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Optimal Second-Order Kalman Filter for Pulse Radar Tracking Using Acceleration Information

Kenshi Saho
Dept. of Intelligent Robotics, Toyama Prefectural University, Japan

Abstract— In this paper, an optimal moving object tracking method using second-order Kalman filter for pulse radars is proposed. The proposed method determines optimal covariance matrices of the process noises in a Kalman filter tracking based on minimization of steady-state prediction errors. The optimal matrix of the process noises is adaptively determined using target's acceleration information, and the empirical setting of the process noise is thus unnecessary. Numerical simulations verify that the proposed method achieves smaller tracking error than the conventional Kalman filter trackers using fixed process noises.

Index Terms— **moving object tracker, Kalman filter, pulse radar, acceleration.**

I. INTRODUCTION

Accurate moving object tracking is an essential function for monitoring in autonomous vehicles, intelligent robots, and security systems. For such applications, pulse radar is promising technique because of its high-range-resolution capability and applicability in low-light conditions [1]–[5]. Since study on moving object tracking using pulse radar have long history, many efficient algorithms based on Kalman filter (KF) and its extensions (e.g., extended Kalman filter, two-stage Kalman filter, and particle filter) are proposed and applied [5]–[8]. To achieve accurate tracking using KF-based algorithms, appropriate design of motion model is important.

As a motion model in pulse radar tracking using KF, a nearly constant velocity (NCV) model is generally used [7], [9], [10]. The NCV model assumes that the target velocity is almost constant during the sampling interval, and degree of the ambiguity of the motion model is expressed by random noises, which is known as process noises. Selection of the process noise type and its parameters determine a performance of the tracker using the NCV model. However, the design of the process noises is often conducted empirically, and is not sufficiently discussed [11], [12]. To address this concern, we have clarified the theoretical relationship between covariance matrix of the process noise and a steady-state root-mean-square (SS-RMS) prediction error which is one of the general performance indices of tracking systems [13]. Based on the clarified relationship, we have verified that the covariance matrix of the process noise (CM-PN) that minimize SS-RMS prediction error is determined from only two parameters: target acceleration and variance of measurement errors. However, in this conventional study, use of fixed predicted acceleration is assumed, and this means that fixed covariance matrix of process noises is used for the tracking.

This paper proposes the optimal Kalman filter tracking method for pulse radars based on the previous work in [13]. The proposed method adaptively switch the CM-PN using target acceleration information and does not require empirical parameter settings. Two types of target acceleration acquisition methods are considered and compared: an acceleration estimation using the velocity output and a measurement of the acceleration assuming communication of the radar and an accelerometer embedded to the target. Numerical simulations verify the effectiveness of the proposed method compared with the conventional empirically designed KF tracker with respect to both the tracking accuracy and simplicity of the parameter setting.

II. SYSTEM MODEL AND PROBLEM DEFINITION

A. Motion and Measurement Models

Assumed tracking system model is depicted in Fig. 1. For simplicity, this paper assumes that the single pulse radar measures target range (the distance between the radar and the target) and tracking along range direction is conducted. The accelerometer is embedded to the target and its data is used in one of the two proposed tracking methods in this paper to improve the tracking accuracy (presented in Section IV B). The tracking algorithm is a typical second-order KF tracker. The KF tracker estimates a state vector composed of range and radial velocity of the target based on dynamic and measurement models. The dynamic model is the NCV model which is expressed as:

$$\mathbf{x}_{tk+1} = \mathbf{\Phi}\mathbf{x}_{tk} + \mathbf{w}_k, \tag{1}$$

Where $\mathbf{x}_{tk} = (R_{tk}\ v_{dtk})^T$ (^{T} denotes the transpose) is true target state vector at time kT, T is the sampling interval, R_{tk} and v_{dtk} are true range and radial velocity. $\mathbf{w}_k$ is the process noise with covariance matrix $\mathbf{Q}$, and $\mathbf{\Phi}$ is the transition matrix from kT to $(k+1)T$ which is expressed as

$$\mathbf{\Phi} = \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix}. \tag{2}$$

