

# Linear Array of Non-Uniformly Spaced Antennas with Non-Uniform Excitation Amplitude

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**Abstract:** A numerically efficient approach based on the Method of Moments (MoM) analysis is described for a non-uniform linear array antenna with unequal inter-element spacing. The MoM provides the excitation amplitude of the antenna. This method of solution is robust and reliable because of the large degree of freedom created by the varying inter element spacing to achieve its solution. This helps to generate any radiation pattern to suit the application under consideration. The impact of the variations in the inter-element spacing, excitation amplitude and number of the constituting antenna elements in the array is investigated. The results show that few antennas elements can yield radiation with low beam width (improved directivity), thus element reduction can be achieved to yield enhanced radiation pattern. Furthermore by varying the inter element spacing; different radiation pattern can be obtained for the same number of antennas. This aids in realizing radiation patterns with the least side lobe.

**Keywords:** Antenna spacing, Array factor, excitation amplitude, radiation pattern.

## I. INTRODUCTION

More than one antenna element can be arranged in any defined configuration along a co-ordinate system to generate an array antenna. Different types of array antennas exist. They range from the linear, planar, circular, elliptical to the conformal arrays. The essence of the array antenna is to overcome the limitations of the single element antenna. These limitations include low directivity and poor radiation pattern which makes the radiated signal prone to fading and attenuation in long distance communication. Any array model can be generated depending on the design interest and application. The array could serve for point-to-point communication, directional broadcast application [1], [2] or omnidirectional coverage. The radiation pattern of the array antenna is dependent on some input parameters which include, the inter-element spacing between the elements in the array, the radiation pattern of the single antenna element, the excitation amplitude, phase and the number of antenna element constituting the array.

Analysis involving non-uniformly spaced linear array dates back to the works of Unz [3] where he formulated a matrix method to determine the current distribution necessary for radiation from an antenna. The linear array generally is classified into two categories; the first one is the thinned arrays. This form of array requires some of the elements of an equally spaced array to be zeroed out. By this approach, the elements are present but do not form part of the analytical computation. The second category involves linear arrays whose elements are randomly spaced. The latter is considered in this work. Its

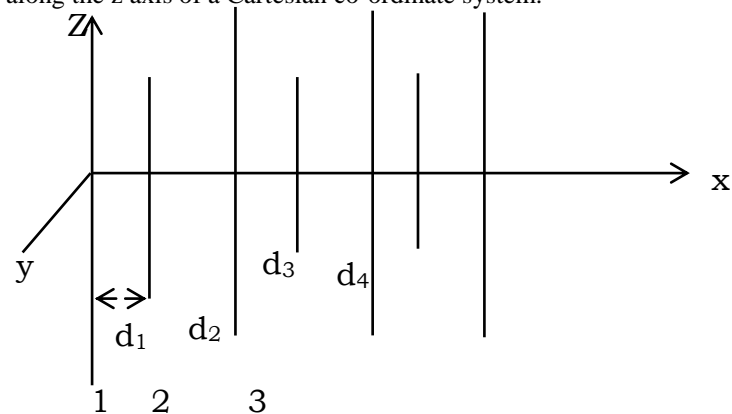
history of application dates back to the works of Harrington [4], where an iterative method was adopted to reduce the side lobe levels of uniform linear antennas. Sandler [5] considered the non-uniformly spaced array as an equivalent uniformly spaced array by exploring each cosine term in the array factor of the non-uniformly spaced array in a Fourier series. Further works involving non-uniformly spaced array which addressed the issue of side lobe reduction and grating lobe suppression were also considered [6]-[8]. Bavelacqua and Balanis [9] obtained optimal array geometrics with optimal weights in other to minimize side lobe levels in wide band array antennas. Non-uniformly spaced array antenna was also investigated by considering the average element spacing such that a relationship between the array length, side lobe and directivity was established [10].

In this paper, a non-uniformly spaced linear array with non-uniform, excitation amplitude is investigated for low side lobe radiation and enhanced directivity. The results reflect the impact of these parameters on the side lobe radiation.

The remaining part of the paper is organized as follows; section II describes the antenna formulation. Section III gives the result of the analysis and the discussions therein. Section IV concludes the work.

## II. ANTENNA FORMULATION

Consider a linear array of un-equal radiating wire antennas with non-uniform inter-element spacing arrayed along the z axis of a Cartesian co-ordinate system.



**Linear array of non uniform length and inter element spacing.**

The antennas are assumed to be thin and are perfect conductors so that the boundary conditions for the electric field exist on the surface of the conductor.

