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Thermal Stress Analysis of a Thin Hollow Cylinder: Transient Problem

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Abstract- This paper is concerned with the determination of temperature distribution, displacement function and thermal stresses of a thick annular disc occupying the space $D: a \le r \le b, -h \le z \le h$. The governing heat conduction equation has been solved by using Marchi-Zgrablich transform and Marchi-Fasulo transform techniques.

Key words: **Transient thermo elastic problem, thermal stresses, hollow cylinder, integral transform.**

I. INTRODUCTION

Khobragade et al. [8-10] have investigated temperature distribution, displacement function, and stresses of a thin as well as thick hollow cylinder and **Khobragade et al. [16]** have established displacement function, temperature distribution and stresses of a semi-infinite cylinder.

In the present chapter, an attempt is made to study the theoretical solution for a thermoelastic problem to determine the temperature distribution, displacement and stress functions of a hollow cylinder with boundary conditions occupying the space $D = \{(x, y, z) \in R^3 : a \le (x^2 + y^2)^{1/2} \le b, 0 \le z \le h\}$, where $r = (x^2 + y^2)^{1/2}$. A transform defined by **Zgrablich et al. [12]** is used for investigation which is a generalization of Hankel's double radiation finite transform and used to treat the problem with radiation type boundaries conditions.

II. FORMULATION OF THE PROBLEM

Consider a hollow cylinder as shown in the figure 1. The material of the cylinder is isotropic, homogenous and all properties are assumed to be constant. We assume that the cylinder is of a small thickness and its boundary surfaces remain traction free. The initial temperature of the cylinder is the same as the temperature of the surrounding medium, which is kept constant.

The displacement function $\phi(r, z, t)$ satisfying the differential equation as **Khobragade [8]** is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu}\right) a_t T \qquad (1)$$

with $\phi = 0$ at $r = a$ and $r = b$ (2)

where ν and a_t are Poisson ratio and linear coefficient of thermal expansion of the material of the cylinder respectively and $T(r, z, t)$ is the heating temperature of the cylinder at time t satisfying the differential equation as **Khobragade [8]** is

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] + \frac{g(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (3)$$

where $\kappa = K/\rho c$ is the thermal diffusivity of the material of the cylinder, K is the conductivity of the medium, c is its specific heat and ρ is its calorific capacity (which is assumed to be constant) respectively, subject to the initial and boundary conditions

$$M_t(T, 1, 0, 0) = F \quad \text{for all } a \le r \le b,\ 0 \le z \le h \qquad (4)$$

$$M_r(T, 1, \bar{\bar{k}}_1, a) = F_1(z, t), \text{ for all } 0 \le z \le h,\ t > 0 \qquad (5)$$

$$M_r(T, 1, \bar{\bar{k}}_2, b) = F_2(z, t) \quad \text{for all } 0 \le z \le h,\ t > 0 \qquad (6)$$

$$M_z(T, 1, k_3, -h) = F_3(r, t) \quad \text{for all } a \le r \le b,\ t > 0 \qquad (7)$$

$$M_z(T, 1, k_4, h) = G(r, t) \quad \text{for all } a \le r \le b,\ t > 0 \qquad (8)$$

being:

$$M_\vartheta(f, \bar{k}, \bar{\bar{k}}, \$) = (\bar{k}\, f + \bar{\bar{k}}\, \hat{f})_{\vartheta = \$}$$

where the prime (^) denotes differentiation with respect to ϑ, radiation constants are $\bar{k}$ and $\bar{\bar{k}}$ on the curved surfaces of the plate respectively.

The radial and axial displacement U and W satisfy the uncoupled thermoelastic equation as **Khobragade [8]** are

$$\nabla^2 U - \frac{U}{r^2} + (1-2\nu)^{-1}\frac{\partial e}{\partial r} = 2\left(\frac{1+\nu}{1-2\nu}\right) a_t \frac{\partial T}{\partial r} \qquad (9)$$

$$\nabla^2 W + (1-2\nu)^{-1}\frac{\partial e}{\partial z} = 2\left(\frac{1+\nu}{1-2\nu}\right) a_t \frac{\partial T}{\partial z} \qquad (10)$$