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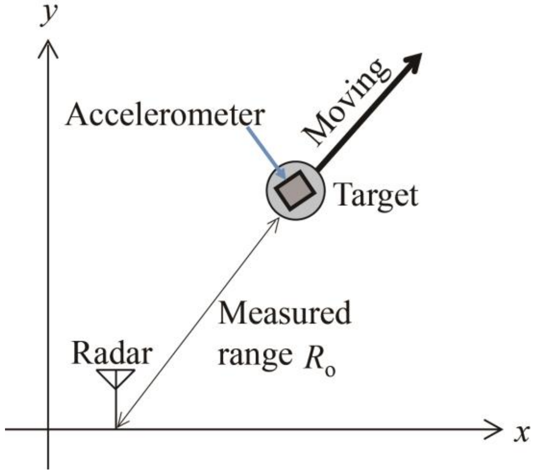

Fig. 1: Tracking system model.

The pulse radar measures the range of the target. Thus, the measurement model is:

$$R_{ok} = \mathbf{H}\mathbf{x}_{tk} + v_k , \tag{3}$$

where R_{ok} is the target range measured by the pulse radar, v_k is the measurement noise with variance B_r, and $\mathbf{H}$ is the measurement matrix that is expressed as

$$\mathbf{H} = (1 \quad 0). \tag{4}$$

B. KF Tracker

Based on the models of (1) and (3), the KF tracker sequentially estimates state vectors via the iterative calculations of following two equations:

$$\tilde{\mathbf{x}}_k = \mathbf{\Phi}\hat{\mathbf{x}}_{k-1} , \tag{5}$$

$$\hat{\mathbf{x}}_k = \mathbf{K}_k (R_{ok} - \mathbf{H}\tilde{\mathbf{x}}_k) , \tag{6}$$

Where predicts and estimates are denoted by ~ and ^, and $\mathbf{K}_k$ is the Kalman gain that minimizes the errors in the estimated state vectors. $\mathbf{K}_k$ is calculated by:

$$\mathbf{K}_k = \tilde{\mathbf{P}}_k \mathbf{H}^{\mathrm{T}} (\mathbf{H}\tilde{\mathbf{P}}_k \mathbf{H}^{\mathrm{T}} + B_r)^{-1} , \tag{7}$$

where $\mathbf{P}_k$ is the error covariance matrix determined from:

$$\tilde{\mathbf{P}}_k = \mathbf{\Phi}\hat{\mathbf{P}}_{k-1}\mathbf{\Phi}^{\mathrm{T}} + \mathbf{Q} , \tag{8}$$

$$\hat{\mathbf{P}}_k = \tilde{\mathbf{P}}_k - \mathbf{K}_k \mathbf{H}\tilde{\mathbf{P}}_k . \tag{9}$$

The iterative calculations of (8) and (9) determine the error covariance matrix to determine the Kalman gain using (7). Equations (5)–(9) are known as the KF equations.

C. Problem Definition

The purpose of the above range tracking problem is the acquisition of the predicted range calculated by (5) with small errors. As indicated in (2), (4), and (5)–(9), the design parameters of the KF tracker are B_r and $\mathbf{Q}$. In the pulse radar system, B_r is easily determined based on its general performance (Signal-to-noise ratio and a nominal range resolution determined by a bandwidth) [3]. Thus, the parameter that we must set is the elements of $\mathbf{Q}$. In summary, this paper deals with the problem that "set appropriate $\mathbf{Q}$ to predict the target range with minimal errors".

III. CONVENTIONAL PROCESS NOISE SETTING METHODS

A. Empirical Selection of Random Acceleration Process Noise and Its Variance

Many conventional KF-based tracking system selected the random acceleration process noise whose covariance matrix is expressed as [9], [12]:

$$\mathbf{Q}_{ra} = \begin{pmatrix} T^4/4 & T^3/4 \\ T^3/2 & T^2 \end{pmatrix} q , \tag{10}$$

In this model, wk = (T2/2 T)Twq, where wq is white Gaussian acceleration with variance q. Although various other settings of Q is known, Qra is most popularly used because it realizes relatively accurate tracking with only one design parameter q. q is set based on the assumed target motion. For example, if the target has a relatively complicated motion (i.e., the target has a large range of acceleration), a relatively large value of q is selected. One of the efficient setting methods of q is the use of the tracking index proposed by Kalata [14].

