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Characterizing Solution of Fuzzy Complex Programming Using Lexicographic Order

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Abstract. ***In this paper we will discuss the solution of complex programming problem via Lexicographic order. The complex programming problem will be divided into two real subproblems which from their solutions, we can deduce the solution of complex programming problem. Also the fuzzy complex nonlinear programming problem is discussed in light of Lexicographic order. An illustrative example to classify the developed result is given.***

Keywords: **Complex programming, Lexicographic order, fuzzy set, fuzzy number, fuzzy nonlinear programming, fuzzy complex nonlinear programming, membership function, fuzzy decision making.**

I. INTRODUCTION [1, 4, 8, 11, 12, 13].

In many earlier works in complex programming problem, the researchers consider the real part of complex objective function as the objective function. The constraints of the problem was considered a cone in complex space C^n. Since many applications in mathematics, physics and engineering, the imaginary part of the objective functions plays an important role, so, the authors consider the two parts (real and imaginary) of the objective function. Since the comparison between any two complex numbers has no fixed manner, so we will use the Lexicographic order to compare between two complex numbers in which the comparison will be done between the two real parts and the two imaginary parts separately. In this paper we will assume the objective function $f(x)=u(x)+iv(x)$ and the constraints $g_r(x)=l_r(x)+ih_r(x)$, which are defined on $M=\{x\in R^n : g_r(x)=l_r(x)+ih_r(x)\le b_r+ib_r'=(b_r,b_r'),\ i=\sqrt{-1}\}$, and their ranges in the complex space. The main aim of this paper is to characterize solution of fuzzy complex nonlinear programming problem via Lexicographic order. The concept of fuzzy decision making [13] and the maximum decision [1] is developed to find the optimal solution for the considered problem.

II. PROBLEM FORMULATION

Let us consider the following complex programming problem

$$P_c\begin{cases}\min f(x)=u(x)+iv(x)\\ \text{s.t.}\\ M=\{x\in R^n : g_r(x)=l_r(x)+ih_r(x)\le(b_r,b_r'),\ r=1,2,\dots,m\}\end{cases}$$

where $u,v: R^n\to R,\ l_r,h_r : R^n\to R,\ r=1,2,\dots,m$ are convex functions on M.

Definition 2.1. (Lexicographic Order). Lexicographic order of two complex numbers $z_1=a+ib$ and $z_2=c+id$ is $z_1\le z_2 \Leftrightarrow a\le c$ and $b\le d$.

To characterize the solution of problem P_c, let us divide it into the following two subproblems:

$$P_u=\begin{cases}\min u(x)\\ \text{s.t.}\\ M=\{x\in R^n : l_r(x)\le b_r,\ h_r(x)\le b_r',\ r=1,2,\dots,m\}\end{cases}$$

and

$$P_v=\begin{cases}\min v(x)\\ \text{s.t.}\\ M=\{x\in R^n : l_r(x)\le b_r,\ h_r(x)\le b_r',\ r=1,2,\dots,m\}\end{cases}$$

Definition 2.2. $\bar{x}\in M$ is called an optimal solution for P_c if and only if $u(\bar{x})\le u(x)$ and $v(\bar{x})\le v(x)$, for each $x\in M$.

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Denote S_u and S_v the set of solutions for problems P_u and P_v respectively, i.e.,

$$S_u = \{\bar{x} \in M : u(\bar{x}) \le u(x)\ \forall\ x \in M\},$$

$$S_v = \{x^* \in M : v(x^*) \le v(x)\ \forall\ x \in M\}.$$

Proposition 2.1. Let $S_u \cap S_v \ne \phi$. Any solution of problem P_c is in $S_u \cap S_v$.

Proof. Let $\hat{x}$ be a solution for problem P_c, then $u(\hat{x}) \le u(x)\ \forall\ x \in M$, i.e. $\hat{x} \in S_u$. Similarly, $v(\hat{x}) \le v(x)\ \forall\ x \in M$, i.e. $\hat{x} \in S_v$ and then $\hat{x} \in S_u \cap S_v$ i.e. every solution of P_c lies in the set of intersection of S_u and S_v. In the following we study the case if $S_u \cap S_v = \phi$.

Proposition 2.2. If S_u and S_v are open, $S_u \cap S_v = \phi$, and *u*, *v* are strictly convex functions then $\bar{x} \in S_u$ is a solution of a conjugate function $\bar{f} = u(x) - i\,v(x)$.

Proof. Since $\bar{x} \in S_u$, so $u(\bar{x}) \le u(x)\ \forall\ x \in M$. Also

$$u(\bar{x}) \le u(x^*)\ \ \forall\ x^* \in S_v \subset M \qquad (*)$$

But, $x^* \in S_v$ means that

$$v(x^*) \le v(\bar{x})\ \ \forall\ \bar{x} \in S_u \subset M \text{ and } -i\,v(\bar{x}) \le -i\,v(x^*) \qquad (**)$$

By adding (*) and (**)

$$u(\bar{x}) - i\,v(\bar{x}) \le u(x^*) - i\,v(x^*)\ \ \forall\ x^* \in S_v.$$

i.e. $\bar{x} \in S_u$ is a solution of a conjugate function $\bar{f}(x)$.

Now, we will prove that there is no $\hat{x} \in M$ and $\hat{x} \notin S_u$ such that

$$\bar{f}(\hat{x}) = u(\hat{x}) - i\,v(\hat{x}) \le \bar{f}(\bar{x}) = u(\bar{x}) - i\,v(\bar{x}).$$

We have two cases:

Case 1: (i) Let us assume that, there exists such point $\hat{x} \in M$, $\hat{x} \notin S_u$, $\hat{x} \in S_v$ and $u(\hat{x}) - iv(\hat{x}) \le u(\bar{x}) - i\,v(\bar{x})$, i.e. $v(\bar{x}) \le v(\hat{x})$ we have from strictly convexity of *v* that

$$v(\lambda\hat{x} + (1-\lambda)\bar{x}) < \lambda v(\hat{x}) + (1-\lambda)v(\bar{x}),\ 0 \le \lambda \le 1 \Rightarrow$$

$$v(\lambda\hat{x} + (1-\lambda)\bar{x}) < \lambda v(\hat{x}) + v(\hat{x}) - \lambda v(\hat{x}), \text{ i.e.}$$

for certain λ such that $\lambda\hat{x} + (1-\lambda)\bar{x} \in S_v$, we get

$$v(\lambda\hat{x} + (1-\lambda)\bar{x}) < v(\hat{x})$$

which is a contradiction with $\hat{x} \in S_v$.

i.e. there is no $\hat{x} \in M$, $\hat{x} \notin S_u$, $\hat{x} \in s_v$ such that $\bar{f}(\hat{x}) \le \bar{f}(\bar{x})$.