where

$$e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial r} \qquad (11)$$

$$U = \frac{\partial \phi}{\partial r}, \qquad (12)$$

$$W = \frac{\partial \phi}{\partial z} \qquad (13)$$

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The stress functions are given by

$$\tau_{rz}(a, z, t) = 0, \tau_{rz}(b, z, t) = 0, \tau_{rz}(r, 0, t) = 0 \tag{14}$$

$$\sigma_r(a, z, t) = p_i, \sigma_r(b, z, t) = -p_o, \sigma_z(r, 0, t) = 0 \tag{15}$$

where p_i and p_o are the surface pressure assumed to be uniform over the boundaries of the cylinder.

The stress functions are expressed in terms of the displacement components by the following relations as **Khobragade [8]** are

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right) \tag{16}$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left(\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z}\right) \tag{17}$$

$$\tau_{rz} = G\left(\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right) \tag{18}$$

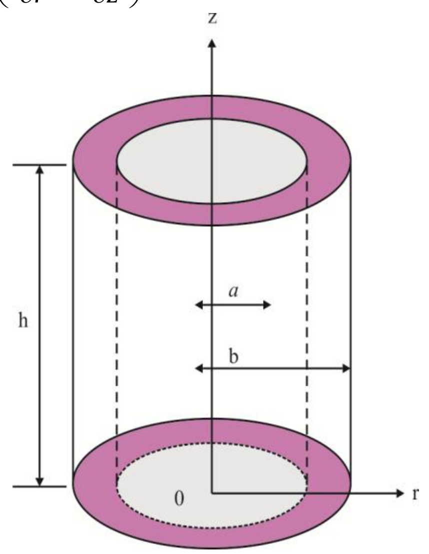

Fig 1: Geometry of the problem

where $\lambda = 2G\nu/(1-2\nu)$ is the Lame's constant, G is the shear modulus and U, W are the displacement components.

Equations (1)-(18) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE OF THE PROBLEM

Applying transform defined in **[12]** to the equations (3), (4) and (6) over the variable r having $p = 0$ with responds to the boundary conditions of type (4) and taking the Laplace transform , one obtains

$$\overline{T}^*(n, z, s) = e^{-\alpha p^2 t}\left[\overline{F}^* + \int_0^t \Pi\, e^{\alpha p^2 t'} dt'\right]$$

where constants involved $\overline{T}^*(n, z, s)$ are obtained by using boundary conditions (4). Finally applying the inversion theorems of transform defined in [**12**] and inverse Laplace transform by means of complex contour integration and the residue theorem, one obtains the expressions of the temperature distribution $T(r, z, t)$ for heating processes as

$$T(r, z, t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_m(z) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_m^2 \lambda_m} \times e^{-\alpha p^2 t}\left[\overline{F}^* + \int_0^t \Pi\, e^{\alpha p^2 t'} dt'\right] \tag{20}$$

Where n is the transformation parameter as defined in appendix, m is the Fourier sine transform parameter.

IV. DETERMINATION OF DISPLACEMENT AND STRESS FUNCTION

Substituting the value of temperature distribution from (20) in equation (1) one obtains the thermo elastic displacement function $\phi(r, z, t)$ as

$$\phi(r, z, t) = \frac{r^2 a_t}{4}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{P_m(z) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_m^2 \lambda_m} \times e^{-\alpha p^2 t}\left[\overline{F}^* + \int_0^t \Pi\, e^{\alpha p^2 t'} dt'\right] \tag{21}$$

Using (21) in the equations (12) and (13) one obtains

$$U = \frac{k a_t (1+\nu)}{2\xi(1-\nu)}\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\varphi_{nm}\begin{bmatrix} 2S_0(k_1, k_2, \mu_n r) \\ + rS'_0(k_1, k_2, \mu_n r)\end{bmatrix} \tag{22}$$

$$W = \frac{r^2 k a_t (1+\nu)}{2\xi(1-\nu)}\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\lambda_m \varphi_{nm} S_0(k_1, k_2, \mu_n r) \tag{23}$$