NCV model with Qra is generally used for the radar tracking systems. The selection of Qra from various candidates of Q and the setting of q are both empirically conducted in many conventional studies. However, Ekstr and pointed out in that these were not optimal and the appropriateness of the empirical settings were not sufficiently discussed [12]. In addition, there are same problems in advanced KF algorithms such as the extended and two-stage KFs [11].

B. Minimization of SS-RMS Prediction Errors

To resolve above problems, our previous work derived an analytical relationship between general $\mathbf{Q}$ and the SS-RMS prediction error and proposed an automatic setting method of $\mathbf{Q}$ by minimizing this error [13]. In the derivation of the SS-RMS prediction error, a constant acceleration target is assumed. This is because that sufficiently small tracking error for accelerating targets are expected in the KF tracking using the NCV model. The SS-RMS prediction error is defined as:

$$\varepsilon \equiv \lim_{k \to \infty} \sqrt{\mathrm{E}[a_c T^2/2 - \tilde{R}_k]} , \tag{11}$$

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Where $a_c T^2/2$ is the true range of the target moving with the constant acceleration a_c, $\tilde{R}_k$ is the predicted range (the tracking result, first component of $\tilde{\mathbf{x}}_k$), and E[ ] denotes the mean with respect to k. The evaluating function to determine optimal **Q** is the SS-RMS prediction error normalized by B_r, which is defined as

$$\mu \equiv \varepsilon / \sqrt{B_r} \,. \tag{12}$$

The arbitrary **Q** is now considered, which is defined as

$$\mathbf{Q}_{gen} \equiv \begin{pmatrix} a & b \\ b & c \end{pmatrix} . \tag{13}$$

The analytical relationship between μ and $\mathbf{Q}_{gen}$ is [13]:

$$\mu = \sqrt{\frac{2\alpha^2 + 2\beta + \alpha\beta}{\alpha(4 - 2\alpha - \beta)} + \frac{a_D{}^2}{\beta^2}} \,, \tag{14}$$

where

$$a_D = a_c T^2 / \sqrt{B_r} \,. \tag{15}$$

α and β are components of the steady-state Kalman gain $\mathbf{K}=(\alpha, \beta/T)^T$ calculated by

$$\beta = \frac{C + \sqrt{C(16 + 4A - 4B + C)}}{4} - \sqrt{\frac{C^2(16 + 4A - 4B + C)}{8\sqrt{C(16 + 4A - 4B + C)}} + \frac{C(2A - 2B + C}{8}} \,, \tag{16}$$

where,

$$A = a / B_r, \quad B = bT / B_r, \quad C = cT^2 / B \,, \tag{17}$$

$$\alpha = 1 - \beta^2 / C \,. \tag{18}$$

The optimal elements of $\mathbf{Q}_{gen}$ is determined by solving:

$$\arg\min_{a,b,c} \ \mu(a, b, c, a_D)$$
$$\text{sub. to.} \quad a > 0,\ b > 0,\ c > 0 \tag{19}$$

Note that the analytical formulation of the solution to this problem is not obtained. However, (19) is easily solved by the simple gradient descent method with several initial values.

The author's previous work has already verified that the using ***Q*** determined by (19) achieves accurate tracking compared with the use of the conventional $\mathbf{Q}_{ra}$ that is empirically set [13]. Moreover, the preset parameter is only a_D defined as (15), and it is easily determined its appropriate value based on the target acceleration and the radar measurement performance.

IV. PROPOSED ALGORITHM

As indicated in the previous section, the minimum (optimal) SS-RMS prediction error is depending on a_D. However, the previous study assumed a_D is fixed. Apparently, adaptively set a_D can enhance the tracking accuracy. This section proposes the optimal KF tracker based on the above idea.