Case (2): (ii) Let us assume that, there exists such point $\hat{x} \in M$, $\hat{x} \notin S_u$, $\hat{x} \notin S_v$ and

$$u(\hat{x}) - i\,v(\hat{x}) \le u(\bar{x}) - i\,v(\bar{x})$$

i.e. $u(\hat{x}) \le u(\bar{x})$ and $v(\bar{x}) \le v(\hat{x})$.

From strictly convexity of $u(x)$,

$$u(\lambda\bar{x} + (1-\lambda)\hat{x}) < \lambda u(\bar{x}) + (1-\lambda)u(\hat{x}), 0 \le \lambda \le 1$$

$$\Rightarrow u(\lambda\bar{x} + (1-\lambda)\hat{x}) < \lambda u(\bar{x}) + u(\bar{x}) - \lambda u(\bar{x}),$$

i.e. for certain λ such that $\lambda\bar{x} + (1-\lambda)\hat{x} \in S_u$, we get

$$u(\lambda x + (1-\lambda)\hat{x}) < u(\bar{x}).$$

which contradicts with $\bar{x} \in S_u$.

So there is no $\hat{x} \in M$, such that

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$u(\hat{x}) - i\,v(\hat{x}) \le u(\bar{x}) - i\,v(\bar{x})$.

The following example illustrate Proposition 2.1.

Example 2.1.

$$P_c = \begin{cases} \min(\cos x + i \sin x) \\ \text{s.t.} \\ M = \{x \in R : 0 \le x \le \pi\}. \end{cases}$$

To characterize the solution of problem P_c, let us divide it into the following two subproblems:

$$P_u = \begin{cases} \min \cos x \\ \text{s.t.} \\ M = \{x \in R : 0 \le x \le \pi\} \end{cases}$$

and

$$P_v = \begin{cases} \min \sin x \\ \text{s.t.} \\ M = \{x \in R : 0 \le x \le \pi\} \end{cases}$$

The solution of problem P_u occurs at $x = \pi$ and $x = \pi \in S_u$ i.e. $S_u = \{\pi\}$.

Similarly, the solution of problem P_v at $x = 0$ and $x = \pi$ i.e. $S_v = \{0, \pi\}$, and the optimal solution of problem P_c occurs at $x = \pi \in S_u \cap S_v$ .In the following example, we show that the condition strictly convexity of $u(x)$ and $v(x)$ is necessary in Proposition 2.2

Example 2.2.

$$P_c = \begin{cases} \min(3x_1 + x_2) + i(5x_1 - 11x_2) \\ \text{s.t.} \\ x_1^2 + x_2^2 + i(x_1 - x_2) \le 5 + i = (5, 1) \end{cases}$$

According to Lexicographic order, the problem can be divided into the following two subproblems

$$P_u = \begin{cases} \min(3x_1 + x_2) \\ \text{s.t.} \\ x_1^2 + x_2^2 \le 5 \\ x_1 - x_2 \le 1 \end{cases}$$

and

$$P_v = \begin{cases} \min(5x_1 - 11x_2) \\ \text{s.t.} \\ x_1^2 + x_2^2 \le 5 \\ x_1 - x_2 \le 1 \end{cases}$$

By solving these problems by using Kuhn-Tucker conditions [10] we can find that, the set of solutions of P_u is

$$S_u = \{(-2, -1)\}$$

and the optimal value of P_u is -7 occurs at $(-2, -1)$.

Similarly, the set of solution of P_v is

$$S_v = \{(-2, 1)\},$$

Therefore $S_u \cap S_v = \phi$ and $(-2, -1)$ is not a solution for conjugate function $u = i\,v$, since both u and v are not strictly convex..

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III. FUZZY COMPLEX NONLINEAR PROGRAMMING PROBLEM VIA LEXICOGRAPHIC ORDER

In this section we discuss the optimization fuzzy complex nonlinear programming problem with fuzzy inequalities $(\tilde{\leq})$ characterized by linear membership function.

Definition 3.1. [9, 13]. Let X denote a universal set. Then a fuzzy subset $\tilde{A}$ of X is defined by its membership function

$$\mu_{\tilde{A}} : X \to [0, 1]$$

which assigns to each element $x \in X$ a real number $\mu_{\tilde{A}}(x)$ in the interval $[0, 1]$, where the value of $\mu_{\tilde{A}}(x)$ at x represents the grade of membership of x in $\tilde{A}$.

A fuzzy set $\tilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\tilde{A}}(x)$ and is often written

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}.$$

When the membership function $\mu_{\tilde{A}}(x)$ contains only the two points 0 and 1, then $\mu_{\tilde{A}}(x)$ is identical to the characteristic function $C_A : X \to \{0, 1\}$, and hence $\tilde{A}$ is no longer a fuzzy subset, but an ordinary set.

Definition 3.2. [2, 3]. The function $L : X \to [0, 1]$ is a function with two parameters defined as:

$$L(x; \alpha, \beta) = \begin{cases} 1 & \text{if} \quad x \leq \alpha \\ \dfrac{\alpha + \beta - x}{\beta} & \text{if} \quad \alpha \leq x < \alpha + \beta \\ 0 & \text{if} \quad x \geq \alpha + \beta \end{cases}$$

It is called the trapezoidal linear membership function.

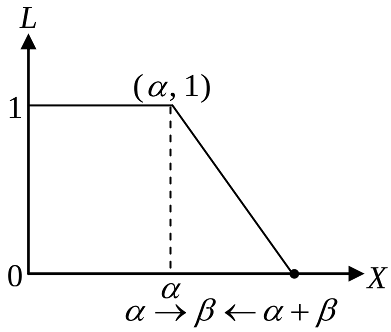

Fig. 1. Represent L-function of the membership function.

