

# Optimal LQG Controller Based on Genetic Algorithm Technique For Speed Control of DC Motor

Jamal M. Ahmed

**Abstract**— In this paper the application of soft-computing algorithm design based on liner quadratic Gaussian (LQG) controller parameters optimization through the genetic algorithms (GAs) has been applied to control the speed of DC motor. The Optimum LQG Controller, which is composed of the feedback gain K and the Kalman filter has been shown to be a powerful method that could efficiently achieve the desired performance. This increases the flexibility and robustness of the system and gives a better performance. Obtained results shows that the selected approach used in the choice of matrix Q, R, Qe and Re achieves very good performance. The insertion of the GAs to be a part of the liner quadratic Gaussian controller design increases its applicability to be a tremendous off-line or on-line design process. Simulation results reflect the efficiency and reachability to the design requirements of the proposed approach.

**Index Terms**— Linear Quadratic Gaussian (LQG), DC motor, Kalman filter, GA.

## I. INTRODUCTION

Linear Quadric Gauss optimal control (LQG) are employed in many control engineering fields as it has been shown to be a considerable approach that efficiently achieve sub optimum performance. The idea of sub optimality raises from the fact that this control technique minimizes a predetermined performance index that reshape the trace of the system's states and the control surface. However, the LQG technique requires to carefully tune the Q and R parameters that influence on shape of the states and control signal traces respectively. Many researchers tried to find a way to tune the LQG controller and they presented either complicated or try-and-error procedure to fix these parameters.

On the other hand, Genetic Algorithms (GAs) are an optimization technique that can deals with optimization problems in a simple manner if the designer managed to formulate his/her problem as the GAs requested. GAs act as a natural selection mechanism, where the chromosomes are related to the engineering problem as the set of its unknown variables. The best individual transcends their genes into the next generation so that the new population can have better reachability to the required goal. Likewise, the unknown variables are optimized so that they lead to a better solution of the problem [1].

This paper presents new approach to liner quadratic Gaussian (LQG) controller design based on genetic Algorithms (GAs). The proposed technique was applied to design controller of a DC motor. The DC motors are used extensively in many applications requiring regulated speed as

in rolling and paper mills, hoists, CNC machines, robots, excavators and cranes. Here, the aim is to design a speed controller capable of meeting the following requirements [2],[3].

- A. Settling time,  $T_s$ , less than 2 seconds.
- B. Maximum peak overshoot less than 10%
- C. Rise time  $T_r$  less than 1 seconds.

## II. CONTROL PLANT MODEL

The armature controlled DC motor with the block diagram as shown in Figure. 1 is chosen. The overall transfer function of DC motor is given by [4].

$$\frac{w(s)}{Ea(s)} = \frac{K_T}{[(R+Ls)(Js+B)+K_TK_f]} \quad (1)$$

After applying the parameter values of DC motor as shown in the table.1 the final transfer function of DC motor becomes:

$$\frac{w(s)}{Ea(s)} = \frac{0.023}{0.005s^2+0.01s+0.000559} \quad (2)$$

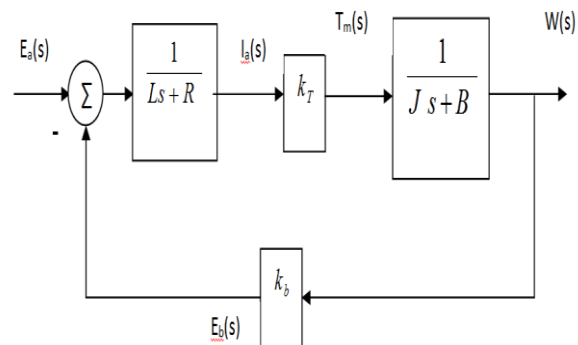


Fig. 1 DC-Motor System Block Diagram for speed.

Table.1 Parameter values of DC motor [4]

Specification of DC motor	
L	0.5 H
R	1 Ω
K	0.023
J	0.01 k.m <sup>2</sup>
B	0.00003 Nm*s/rad

## III. DESIGN OF LQG CONTROLLER

The classical LQG Controller has structure illustrated in Fig. (2) [5]. It composites of vector gain K and the Kalman filter.