Substitution the value of (22), (23) in (15) to (18) one obtains the stress functions as

$$\sigma_r = \frac{k a_t (1+\nu)}{2\xi(1-\nu)}\sum_{m,n=1}^{\infty}\frac{\varphi_{nm}}{C_n}\begin{bmatrix}(\lambda + 2G)(r^2 S''_0(k_1, k_2, \mu_n r)) + 4rS'_0(k_1, k_2, \mu_n r) \\ + 2S_0(k_1, k_2, \mu_n r)\end{bmatrix} \times \lambda[2 S_0(k_1, k_2, \mu_n r) + r S'_0(k_1, k_2, \mu_n r)] - r^2\lambda_m S_0(k_1, k_2, \mu_n r) \tag{24}$$

$$\sigma_z = -\frac{k a_t (1+\nu)}{2\xi(1-\nu)}\sum_{m,n=1}^{\infty}\left(\frac{\varphi_{nm}}{C_n}\right)[(\lambda + 2G) r^2 \lambda_m^2 S_0(k_1, k_2, \mu_n r) - \lambda(r^2 S''_0(k_1, k_2, \mu_n r) + 5rS'_0(k_1, k_2, \mu_n r) + 4S_0(k_1, k_2, \mu_n r))] \tag{25}$$

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$$\sigma_\theta = \frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\varphi_{nm}}{C_n}\right) (\lambda + 2G)$$
$$[rS_0'(k_1,k_2,\mu_n r) + 2S_0(k_1,k_2,\mu_n r)]$$
$$+\lambda \begin{bmatrix} r^2 S_0''(k_1,k_2,\mu_n r) + 4rS_0'(k_1,k_2,\mu_n r) \\ +(2 - r^2\lambda_m^2) S_0(k_1,k_2,\mu_n r) \end{bmatrix} \quad (26)$$

$$\tau_{rz} = \frac{k a_t G(1+\nu)}{\xi(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\lambda_m \varphi_{nm}}{C_n}\right) \quad (27)$$
$$[r^2 S_0'(k_1,k_2,\mu_n r) + 2rS_0(k_1,k_2,\mu_n r)]$$

V. SPECIAL CASE

Set $f(r,t) = (1-e^{-t})\, \delta(r-r_0)$ (28)

Applying finite transform defined in **Marchi Zgrablich [12]** to the equation (28) one obtains

$$\bar{f}(n,t) = (1-e^{-t})\, r_0\, S_0(k_1,k_2,\mu_n r_0) \quad (29)$$

Substituting the value of (29) in the equations (20) to (28) one obtains

$$T(r,z,t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{P_m(z) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n^2 \lambda_m}$$
$$\times e^{-\alpha p^2 t} \left[\overline{F}^* + \int_0^t \Pi e^{\alpha p^2 t'} dt' \right] \quad (30)$$

$$\phi(r,z,t) = \frac{r^2 a_t}{4} \left(\frac{1+\nu}{1-\nu}\right) \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=1}^{\infty} \frac{P_m(z) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n^2 \lambda_m}$$
$$\times e^{-\alpha p^2 t} \left[\overline{F}^* + \int_0^t \Pi e^{\alpha p^2 t'} dt' \right] \quad (31)$$

$$U = \frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=1}^{\infty} \varphi_{nm} \left[2S_0(k_1,k_2,\mu_n r) + rS'_0(k_1,k_2,\mu_n r)\right]$$
(32)

$$W = \frac{r^2 k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=1}^{\infty} \lambda_m \varphi_{nm} S_0(k_1,k_2,\mu_n r)$$
(33)

$$\sigma_r = \frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{m,n=1}^{\infty} \frac{\varphi_{nm}}{C_n}$$
$$\begin{bmatrix} (\lambda+2G)(r^2 S''_0(k_1,k_2,\mu_n r)) + 4rS'_0(k_1,k_2,\mu_n r) \\ +2S_0(k_1,k_2,\mu_n r) \end{bmatrix}$$
$$\times \lambda [2\, S_0(k_1,k_2,\mu_n r) + r\, S'_0(k_1,k_2,\mu_n r)] \quad (34)$$
$$-r^2 \lambda_m\, S_0(k_1,k_2,\mu_n r)$$