A. Procedure

The proposed method adaptively set a_D based on acceleration information by following procedures.

- **Step 1**. Initialization: a_D is set using predicted target acceleration that can be considered as appropriate. The KF tracker is driven using **Q** calculated from preset a_D by solving (19).
- **Step 2**. Reset of a_D: a_D is reset at each k based on acquired acceleration information. Acquisition methods of the target acceleration is discussed in the next subsection.
- **Step 3**. Update of the Kalman gains: Using re-calculated (a, b, c), the solving (19) is re-calculated and KF tracker is driven again.
- **Step. 4**. Iteration: Steps 2 and 3 are iterated until end of radar measurements.

B. Setting methods of a_D

As indicated in (15) a_D is set with the variance of the range measurement error B_r, the sampling interval T, and the target acceleration a_c. Apparently, T is known for the tracker designers. B_r is easily determined based on the relationship between signal-to-noise ratio of the pulse radar signals and the range measurement errors. Thus, determination of a_c achieves optimal tracker using (19). This paper considers the following two methods for acquisition of the target acceleration:

- **Method (A). Estimation using the state vector**: we can obtain a target acceleration at time k using estimated target velocity as:

$$\hat{a}_{ck} = (\hat{v}_{dk} - \hat{v}_{dk-1}) / T \tag{20}$$

  where $\hat{v}_{dk}$ is the estimated velocity (i.e., second component of $\hat{\mathbf{x}}_k$) and $\hat{a}_{ck}$ is the target acceleration. In the method (A), $\hat{a}_{ck_{step}}$ is used for Step 2 of the proposed algorithm.
- **Method (B). Acceleration measurement is assumed**: In the method (B), we assume that the accelerometer is embedded to the target, and measured acceleration is obtained using the communication of the radar and the accelerometer. This is the realistic assumption for recent sensor fusion systems based on Internet of Things

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technology [8]. Although the tracking system becomes complicated, accurate tracking compared with the method (A) is expected when the performance of the accelerometer is sufficiently high.

V. EVALUATION WITH NUMERICAL SIMULATIONS

A. Application Example

This section investigates the performance of the proposed algorithm with numerical simulations. An application example to maneuvering target compares the performance of following three methods:

- **Proposed algorithm A**: the proposed algorithm using the method (A) for setting a_D.
- **Proposed algorithm B**: the proposed algorithm using the method (B) for setting a_D.
- **Conventional algorithm**: the conventional KF tracker presented in [13] (KF tracker using fixed a_D).

Fig. 2 shows true range and acceleration of an assumed maneuvering target. The observed range is obtained from the simulated radar received signals, that are generated by ray-tracing and adding the white Gaussian noises. Transmitting signal is a pulse with center frequency of 26 GHz and bandwidth of 1 GHz. The minimum signal to noise ratio of the received signals is 20 dB. The observed range is obtained by a range interpolation method [3], [4]. These process and setting lead to average of the range error of $\sqrt{B_r}$ = 1.12 cm. The sampling interval T is 50 ms. The tracking performance is evaluated using the RMS prediction error of 100 times Monte Carlo simulations, which is defined as

$$\varepsilon_k = \sqrt{\frac{1}{100}\sum_{m=1}^{100}(R_{tk} - R_{pmk})^2}\,, \tag{21}$$

where R_{pmk} is the predicted range at m-th Monte Carlo simulation.

Fig 3 shows the results. Fig. 3 (a) compares ε_k of the conventional algorithm and proposed algorithm A algorithms. The conventional algorithm set a_c = 0.6 m/s based on the target acceleration of Fig 2 (b). Note that this better setting for the conventional algorithm is not easy because the true target accelerations are unknown in real applications. As shown in Fig. 3 (a), although the proposed algorithm A slightly improve the tracking accuracy compared with the conventional algorithm, their RMS prediction errors are almost same. This is because that the proposed algorithm A estimates the target acceleration using the simple derivation of the velocity and this leads to relatively large noises in the estimated acceleration. However, the proposed algorithm A does not require the empirical parameter presetting.