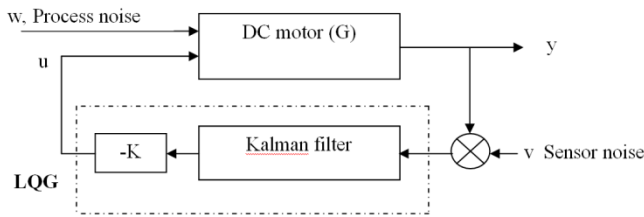


Fig.2 The main parts of the LQG controller

In Fig. (2), w is the input disturbance noise, v is the output or measurement additive corrupted noise sensor. Notice that w and v are assumed to be Gauss process with zero mean and covariance defined as

$$E[w(t) w^T(t)] = \Theta \geq 0$$

$$E[v(t) v^T(t)] = \Theta \geq 0$$

The state equations can be defined as: [5]

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t)$$

$$y(t) = Cx(t) + v(t) \quad \text{--- (3)}$$

The performance index to have sub-optimum state and control signal performance is defined as:

$$J = E \left\{ \int_0^{\infty} [x^T(t)Qx(t) + u^T Ru(t)] dt \right\} \quad \text{--- (4)}$$

where Q and R are weighting matrices. To minimize J, one could have the control signal as:

$$u = -R^{-1}B^T P x = -K x \quad \text{--- (5)}$$

$$PA + A^T P - PBR^{-1}BP + Q = 0 \quad \text{--- (6)}$$

Equation (5) is called the Riccati algebraic equations and equation (6) represents the control law. In both above equations, P is the constant positive matrix. The system output will tend to stable state using the state feedback gain vector K that leads to zero system error at steady-state.

To reduce the effect of input and measurement noise, Kalman filter is utilized in this proposed technique as a state observer of the LQG controller.

Based on the state model-plant, set x as the estimated state and x' as the estimated error of the state vector. Furthermore, {C, A} were assumed to fulfilled the observability condition. Therefore, the expected error squares could be:

$$J = E\{x^T(t)\tilde{x}(t)\} = \min,$$

Hence, the estimate controller, optimally will be:

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu(t) + L[y(t) - C\tilde{x}(t)]$$

$$= (A - LC)\tilde{x}(t) + Bu(t) + Ly(t) \quad \text{--- (7)}$$

Where,  $L = P_0 C^T R_0^{-1}$  and  $P_0$  is the solution of Riccati equation:

$$AP_0 + P_0 A^T + GQ_0 G^T - P_0 C^T Q_0^{-1} C P_0 = 0 \quad \text{--- (8)}$$

As a summary, the design technique of the LQG controller is given by the following steps: [5]

- A. Find the feedback gain vector 'K' that minimizes J.
- B. Design a Kalman filter to estimate the system's states.
- C. Implement the LQG controller

#### IV. GENETIC ALGORITHM (GA)

The Genetic Algorithms (GAs) are adaptive sub-optimal search technique depending on the natural selection ideology. It is defined as sub-optimal since it finds an optimal solution within the predetermined hyper surface. The main GAs operations are the selection, crossover and mutation. Better parents (solutions) could be obtained by continually repeating these three operations until reaching individuals (solution parameters) that satisfies the fitness function (reach to the optimal solution) [6]. Fig. (3) Illustrate graphically the GAs loop. [7]

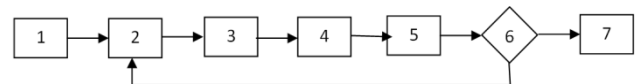


Fig. 3 generation of the Genetic algorithm

1. Randomly selection of first population.
2. Individuals' selection for mating.
3. Producing offspring individuals.
4. Perform offspring Mutation.
5. Return back the new individuals into population.
6. Is the stopping condition satisfied?
7. Stop searching.