$$\sigma_z = -\frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\varphi_{nm}}{C_n}\right) [(\lambda+2G) r^2 \lambda_m^{\,2} S_0(k_1,k_2,\mu_n r)$$
$$-\lambda \begin{pmatrix} r^2 S_0''(k_1,k_2,\mu_n r) + 5rS_0'(k_1,k_2,\mu_n r) \\ +4S_0(k_1,k_2,\mu_n r) \end{pmatrix}] \quad (35)$$

$$\sigma_\theta = \frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\varphi_{nm}}{C_n}\right) (\lambda+2G) \begin{bmatrix} rS_0'(k_1,k_2,\mu_n r) \\ +2S_0(k_1,k_2,\mu_n r) \end{bmatrix}$$
$$+\lambda \begin{bmatrix} r^2 S_0''(k_1,k_2,\mu_n r) + 4rS_0'(k_1,k_2,\mu_n r) \\ +(2 - r^2\lambda_m^2) S_0(k_1,k_2,\mu_n r) \end{bmatrix} \quad (36)$$

$$\tau_{rz} = \frac{k a_t G(1+\nu)}{\xi(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\lambda_m \varphi_{nm}}{C_n}\right) \begin{bmatrix} r^2 S_0'(k_1,k_2,\mu_n r) \\ +2rS_0(k_1,k_2,\mu_n r) \end{bmatrix}$$
(37)

VI. NUMERICAL RESULTS

Set $a = 2,\ b = 4,$ t= 1 sec

Substituting this value in Equations (30) to (37) we get

$$T(r,z,t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{P_m(z) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n^2 \lambda_m}$$
$$\times e^{-\alpha p^2} \left[\overline{F}^* + \int_0^1 \Pi e^{\alpha p^2 t'} dt' \right] \quad (38)$$

$$\phi(r,z,t) = \frac{r^2 a_t}{4} \left(\frac{1+\nu}{1-\nu}\right) \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=1}^{\infty} \frac{P_m(z) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n^2 \lambda_m}$$
$$\times e^{-\alpha p^2} \left[\overline{F}^* + \int_0^1 \Pi e^{\alpha p^2 t'} dt' \right] \quad (39)$$

$$U = \frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=1}^{\infty} \varphi_{nm} \begin{bmatrix} 2S_0(k_1,k_2,\mu_n r) \\ +rS'_0(k_1,k_2,\mu_n r) \end{bmatrix} \quad (40)$$

$$W = \frac{r^2 k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=1}^{\infty} \lambda_m \varphi_{nm} S_0(k_1,k_2,\mu_n r) \quad (41)$$

$$\sigma_r = \frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{m,n=1}^{\infty} \frac{\varphi_{nm}}{C_n}$$

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$$\begin{bmatrix}(\lambda+2G)\left(r^2 S''_0(k_1,k_2,\mu_n r)\right)+4rS'_0(k_1,k_2,\mu_n r)\\ +2S_0(k_1,k_2,\mu_n r)\end{bmatrix}$$

$$\times\lambda\left[2\,S_0(k_1,k_2,\mu_n r)+r\,S'_0(k_1,k_2,\mu_n r)\right] \qquad (43)$$
$$-r^2\lambda_m\,S_0(k_1,k_2,\mu_n r)$$

$$\sigma_z=-\frac{k\,a_t\,(1+\nu)}{2\xi\,(1-\nu)}\sum_{m,n=1}^{\infty}\left(\frac{\varphi_{nm}}{C_n}\right)\Big[(\lambda+2G)r^2\lambda_m^{\,2}S_0(k_1,k_2,\mu_n r)$$
$$-\lambda\begin{pmatrix}r^2 S''_0(k_1,k_2,\mu_n r)+5rS'_0(k_1,k_2,\mu_n r)\\ +4S_0(k_1,k_2,\mu_n r)\end{pmatrix}\Big] \qquad (44)$$

$$\sigma_\theta=\frac{k\,a_t\,(1+\nu)}{2\xi\,(1-\nu)}\sum_{m,n=1}^{\infty}\left(\frac{\varphi_{nm}}{C_n}\right)(\lambda+2G)$$
$$\left[rS'_0(k_1,k_2,\mu_n r)+2S_0(k_1,k_2,\mu_n r)\right]$$
$$+\lambda\begin{bmatrix}r^2 S''_0(k_1,k_2,\mu_n r)+4rS'_0(k_1,k_2,\mu_n r)\\ +\left(2-r^2\lambda_m^2\right)S_0(k_1,k_2,\mu_n r)\end{bmatrix} \qquad (45)$$