Definition 3.3. [1, 9, 13]. Given a fuzzy goal (fuzzy objective function) $\tilde{G}$ and fuzzy constraints $\tilde{C}$ in a space of alternatives X. The $\tilde{G}$ and $\tilde{C}$ combine to form a decision, $\tilde{D}$, which is a fuzzy set resulting from intersection of $\tilde{G}$ and $\tilde{C}$. Furthermore, $\tilde{D} = \tilde{G} \cap \tilde{C}$ is the membership function of $\tilde{D}$ can be defined as $\mu_{\tilde{D}} = \min\{\mu_{\tilde{G}}, \mu_{\tilde{C}}\}$. In general if we have n goals $\tilde{G}_1, \ldots, \tilde{G}_n$ and m constraints $\tilde{C}_1, \ldots, \tilde{C}_m$, then, the resultant decision can be defined as

$$\tilde{D} = \tilde{G}_1 \cap \ldots \cap \tilde{G}_n \cap \tilde{C}_1 \cap \ldots \cap \tilde{C}_m .$$

Therefore, for $j = 1, \ldots, n$ and $i = 1, \ldots, m$ it can be written as follows:

$$\begin{aligned} \mu_{\tilde{D}} &= \min\{\min\{\mu_{\tilde{G}_j}\}, \min\{\mu_{\tilde{C}_i}\}\} \\ &= \min\{\mu_{\tilde{G}_1}, \ldots, \mu_{\tilde{G}_n}, \mu_{\tilde{C}_1}, \ldots, \mu_{\tilde{C}_m}\} \\ &= \min\{\mu_{\tilde{G}_j}, \mu_{\tilde{C}_i}\}. \end{aligned}$$

Now, consider the following complex nonlinear programming problem

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$$P_c \begin{cases} \min f(x) = u(x) + i\,v(x) \\ \text{s.t.} \\ M = \{x \in R^n : g_r(x) = l_r(x) + i\,h_r(x) \le (b_r, b_r'), \\ \qquad r = 1, 2, \dots, m\} \end{cases} \tag{3.1}$$

The fuzzy version of this problem is:

$$\tilde{P}_c \begin{cases} \widetilde{\min} f(x) = u(x) + i\,v(x) \\ \text{s.t.} \\ M = \{x \in R^n : l_r(x) + i\,h_r(x) \tilde{\le} b_r + i\,b_r' = (b_r, b_r'), \\ \qquad r = 1, 2, \dots, m\} \end{cases} \tag{3.2}$$

The problem $\tilde{P}_c$ can be divided into the following two subproblems

$$\tilde{P}_u \begin{cases} \widetilde{\min}\, u(x) \\ \text{s.t.} \\ l_r(x) \le b_r,\ h_r(x) \tilde{\le} b_r',\ r = 1, 2, \dots, m \end{cases} \tag{3.3}$$

and

$$\tilde{P}_v \begin{cases} \widetilde{\min}\, v(x) \\ \text{s.t.} \\ l_r(x) \le b_r,\ h_r(x) \tilde{\le} b_r',\ r = 1, 2, \dots, m \end{cases} \tag{3.4}$$

The sign "~" denotes a fuzzy satisfaction of the constraints. The fuzzy min corresponds to achieving the lowest possible aspiration level for the general $u(x)$, $v(x)$. For the problem $\tilde{P}_u$ it can be solved by using the properties of fuzzy decision making problems as follows [5, 6, 7]:

Step 1: Fuzzify the objective function by calculating the lower and the upper bounds of the optimal values. The bounds of optimal values (lower and upper) u_l and u_u can be obtained by solving the standard crisp nonlinear programming problem NLPP as follows:

$$u_1 = \min u(x)$$

Subject to (3.5)

$$l_r(x) \le b_r,\quad h_r(x) \le b_r',\quad r = 1, 2, \dots, m$$

For all $x \in R^n$ and $x \ge 0$.

$$u_2 = \min u(x)$$

Subject to

$$l_r(x) \le b_r + q_r,$$
$$h_r(x) \le b_r' + q_r',\ r = 1, 2, \dots, m.$$

For all $x \in R^n$ and $x \ge 0$. Where the objective function takes the values between u_1 and u_2.

Let $u_l = \min(u_1, u_2)$ and $u_u = \max(u_1, u_2)$. Where u_l and u_u are the lower and upper bounds of the optimal values.

Suppose that $\tilde{S}_u$ is the fuzzy set representing the objective function $u(x)$ such that

$$\tilde{S}_u = \{(x, \mu_{\tilde{S}_u}(x)) : x \in R^n\}$$

Where

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$$\mu_{\tilde{S}_u}(x) = \begin{cases} 1 & \text{if} \quad u_u \le u(x) \\ \dfrac{u(x) - u_l}{u_u - u_l} & \text{if} \quad u_l < u(x) \le u_u \\ 0 & \text{if} \quad u(x) \le u_l \end{cases} \tag{3.6}$$

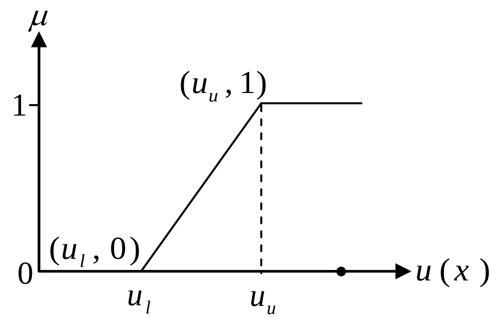

Fig. 2. **Represents the membership function of the fuzzy set of the objective function** $u(x)$

Note that: (1) q_r and q'_r are vectors of relaxation and can be determined by fuzzyifying b_r (denoted by $\tilde{b}_r$) and b'_r by (denoted by $\tilde{b}'_r$) by using the definition of (3.2) of L-function of the membership function as follows

$$\tilde{b}_r = \{(x, \mu_{\tilde{b}_r}(x)) : x \in R^n\}$$

where

$$\mu_{\tilde{b}_r}(x) = \begin{cases} 1 & \text{if} \quad x \le b_r \\ \dfrac{b_r + q_r - x}{q_r} & \text{if} \quad b_r \le x < b_r + q_r \\ 0 & \text{if} \quad x \ge b_r + q_r \end{cases} \tag{3.7}$$

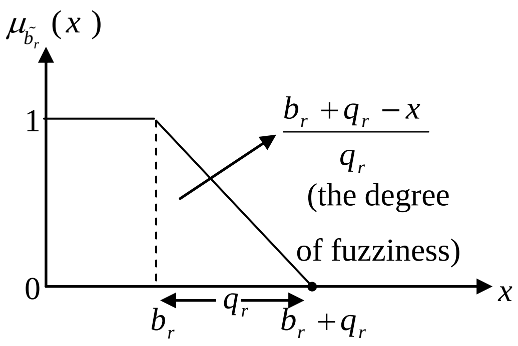

Fig. 3. Represents $\mu_{\tilde{b}_r}(x)$ **membership function of the fuzzy set** $\tilde{b}_r$.