The genetic algorithms tuning procedure starts with the determination of the chromosome representation. Each chromosome represented in real valued form as shown in Fig.4, the chromosome consists seven values that correspond to the weight matrices  $Q_c$ ,  $R_c$ , of LQR and five values that correspond to the weight matrices  $Q_e$  and  $R_e$  of kalman filter. The values  $q_{c11}$ ,  $q_{c22}$ ,  $R_c$ ,  $Q_e$  and  $R_e$  must be positive numbers and represent the unknowns to be evaluated. The block diagram of LQG-GA controller is illustrated in Fig.5.[8]

$q_{c11}$	$q_{c22}$	$R_c$	$Q_e$	$R_e$
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Fig.4 Chromosome Definition

The fitness function is the measure of how certain chromosome reflect a desired solution. It is totally depending on the designer favor in interpreting his/her required system performance. In this paper the fitness function was selected to put a maximum limit on the key factors of the system time-domain characteristics which are mainly the rise and settling time values and the peak overshoot value. Therefore, the fitness function was defined as:

Minimize J

$$J = S.RiseTime * RT + S.SettlingTime * ST + S.Overshoot * OV;$$

Where,

S is closed loop transfer function, RT is max. Settling time, ST is max. rising time and OV is the maximum over shoot.

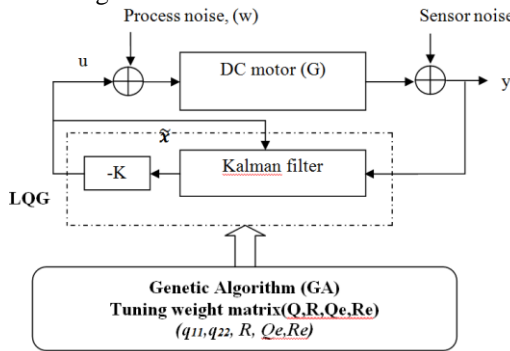


Fig.5 Structure of GA-LQG controller

The proposed approach utilized the following GA parameters as shown in Table.2

Table 2. GA's parameters

GA operation	Value/Procedure
Size of Population	20
Maximum number of Generations	100
Selection technique	Rolette Wheel
Probability of Selection	0.05
Crossover technique	scattering
Probability of Crossover	0.2
Mutation technique	Uniform Mutation
Probability of Mutation	0.01

### V. SIMULATION AND RESULTS

A DC motor is simulated using genetic algorithm (GA) and the related simulation results are presented. Figure 6 shows the speed response of the DC motor using GA-LQG controller with optimum LQR and the Kalman filter parameters  $Q_c$ ,  $R_c$ ,  $Q_e$  (process noise) and  $R_e$  (sensor noise).

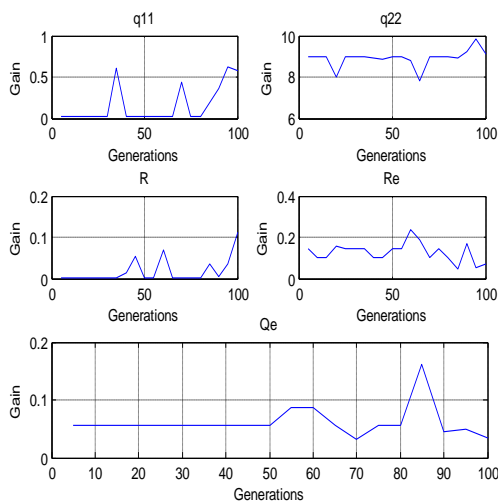


Fig. 6 Step response of DC motor using GA-LQG controller.

Step response of the DC motor with LQG controller

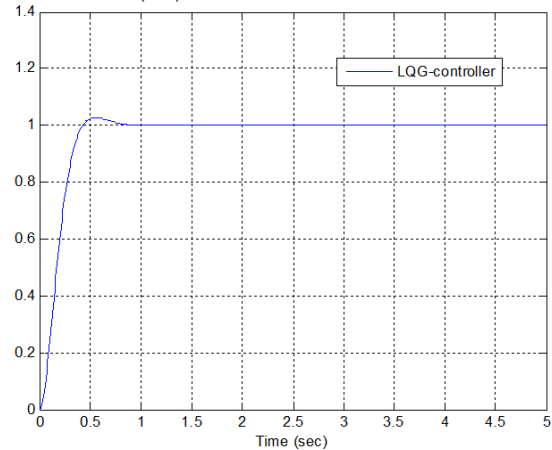


Fig. 7 Illustration of GA Converging Through Generations.