Fig 3 (b) compares ε_k of the conventional algorithm and the proposed algorithm B with a standard deviation of the acceleration measurement σ_a of 0.3 m/s^2. These results indicate that the proposed algorithm B achieves significant improvement of the tracking accuracy. This is because that the proposed algorithm optimizes the SS-RMS prediction errors using sufficiently accurate acceleration information. These results verify that the proposed algorithm achieves accurate tracking without empirical presetting of its parameters.

B. Discussion on Effect of Errors in a_c

The essential difference between the proposed algorithm A and B is in the accuracy of the obtained a_c. Thus, this subsection discusses the relationship between the error of a_c (σ_a) and the performance of the proposed algorithm.

It is confirmed that the results of the proposed algorithm B in Fig. 3 (b) is almost equal to that with σ_a of 0. Thus, σ_a of 0.3 m/s^2 is sufficiently accurate for above situation. In contrast, in the proposed algorithm A presented in Fig. 3 (a), σ_a is approximately 2.2 m/s^2 which is quite worse than the case of Fig. 3 (b). This is the reason for the performance difference of the proposed A and B algorithms presented in Fig 3.

To clarify the relationship between σ_a and the SS-RMS prediction error, simulations assuming the constant acceleration target are now conducted. The true target range is $0.5a_c(kT)^2$. We assume that T, B_r, and a_c are normalize to 1. The proposed algorithm B is used for the simulation, and mean of RMS prediction error E[ε_k] is calculated with 100 times Monte Carlo simulations. White Gaussian noises are added to a_D in the optimization process of (19). Fig. 4 shows the evaluation results. As shown in this figure, when the standard deviation of setting error of a_D is larger than approximately 1, the tracking accuracy of the proposed algorithm is deteriorated. However, when it is smaller than 0.5 (approximately), the proposed algorithm achieves accurate tracking whose error is almost same as the case that correct a_D is set. This setting is easily realized by use of generally used accelerometer in various applications such as tracking of pedestrians and robots using millimeter wave pulse radars. Thus, the proposed algorithm is applicable even when the obtained acceleration information has some errors.

VI. CONCLUSIONS

This paper has proposed an optimal second-order KF tracker for pulse radar systems. The proposed algorithm adaptively determines the covariance matrix of process noises (**Q**) that minimizes the SS-RMS prediction error variance. The proposed algorithm uses estimated or measured acceleration information and is not require empirical parameter preset. Numerical simulations assuming range tracking using pulse radar show that the proposed algorithm achieves accurate tracking when the measured acceleration is sufficiently accurate. Moreover, the proposed algorithm using the acceleration estimated from the state vector realizes the almost same accuracy of the conventional algorithm, even though it does not require the preset of **Q**. The relationship between the tracking accuracy and the acceleration error is also clarified, and this indicates that the proposed algorithm can achieve accurate tracking in many realistic applications. However, application of the proposed algorithm to other KF based trackers such as extended and two-stage KFs is important future task.

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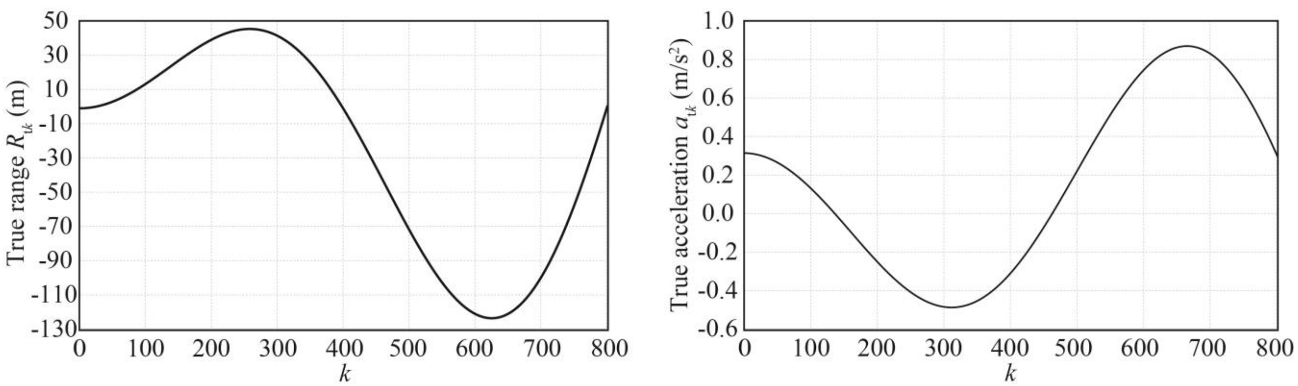
AUTHOR BIOGRAPHY