$$\tau_{rz}=\frac{k\,a_t G(1+\nu)}{\xi\,(1-\nu)}\sum_{m,n=1}^{\infty}\left(\frac{\lambda_m\,\varphi_{nm}}{C_n}\right)\left[r^2 S'_0(k_1,k_2,\mu_n r)+2rS_0(k_1,k_2,\mu_n r)\right]$$
(46)

VII. NUMERICAL RESULTS, DISCUSSION AND REMARKS

To interpret the numerical computation we consider material properties of low carbon steel (AISI 1119), which can be used for medium duty shafts, studs, pins, distributor cams, cam shafts, and universal joints having mechanical and thermal properties:

$\kappa=13.97\,[\mu m/s^2]$ $\upsilon=0.29$,

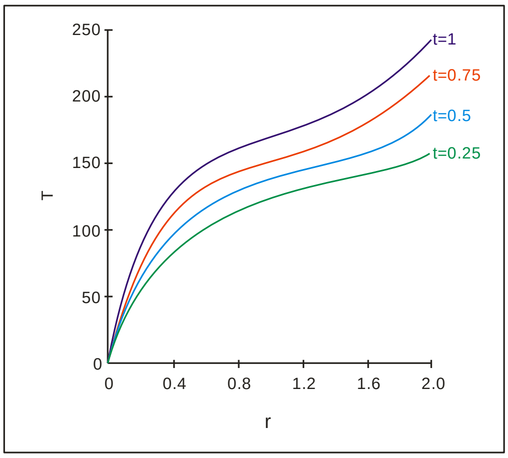
$\lambda=51.9\,[W/(m-K)]$ and

$a_t=14.7\,\mu m/m-{}^0C$.

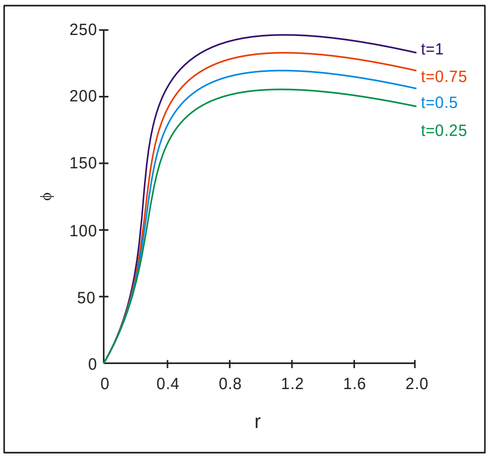
Setting the physical parameter with $a=0.5$, $b=1$ and $h=3$.

VIII. CONCLUSION

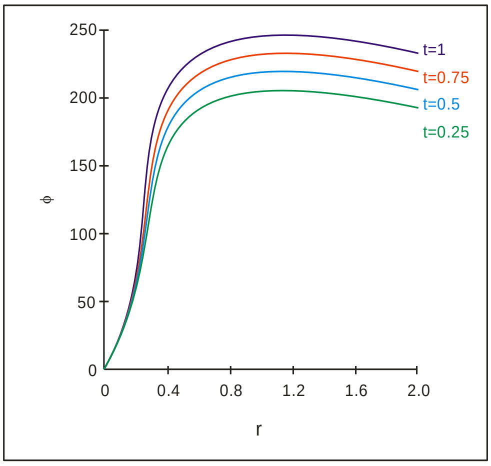
In this paper, we modify the conceptual idea proposed by **Khobragade et al**. [**8**] for hollow cylinder and the temperature distributions, displacement and stress functions at the edge $z=h$ occupying the region of the cylinder $a\le r\le b,\ \ 0\le z\le h$ have been obtained with the known boundary conditions. We develop the analysis for the temperature field by introducing the transformation defined by **Zgrablich et al. [12]**, finite Fourier sine transform and Laplace transform techniques with boundaries conditions of radiations type. The series solutions converge provided we take sufficient number of terms in the series. Since the thickness of cylinder is very small, the series solution given here will be definitely convergent. Assigning suitable values to the parameters and functions in the series expressions can derive any particular case. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications.