The generation trace of convergence for each gain for  $q_{11}, q_{22}, R, Q_e$  and  $R_e$  are plotted to show the convergence of the GA to the optimal final values as in Figure 7. The control law  $u(t) = -K \hat{x}(t)$  of GA-LQG controller is illustrated in Figure 8.

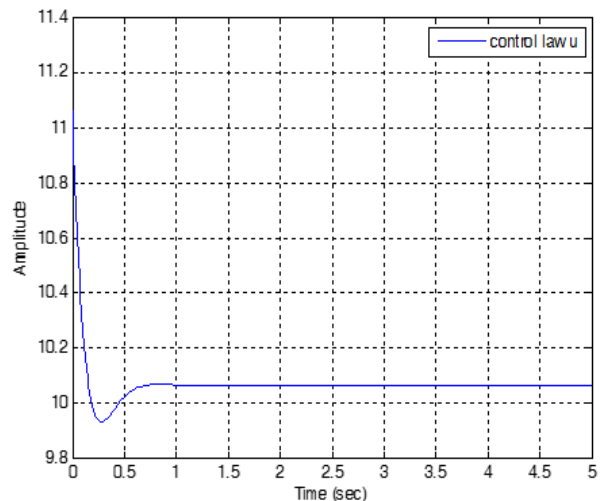


Fig. 8 Time trace of the control signal

The GA-LQG controller parameters that are obtained for the LQR and the Kalman filter ( $Q_c$ ,  $R_c$ ,  $Q_e$  (process noise) and  $R_e$  (sensor noise)) are found to be as follows:

$$Q_c = \begin{bmatrix} 0.0256885 & 0 \\ 0 & 8.9706 \end{bmatrix}, R_c = [0.00158016]$$

$$R_e = 0.141974, Q_e = 0.056584$$

The elements of optimum feedback gain matrix  $K = [k_1 k_2]$  obtained by GA-LQG controller is.

$$K = [11.062800239848659 \quad 75.234209667297492]$$

The solution of Riccati equation  $p_0$  is:

$$P_0 = \begin{bmatrix} 0.017481035882327 & 0.118882370670966 \\ 0.118882370670966 & 1.555366569408800 \end{bmatrix}$$

The estimator gain matrix L is

$$L = \begin{bmatrix} 0.000001991719513628 \\ 0.000930572407674828 \end{bmatrix}$$

The closed loop poles E are [p1; p2]

$$p1 = -6.533400119924336 + 5.714953415406837i$$

$$p2 = -6.533400119924336 - 5.714953415406837i$$

The time response specifications for the DC motor under consideration equipped with the proposed controller are given in Tables 4.

**Table 4. Performance Characteristics for DC motor.**

Time response specifications	GA-LQG Controller
Settling Time $T_s$ (Sec)	0.658
Rise Time $T_r$ (Sec)	0.256
Over shoot (Mp %)	2.75
Steady state error $e_{ss}$	0

## VI. CONCLUSIONS

Speed control of a DC motor is studied in this paper in which a design method to determine the optimal speed control using GA-LQG controller is presented. According to the simulation results, one can conclude that this controller is adequate to control the speed of the DC motor effectively. The motor response shows a satisfaction time-domain characteristic in terms of minimum rise and settling time values and peak overshoot value as required by the designer. Moreover, the generated control signal is also presents a smooth and non-harmful signal that can be practically implemented.

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## AUTHOR BIOGRAPHY

Jamal M. Ahmed. Lecturer in Electronic Engineering Department, College of Electronics Engineering, Ninawah University, Iraq. Main research area of interest is artificial intelligent techniques in designing engineering systems.