Similarly we can find

$$\tilde{b}'_r = \{(x, \mu_{\tilde{b}'_r}(x) : x \in R^n\}$$

where

$$\mu_{\tilde{b}'_r}(x) = \begin{cases} 1 & \text{if} \quad x \le b'_r \\ \dfrac{b'_r + q'_r - x}{q'_r} & \text{if} \quad b_r \le x < b'_r + q'_r \\ 0 & \text{if} \quad x \ge b'_r + q'_r \end{cases} \tag{3.8}$$

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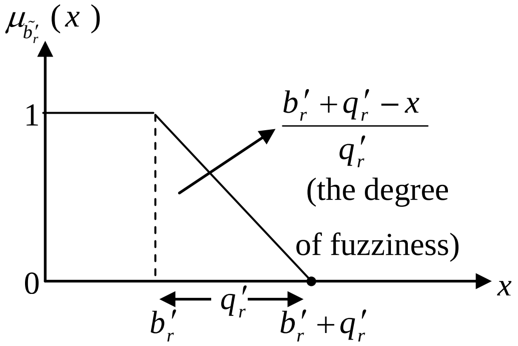

Fig. 4. Represents $\mu_{\tilde{b}'_r}(x)$ membership function of the fuzzy set $\tilde{b}'_r$.

Step 2: Fuzzify the constrains $l_r(x) \le b_r$, $h_r(x) \le b'_r$, $r = 1, 2, ..., m$.

Let $\tilde{c}_r$ and $\tilde{c}'_r$ are the fuzzy sets for r^{th} constraints such that

$\tilde{c}_r = \{(x, \mu_{\tilde{c}_r}(x)) : x \in R^n\}$ and $\tilde{c}'_r = \{(x, \mu_{\tilde{c}'_r}(x)) : x \in R^n\}$, where

$$\mu_{\tilde{c}_r}(x) = \begin{cases} 1 & \text{if} \quad l_r(x) \le b_r \\ \dfrac{b_r + q_r - l_r(x)}{q_r} & \text{if} \quad b_r \le l_r(x) < b_r + q_r \\ 0 & \text{if} \quad l_r(x) \ge b_r + q_r \end{cases} \qquad (3.9)$$

and

$$\mu_{\tilde{c}'_r}(x) = \begin{cases} 1 & \text{if} \quad h_r(x) \le b'_r \\ \dfrac{b'_r + q'_r - h_r(x)}{q'_r} & \text{if} \quad b'_r < h_r(x) < b'_r + q'_r \\ 0 & \text{if} \quad h_r(x) \ge b'_r + q'_r \end{cases} \qquad (3.10)$$

Let $\tilde{D}_u$ be the fuzzy decision set, where

$$\tilde{D}_u = \tilde{S}_u \cap \tilde{c}_r \cap \tilde{c}'_r, \ r = 1, ..., m \qquad (3.11)$$

Therefore, $\tilde{D}_u = \tilde{S}_u \cap \tilde{c}_1 \cap ... \cap \tilde{c}_m \cap \tilde{c}'_1 \cap ... \cap \tilde{c}'_m$ and $\tilde{D}_u = \{(x, \mu_{\tilde{D}_u}(x)) : x \in R^n\}$. Then we have

$$\mu_{\tilde{D}_u}(x) = \min\{\mu_{\tilde{S}_u}(x), \ \min\{\mu_{\tilde{c}_1}(x), ..., \mu_{\tilde{c}_m}(x)\}, \ \min\{\mu_{\tilde{c}'_1}(x), ..., \mu_{\tilde{c}'_m}(x)\}.$$

Now, if we suppose

$$\lambda = \min\{\mu_{\tilde{S}_u}(x), \ \min\{\mu_{\tilde{c}_1}(x), ..., \mu_{\tilde{c}_m}(x)\}, \ \min\{\mu_{\tilde{c}'_1}(x), ..., \mu_{\tilde{c}'_m}(x)\}, \qquad (3.12)$$

then we have the optimal decision:

$$x^* = \max \lambda, \quad x^* \in R^n.$$

Now, the problem $\tilde{P}_u$ (3.3) becomes the following crisp nonlinear programming problem

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$$\left.\begin{array}{ll} \max \lambda & \\ \text{subject to} & \\ \bar{u}_1 & : \lambda - \mu_{\tilde{S}_u}(x) \le 0 \\ \bar{u}_2 & : \lambda - \mu_{\tilde{c}_1}(x) \le 0 \\ \vdots & \\ \bar{u}_m & : \lambda - \mu_{\tilde{c}_{m-1}}(x) \le 0 \\ \bar{u}_{m+1} & : \lambda - \mu_{\tilde{c}_m}(x) \le 0 \\ \bar{u}_{m+2} & : \lambda - \mu_{\tilde{c}'_1}(x) \le 0 \\ \vdots & \\ \bar{u}_{2m} & : \lambda - \mu_{\tilde{c}'_m-1}(x) \le 0 \\ \bar{u}_{2m+1} & : \lambda - \mu_{\tilde{c}'_m}(x) \le 0 \end{array}\right\} \quad (3.13)$$