The vector potential (A) of the field using the thin wire approximate kernel is given by

$$A = \mu \int_{-l/2}^{+l/2} I_z(z') \frac{e^{-jkr}}{4\pi r} dz' \quad 1$$

$$r_m = \sqrt{a^2 + (z - z')^2}$$

$$m=1,2,3,\dots,N+1$$

Where;

$$\frac{e^{-jkr}}{4\pi r} = G = \text{kernel}, \quad I_z = \text{current}$$

The radiating electric field is thus given by

$$\bar{E} = -j\omega A - \frac{j\partial^2 A_z}{\omega\mu\epsilon\partial z^2} \quad 2$$

With zero tangential electric field equation 2 becomes

$$\bar{E}_z^i = \frac{j}{\omega\mu\epsilon} \left[ \frac{\partial^2}{\partial z^2} + K^2 \right] A_z \quad 3$$

$$\bar{E}^i = \frac{j}{\omega\mu\epsilon} \int_{-l/2}^{l/2} I_z(z') \left[ \frac{\partial^2}{\partial z^2} + K^2 \right] \frac{e^{-jkn}}{4\pi r} dz' \quad 4$$

For a symmetrical conductor around the mid point axis of the antenna structure, current density  $J_z(z) = -J_z(z)$  so that the potential  $A_z$  is also symmetrical. This implies that;

$$A_z(z') = -j\sqrt{\mu\epsilon} (B_1 \cos(Kz) + C_1 \sin(K/z)) \quad 5$$

Where;  $B_1$  and  $C_1$  are constants.  $B_1$  is determined from the condition that current tends towards zero at the wire antenna ends.  $C_1$  is half the input voltage. Equating (1) and (5), the Hallen's equation is obtained thus

$$\int_{-l/2}^{+l/2} I_z(z') \frac{e^{-jkr}}{4\pi r} dz' = -j\sqrt{\mu\epsilon} [(B_1 \cos(Kz) + C_1 \sin(K/z))] \quad 6$$

A pulse function is applied to expand the basis function and a derac delta testing function is applied as the weighting function in other to implement the method of moment solution. This is given by equation 7

$$\sum_{n=1}^N I_n \int_{z_n - \Delta z/2}^{z_n + \Delta z/2} \frac{e^{-jKr_m}}{4\pi r_m} dz' = \frac{-j}{\eta_0} \left[ B_1 \cos(Kz_m) + \frac{VT}{2} \sin(K/z_m) \right] \quad 7$$

Where  $\frac{1}{\eta_0} = \sqrt{\mu\epsilon}$

The current distribution is obtained from the Hallen's equation while the peak current at the feed point of the antenna describes its excitation amplitude.

The electric field pattern of the array is determined by applying the superposition theorem. The antenna elements are considered to operate in the far field region and are seen as point sources. As point sources, the normalized field pattern of the element is equivalent to  $\sin\theta$ . Where  $\theta$  takes values of the angles either for broadside or end fire application. This is represented by the expression given by.

$$E_\theta = \sin\theta \sum_n^N a_n \cos \left[ K \sum_{dn=0}^{dn} dn \cos\theta \right]$$

Where

$a_n$ =excitation amplitude determined from moment method

$dn$ = Inter-element distances

$\theta$  = Elevation angle

$K$  = wave number

These source elements are weighted by their respective excitation amplitude. This is as a result of the unequal length of antenna elements in the array.

### III. RESULT/DISCUSSION

The excitation amplitude of each antenna is generated using method of moment approach. It is observed that for different antenna length, the corresponding excitation amplitude can be determined. The first antenna is placed along the origin of the axis and there is no phase difference between each antenna elements. It is observed that the longer the antenna length, the smaller the excitation amplitude. This outcome also depends on the size of the expansion function used in the method of moment analysis. It can then be concluded that a reasonable expansion function would yield very reliable result.

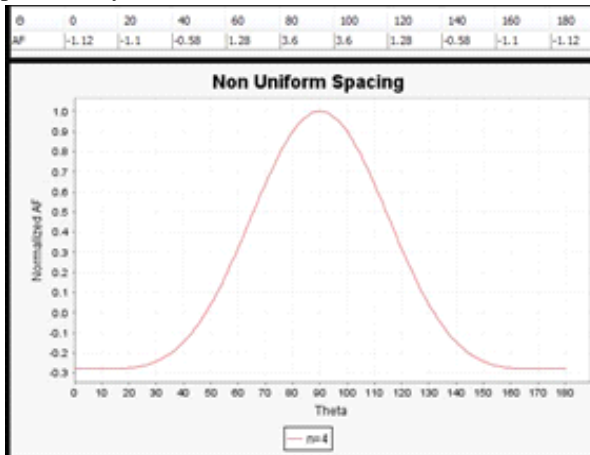
The elements of the array are seen to operate in the far field region as such they can be seen as point sources weighted by their excitation amplitude.

