing two copies of  $P_4$  at its pendent vertex with any of  $W_4$  vertex

6)  $P_m(G)$  is e-cordial iff  $m$  not congruent to  $2 \pmod{4}$  where  $G = \text{tail}(W_4, 2P_3)$  is obtained by fusing two copies of  $P_4$  at its pendent vertex with any of  $W_4$  vertex.

We also observe certain pattern repeated in construction of these graphs and another thing that a graph on  $n$  vertices is not e-cordial if  $n \equiv 2 \pmod{4}$ .

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