where $0 \le \lambda \le 1$ and $x \in R^n$. This is equivalent to the problem

$$\left.\begin{array}{lll} \max \lambda & & \\ \text{subject to} & & \\ \bar{u}_1 & : & \lambda - \left(\dfrac{u(x) - u_l}{u_u - u_l}\right) \le 0 \\ \bar{u}_2 & : & \lambda - \left(\dfrac{b_1 + q_1 - l_1(x)}{q_1}\right) \le 0 \\ \vdots & & \\ \bar{u}_m & : & \lambda - \left(\dfrac{b_{m-1} + q_{m-1} - l_{m-1}(x)}{q_{m-1}}\right) \le 0 \\ \bar{u}_{m+1} & : & \lambda - \left(\dfrac{b_m + q_m - l_m(x)}{q_m}\right) \le 0 \\ \bar{u}_{m+2} & : & \lambda - \left(\dfrac{b'_1 + q'_1 - h_1(x)}{q'_1}\right) \le 0 \\ \vdots & & \\ \bar{u}_{2m} & : & \lambda - \left(\dfrac{b'_{m-1} + q'_{m-1} - h_{m-1}(x)}{q'_{m-1}}\right) \le 0 \\ \bar{u}_{2m+1} & : & \lambda - \left(\dfrac{b'_m + q'_m - h_m(x)}{q'_m}\right) \le 0 \end{array}\right\} \quad (3.14)$$

where $0 \le \lambda \le 1$ and $x \in R^n$. After that, we can obtain the optimal solution $x_u^* \in R^n$ of the problem $\tilde{P}_u$ with the fuzzy decision set

$$\tilde{D}_u = \tilde{S}_u \cap \tilde{c}_r \cap \tilde{c}'_r, \quad r = 1, 2, \ldots, m \quad (3.15)$$

$$\mu_{\tilde{D}_u}(x) = \min\left(\mu_{\tilde{S}_u}(x), \mu_{\tilde{c}_1}(x), \ldots, \mu_{\tilde{c}_m}(x), \mu_{\tilde{c}'_1}(x), \ldots, \mu_{\tilde{c}'_m}(x)\right) \quad (3.16)$$

The problem is to find the x_u^* which maximizes the minimum membership function i.e.

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$$\mu_{\tilde{D}_u}(x_u^*) = \max \min\, (\mu_{\tilde{S}_u}(x_u^*),\ \mu_{\tilde{c}_1}(x_u^*),\ ...,\ \mu_{\tilde{c}_m}(x_u^*),\ \mu_{\tilde{c}'_1}(x_u^*),\ ...,\ \mu_{\tilde{c}'_m}(x_u^*)) \qquad (3.17)$$

where $\mu_{\tilde{S}_u}(x),\ \mu_{\tilde{c}_r}(x),\ \mu_{\tilde{c}'_r}(x)$ are defined in (3.6), (3.9) and (3.10) respectively. And by substituting by x_u^* in the problem $\tilde{P}_u$ in the objective function, we can find that

$$u_l < u_{AF} < u_u \qquad (3.18)$$

where u_{AF} is the objective function of the problem P_u after fuzziness.

Similarly, we can find the optimal solution $x_v^* \in R^n$ of the problem $\tilde{P}_v$, with the fuzzy decision set

$$\tilde{D}_v = \tilde{S}_v \cap \tilde{c}_r \cap \tilde{c}'_r, \qquad r = 1, 2, ..., m \qquad (3.19)$$

$$\mu_{\tilde{D}_v}(x) = \min\, (\mu_{\tilde{S}_v}(x),\ \mu_{\tilde{c}_1}(x),\ ...,\ \mu_{\tilde{c}_m}(x),\ \mu_{\tilde{c}'_1}(x),\ ...,\ \mu_{\tilde{c}'_m}(x)), \qquad (3.20)$$

and the problem is to find the x_v^* which maximizes the minimum membership function i.e.

$$\mu_{\tilde{D}_v}(x_v^*) = \max \min\, (\mu_{\tilde{S}_v}(x_v^*),\ \mu_{\tilde{c}_1}(x_v^*),\ ...,\ \mu_{\tilde{c}_m}(x_v^*),\ \mu_{\tilde{c}'_1}(x_v^*),\ ...,\ \mu_{\tilde{c}'_m}(x_v^*)) \qquad (3.21)$$

where $\mu_{\tilde{S}_v}(x)$ is defined as

$$\mu_{\tilde{S}_v}(x) = \begin{cases} 1 & \text{if} \quad v(x) \ge v_u \\ \dfrac{v(x) - v_l}{v_u - v_l} & \text{if} \quad v_l < v(x) \le v_u \\ 0 & \text{if} \quad v(x) \le v_l, \end{cases} \qquad (3.22)$$

$\mu_{\tilde{c}_r}(x)$ and $\mu_{\tilde{c}'_r}(x)$ are defined in (3.9) and (3.10) respectively.

And by substituting by x_v^* in problem $\tilde{P}_v$ in the objective function, we can find that

$$v_l < v_{AF} < v_u \qquad (3.23)$$

where v_{AF} is the objective function of the problem P_v after fuzziness.

The optimal solution of the fuzzy complex programming problem $\tilde{P}_c$ is $x^* \in R^n$ with the fuzzy decision set

$$\tilde{D} = \tilde{D}_u \cap \tilde{D}_v \qquad (3.24)$$

$$\tilde{D} = \tilde{S}_u \cap \tilde{S}_v \cap \tilde{c}_r \cap \tilde{c}'_r, \qquad (3.25)$$

$$\mu_{\tilde{D}}(x) = \min(\mu_{\tilde{S}_u}(x),\ \mu_{\tilde{S}_v}(x),\ \mu_{\tilde{c}_r}(x),\ \mu_{\tilde{c}'_r}(x)) \qquad (3.26)$$

where $\mu_{\tilde{S}_u}(x),\ \mu_{\tilde{S}_v}(x),\ \mu_{\tilde{c}_r}(x),\ \mu_{\tilde{c}'_r}(x)$ are defined in (3.6), (3.22), (3.9) and (3.10) respectively, and the problem is to find the x^* which maximizes the minimum membership function values i.e. the problem is to choose x^* such that

$$\mu_D(x^*) = \max \min\, (\mu_{\tilde{D}_u}(x_u^*),\ \mu_{\tilde{D}_v}(x^*)) \qquad (3.27)$$

$$= \max \min\, (\mu_{\tilde{S}_u}(x_u^*),\ \mu_{\tilde{c}_r}(x_u^*),\ \mu_{\tilde{c}'_r}(x_u^*),\ \mu_{\tilde{S}_v}(x_v^*),\ \mu_{\tilde{c}_r}(x_v^*)$$

$$\mu_{\tilde{c}'_r}(x_v^*)), \qquad r = 1, 2, ..., m \qquad (3.28)$$

IV. ILLUSTRATIVE EXAMPLE

In the following example, we will illustrate our problem.