Graph 1: Temperature distribution versus r



Graph 2: Displacement function versus r



Graph 4: Displacement component versus r

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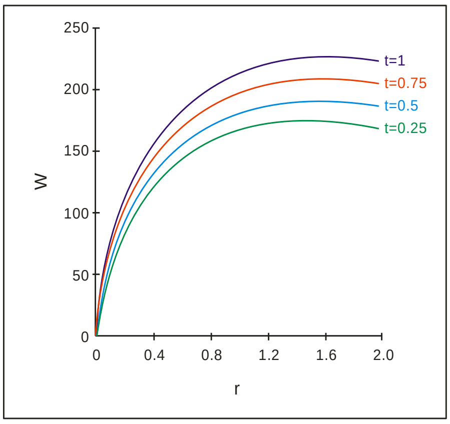
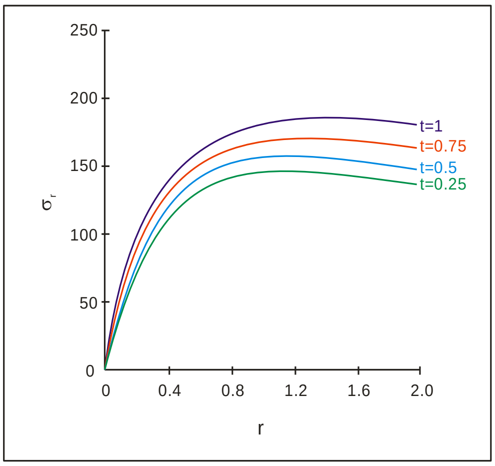
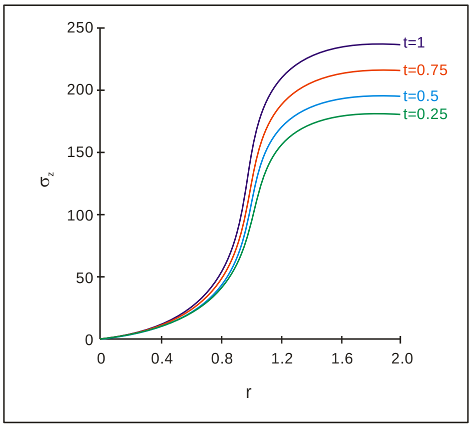


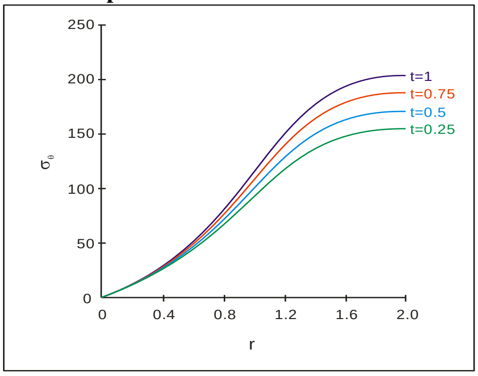
Graph 5: Displacement component versus r



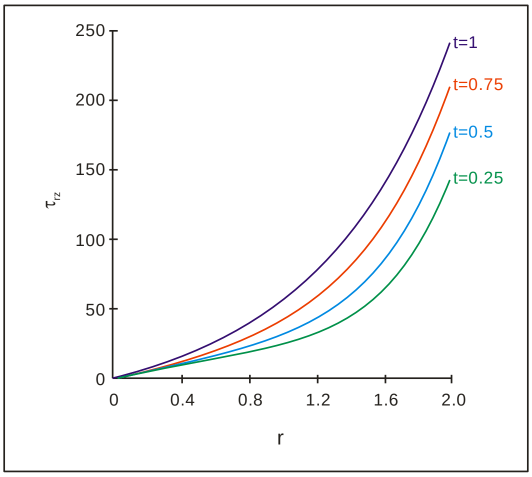
Graph 6: Stress Function versus r



Graph 7: Stress Function versus r



Graph 8: Stress Function versus r



Graph 9: Stress Function versus r

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