Kenshi Saho received Ph. D degree in Informatics from Kyoto University in 2013. Since 2017, he is working as a Junior Associate Professor at Toyama Prefectural University, Japan. His research interest is in signal processing for imaging, tracking, and motion analysis using radar. He is a member of IEEE, IEICE, IEEJ, and ISCIE.



(a) (b)

Fig. 2: True range (a) and acceleration (b) of the target in the numerical simulation.

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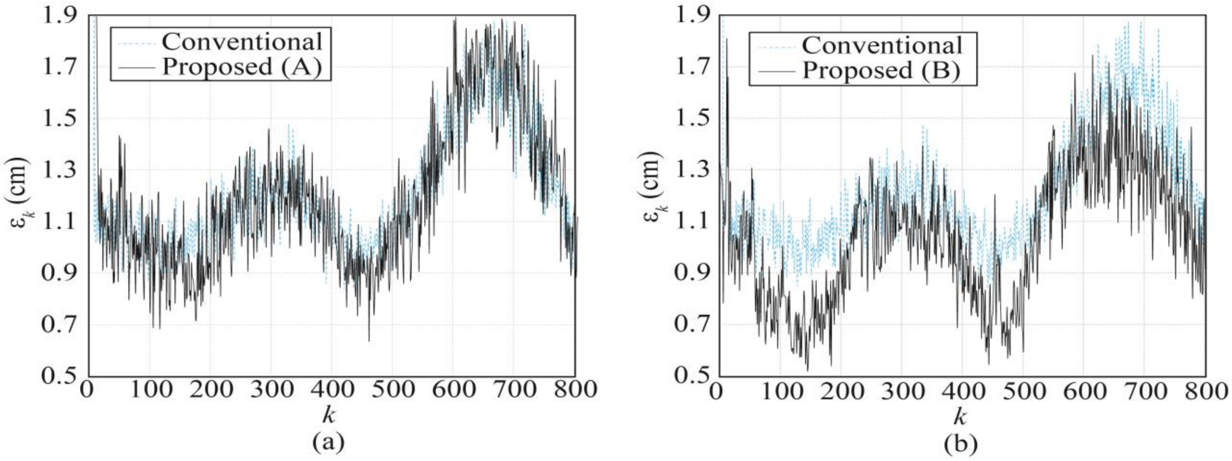

Fig. 3: Simulation results for the proposed algorithm (A) and (B) ((a) and (b), respectively)

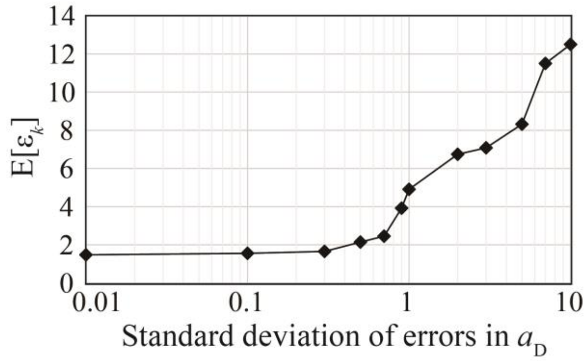

Fig. 4: Relationship between setting error of aD and tracking accuracy.