Example 4.1. Consider the following complex mathematical programming problem

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$$P_c \begin{cases} \min(3x_1 + x_2) + i\,(5x_1 - 11x_2) \\ \text{s.t.} \\ x_1^2 + x_2^2 + i\,(x_1 - x_2) \le 5 + i = (5, 1) \end{cases} \qquad (4.1)$$

The fuzzy version for the problem P_c (4.1) is

$$\tilde{P}_c \begin{cases} \min(3x_1 + x_2) + i\,(5x_1 - 11x_2) \\ \text{s.t.} \\ x_1^2 + x_2^2 + i\,(x_1 - x_2) \tilde{\lesssim} 5 + i = (5, 1) \end{cases} \qquad (4.2)$$

According to Lexicographic order this problem (4.1) can be divide into two subproblems

$$P_u \begin{cases} \min(3x_1 + x_2) = u \\ \text{s.t.} \\ l_1 :\ x_1^2 + x_2^2 \le 5 \\ h_1 :\ x_1 - x_2 \le 1 \end{cases} \qquad (4.3)$$

and

$$P_v \begin{cases} \min(5x_1 - 11x_2) = v \\ \text{s.t.} \\ x_1^2 + x_2^2 \le 5 \\ x_1 - x_2 \le 1 \end{cases} \qquad (4.4)$$

The optimal solution for P_u is $(x_1^*, x_2^*) = (-2, -1)$ and $u_{\text{minmum}} = -7$, and satisfies the constraints of the problem P_u.

The fuzzy version of the problem P_u is

$$\tilde{P}_u \begin{cases} \min u = 3x_1 + x_2 \\ \text{s.t.} \\ l_1(x) :\ x_1^2 + x_2^2 \tilde{\lesssim} 5 \\ h_1(x) :\ x_1 - x_2 \tilde{\lesssim} 1 \end{cases} \qquad (4.5)$$

where the symbol "$\tilde{\lesssim}$" denotes a relaxed or fuzzy version of the ordinary inequality "$\le$". Therefore $b_1 = 5$ and $b_1' = 1$. In order to obtain q_1 and q_1', we have

$$\tilde{5} = \{(x, \mu_{\tilde{5}}(x)) : x \in R^n\}, \qquad (4.6)$$

where

$$\mu_{\tilde{5}}(x) = \begin{cases} 1 & \text{if} \quad x \le 5 \\ \dfrac{8 - x}{3} & \text{if} \quad 5 \le x < 8 \\ 0 & \text{if} \quad x \ge 8 \end{cases} \qquad (4.7)$$

It is shown in figure 5.

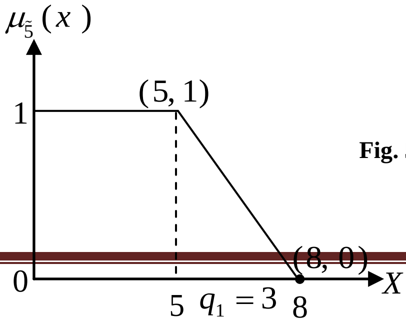

Fig. 5. Membership function of $\mu_{\tilde{5}}(x)$.

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Hence $q_1 = 3$.

Similarly q_1' can be obtained by

$$\tilde{1} = \{(x, \mu_{\tilde{1}}(x)) : x \in R^n\},$$

where

$$\mu_{\tilde{1}}(x) = \begin{cases} 1 & \text{if} \quad x \le 1 \\ \dfrac{4-x}{3} & \text{if} \quad 1 \le x < 4 \\ 0 & \text{if} \quad x \ge 4 \end{cases} \qquad (4.8)$$

we get $q_1' = 3$.

It is shown in the figure 6.

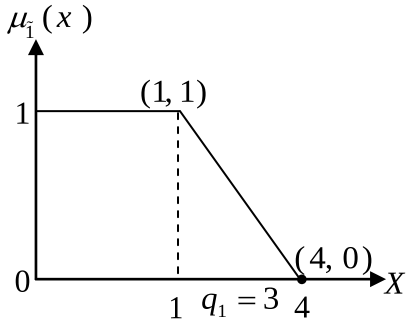

Fig. 6. Membership function of $\mu_{\tilde{1}}(x)$.

Now we can find the lower and upper bound of the optimal value u denoted by u_l and u_u respectively, by solving the two crisp nonlinear programming problems NLPP as follows:

(1) $u_1 = u$: Since the problem is the same first problem P_u, and they have the same solution, therefore

$$u_l = u = -7 \text{ occurs at } (x_1^*, x_2^*) = (-2, -1),$$

(2) $u_2 = \min(3x_1 + x_2)$

s. t. (4.9)

$$x_1^2 + x_2^2 \le 8, \qquad x_1 - x_2 \le 4.$$

The optimal solution is $(x_1^*, x_2^*) = (2.6832816,\ 0.8944272)$ which is satisfies the constraints and the optimal value $u_2 = 4\sqrt{5} = 8.9442719$.

Then $u_l = \min(u_1, u_2) = -7$ and $u_u = \max(u_1, u_2) = 4\sqrt{5} = 8.9442719$.

Let $\tilde{S}_u$ be the fuzzy set of all objective function such that

$$\tilde{S}_u = \{(x, \mu_{\tilde{S}_u}(x) : x \in R\} \qquad (4.10)$$

and

$$\mu_{\tilde{S}_u}(x) = \begin{cases} 1 & \text{if} \quad 4\sqrt{5} \le 3x_1 + x_2 \\ \dfrac{3x_1 + x_2 + 7}{4\sqrt{5} + 7} & \text{if} \quad -7 \le 3x_1 + x_2 < 4\sqrt{5} \\ 0 & \text{if} \quad 3x_1 + x_2 \le -7 \end{cases} \qquad (4.11)$$

In addition, let $\tilde{c}_1$ be the fuzzy set for the constraint $l_1(x)$ such that

$$\tilde{c}_1 = \{(x, \mu_{\tilde{c}_1}(x) : x \in R\} \qquad (4.12)$$

where

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$$\mu_{\tilde{c}_1}(x)=\begin{cases}1 & \text{if} \quad x_1^2+x_2^2\le 5\\ \dfrac{8-x_1^2-x_2^2}{3} & \text{if} \quad 5\le x_1^2+x_2^2<8\\ 0 & \text{if} \quad x_1^2+x_2^2\ge 8,\end{cases} \qquad (4.13)$$

and $\tilde{c}_1'$ be the fuzzy set for the constraint $h_1(x)$ such that

$$\tilde{c}_1'=\{(x,\mu_{\tilde{c}_1'}(x)):x\in R\} \qquad (4.14)$$

where

$$\mu_{\tilde{c}_1'}(x)=\begin{cases}1 & \text{if} \quad x_1-x_2\le 1\\ \dfrac{4-x_1+x_2}{3} & \text{if} \quad 1\le x_1-x_2<4\\ 0 & \text{if} \quad x_1-x_2\ge 4\end{cases} \qquad (4.15)$$

The fuzzy decision making for this problem is

$$\mu_{\tilde{D}_u}(x)=\min\{\mu_{\tilde{S}_u}(x),\ \min\{\mu_{\tilde{c}_1},\ \mu_{\tilde{c}_1'}(x)\}\} \qquad (4.16)$$

Let

$$\lambda=\min\{\mu_{\tilde{S}_u}(x),\ \min\{\mu_{\tilde{c}_1},\ \mu_{\tilde{c}_1'}(x)\}\}, \qquad (4.17)$$

where $0\le\lambda\le 1$, the problem of finding the maximum decision is to choose x^* such that

$$\mu_{\tilde{D}_u}(x^*)=\max\lambda=\max\ \min\{\mu_{\tilde{S}_u}(x),\ \mu_{\tilde{c}_1}(x),\ \mu_{\tilde{c}_1'}(x)\}. \qquad (4.18)$$

In other words, the problem is to find the x^* which maximizes the minimum membership function values. our problem can be transformed into the following crisp NLPP:

$$\max x^*=\lambda$$

subject to

$$\left.\begin{array}{lll}\bar{u}_1: & \lambda-\mu_{\tilde{S}_u}(x) & \le 0\\ \bar{u}_2: & \lambda-\mu_{\tilde{c}_1}(x) & \le 0\\ \bar{u}_3: & \lambda-\mu_{\tilde{c}_1'}(x) & \le 0\end{array}\right\} \qquad (4.19)$$

where $0\le\lambda\le 1$, which is equivalent to the following problem

$$(P_u)_\lambda\begin{cases}\max x^*=\lambda\\ \text{subject to}\\ \quad \bar{u}_1:\ \lambda-\left(\dfrac{3x_1+x_2+7}{4\sqrt{5}+7}\right)\le 0\\ \quad \bar{u}_2:\ \lambda-\left(\dfrac{8-x_1^2-x_2^2}{3}\right)\le 0\\ \quad \bar{u}_3:\ \lambda-\left(\dfrac{4-x_1+x_2}{3}\right)\le 0\end{cases} \qquad (4.20)$$

We can obtain the solution $x_u^*\in R^n$ of the problem $\tilde{P}_u$ with the fuzzy decision set

$$\tilde{D}_u=\tilde{S}_u\cap\tilde{c}_1\cap\tilde{c}_1' \qquad (4.21)$$

$$\mu_{\tilde{D}_u}(x)=\min(\mu_{\tilde{S}_u}(x),\ \mu_{\tilde{c}_1}(x),\ \mu_{\tilde{c}_1'}(x)) \qquad (4.22)$$

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where $\mu_{\tilde{S}_u}(x),\ \mu_{\tilde{c}_1}(x),\ \mu_{\tilde{c}'_1}(x)$ are defined in (4.11), (4.13), and (4.15) respectively.

The problem is to find the x_u^* which maximizes the minimum membership function i.e.

$$\mu_{\tilde{D}_u}(x_u^*) = \max\ \min\ (\mu_{\tilde{S}_u}(x_u^*),\ \mu_{\tilde{c}_1}(x_u^*),\ \mu_{\tilde{c}'_1}(x_u^*)) \qquad (4.23)$$

The optimal solution of the problem $\tilde{P}_u$ is $x_u^* = (1.762333315,\ 0.0913943838)$ and $\lambda^* = 0.7763536896$, which satisfy in the constraints. We can submit x_u^* in the objective function of the crisp nonlinear programming problem. It can be obtained $u_{AF} = 3x_1^* + x_2^* = 5.378394329$, where u_{AF} is the value of the objective function after fuzziness. And we reduce that

$$u_l < u_{AF} < u_u, \qquad (4.24)$$

u_{AF} is an accurate solution.

Similarly, we can solve the problem P_v as follows:

$$P_v \begin{cases} \min(5x_1 - 11x_2) \\ \text{s.t.} \\ x_1^2 + x_2^2 & \le 5 \\ x_1 - x_2 & \le 1 \end{cases} \qquad (4.25)$$

The optimal solution for P_v is

$$(x_1^*, x_2^*) = (-2, 1)$$

and

$$v_{\min} = -10 - 11 = -21 = v_1.$$

The fuzzy version of the problem P_v is

$$\tilde{P}_v \begin{cases} \min(5x_1 - 11x_2) = v \\ \text{s.t.} \\ l_1(x): & x_1^2 + x_2^2 & \tilde{\lesssim} 5 \\ h_1(x): & x_1 - x_2 & \tilde{\lesssim} 1 \end{cases} \qquad (4.26)$$

Therefore $b_1 = 5,\ b'_1 = 1,\ q_1$ and q'_1 as before to find v_2, we solve the problem

$$\left. \begin{array}{l} v_2 = \min(5x_1 - 11x_2) \\ s.t. \\ x_1^2 + x_2^2 \quad \le 8 \\ x_1 - x_2 \quad \le 4 \end{array} \right\} \qquad (4.27)$$

The optimal solution for this problem is $(x_1^*, x_2^*) = (1.1704115,\ -2.5749053)$ and $\min v = v_2 = 34.176016$. The fuzzy set of the objective function $v(x)$ is $\tilde{S}_v = \{(x,\ \mu_{\tilde{S}_v}(x)) : x \in R\}$ and

$$(\mu_{\tilde{S}_v}(x)) = \begin{cases} 1 & \text{if} \quad 5x_1 - 11x_2 \ge 34.176016 \\ \dfrac{5x_1 - 11x_2 + 21}{34.1760616 + 21} & \text{if} \quad -21 < 5x_1 - 11x_2 \le 34.176076 \\ 0 & \text{if} \quad 5x_1 - 11x_2 \le -21 \end{cases} \qquad (4.28)$$

The fuzzy set for the constraints similar to the problem P_u. The fuzzy decision making for this problem is

$$\max x^* = \lambda$$

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subject to

$$\bar{v}_1 : \lambda - (\mu_{S_v}(x)) \leq 0 \quad (4.29)$$
$$\bar{v}_2 : \lambda - (\mu_{\tilde{c}_1}(x))_v \leq 0$$
$$\bar{v}_3 : \lambda - (\mu_{\tilde{c}_1'}(x))_v \leq 0,$$
$$0 \leq \lambda \leq 1,$$

which is equivalent to the following problem

$$(P_v)\begin{cases} \max x^* = \lambda \\ \text{subject to} \\ \bar{v}_1 : \lambda - 3\left(\dfrac{5x_1 - 11x_2 + 21}{55.1760616}\right) \leq 0 \\ \bar{v}_2 : \lambda - \left(\dfrac{8 - x_1^2 - x_2^2}{3}\right) \leq 0 \\ \bar{v}_3 : \lambda - \left(\dfrac{4 - x_1 + x_2}{3}\right) \leq 0 \end{cases} \quad (4.30)$$

We can obtain the optimal solution $x_v^* \in R^n$ of the problem $\tilde{P}_v$ with the fuzzy decision set

$$\tilde{D}_v = \tilde{S}_v \cap \tilde{c}_1 \cap \tilde{c}_1', \quad (4.31)$$
$$\mu_{\tilde{D}_v}(x) = \min(\mu_{\tilde{S}_v}(x), \mu_{\tilde{c}_1}(x), \mu_{\tilde{c}_1'}(x)) \quad (4.32)$$

where $\mu_{\tilde{S}_v}(x)$, $\mu_{\tilde{c}_1}(x)$ and $\mu_{\tilde{c}_1'}(x)$ are defined in (4.28), (4.13), and (4.15) respectively.

The problem is to find the x_v^* which maximizes the minimum membership function i.e.

$$\mu_{\tilde{D}_v}(x_v^*) = \max \min(\mu_{\tilde{S}_v}(x_v^*), \mu_{\tilde{c}_1}(x_v^*), \mu_{\tilde{c}_1'}(x_v^*)) \quad (4.33)$$

The optimal solution of the problem $\tilde{P}_v$ is $x_v^* = (0.9585405344, -1.025648517)$ with $\lambda^* = 0.6719369829$ which satisfy in the constraints. We can submil x_v^* in the objective function of the crisp nonlinear programming problem. It can be obtained $v_{AF} = 16.07483636$ where v_{AF} is the value of the objective function after fuzziness. And we reduce that

$$v_l < v_{AF} < v_u \quad (4.34)$$

which is an accurate solution.

Now, for the problem $\tilde{P}_c$, the optimal solution for the problem $\tilde{P}_c$ is x^* with the fuzzy decision set

$$\tilde{D} = \tilde{D}_u \cap \tilde{D}_v, \quad (4.35)$$
$$\mu_{\tilde{D}}(x) = \min(\mu_{\tilde{D}_u}(x), \mu_{\tilde{D}_v}(x)) \quad (4.36)$$
$$= \min(\mu_{\tilde{S}_u}(x), \mu_{\tilde{S}_v}(x), \mu_{\tilde{c}_1}(x), \mu_{\tilde{c}_1'}(x)) \quad (4.37)$$

where $\mu_{\tilde{S}_u}(x)$, $\mu_{\tilde{S}_v}(x)$, $\mu_{\tilde{c}_1}(x)$, $\mu_{\tilde{c}_1'}(x)$ are defined in (4.11), (4.28), (4.13) and (4.15) respectively.

The problem is to find x^* which maximizes the minimum membership function values such that

$$\mu_{\tilde{D}}(x^*) = \max \min(\mu_{\tilde{S}_u}(x_u^*), \mu_{\tilde{S}_v}(x_v^*), \mu_{\tilde{c}_1}(x_u^*), \mu_{\tilde{c}_1'}(x_u^*), \mu_{\tilde{c}_1}(x_v^*),$$
$$\mu_{\tilde{c}_1'}(x_v^*)) \quad (4.38)$$

By calculating these values of membership functions,

$$\mu_D(x^*) = \max \min(0.7763536899, 0.6719375382, 1.628609451,$$

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$0.7763536896,\ 2.009748388,\ 0.6719369829)$

$= 0.6719369829$

occurs at $x^* = (x_v^*)$ i.e., the fuzzy decision making for this problem is to maximizes the minimum membership function

$$\mu_{\tilde{D}}(x^*) = \max\ \min\ \mu_{\tilde{C}'_1}(x_v^*)$$

i.e., the solution of the problem $\tilde{P}_v = (x_v^*)$ is the solution of the fuzzy complex programming problem $\tilde{P}_c$.

V. CONCLUSION

In this paper, we have introduced a new method to characterize the solution of complex programming problem with two parts of the objective function (real and imaginary) via Lexicographic order. Such that the complex problem can be divided into two real subproblems, and the solution of the main problem can be obtained by solving the two real subproblems. We illustrated our method by some numerical examples. The fuzzy solution of the two real subproblems is given by using the concept of fuzzy decision making and linear membership functions. Finally we illustrated by a numerical example our discussion on fuzzy complex programming problem, also the solutions of the problems after fuzzy are more accurate than the solution in crisp case.

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