**Table 1 Lengths of antenna with their corresponding source amplitude.**

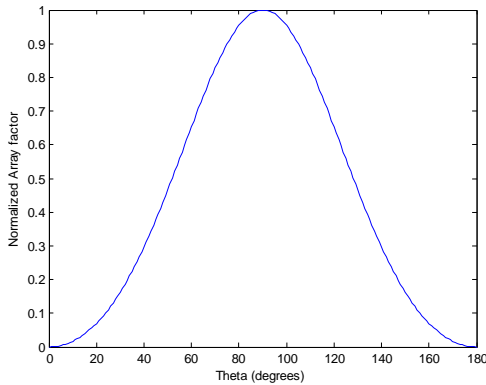
S/No	Length of Antenna ( $\lambda$ )	Excitation Amplitude(A)
1	0.5	0.016

2	1.5	0.014
3	2.5	0.013

From the result (figure 1) it is shown that an array which consist of very few number of antennas ( $N = 4$ ) is expected to have a radiation pattern with large beam width within a large angular direction as well as low directivity. The graph is seen to be devoid of any side lobe radiation for uniformly spaced and non-uniformly spaced array antenna.



(a)

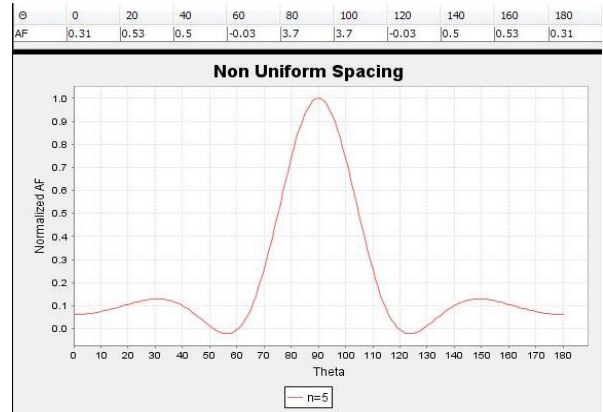


(b)

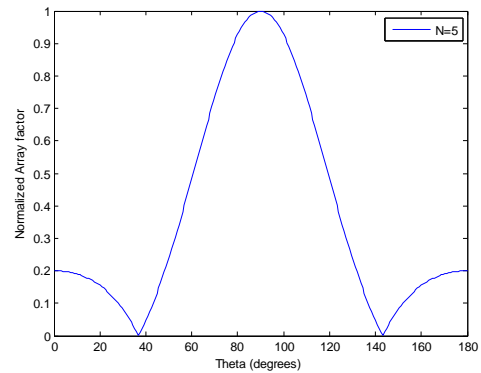
**Fig 1 Array pattern plot for 4 element antenna array. Non uniform array (b) uniform array**

It is further observed that as the number of antenna element increases the radiation pattern obtained shows improved directivity and reduced side lobe radiation for the non-uniformly spaced antennas with non-uniform excitation amplitude as compared to the uniformly spaced array of the same number of elements (figure 2).

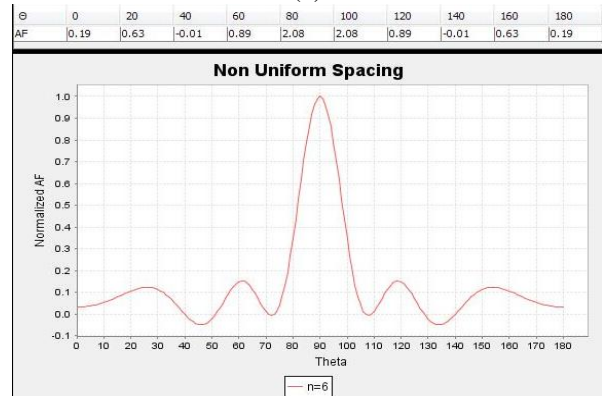
This is because the former is flexible and has a large range of spacing which is employed to yield a good radiation pattern.



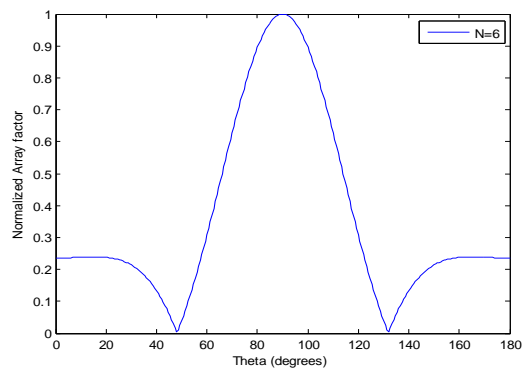
(a)



(b)



(c)



(d)

**Fig.2. Radiation plots for non-uniform and uniform arrays. (a) N = 5 and (c) N = 6 non-uniform linear arrays. (b) N = 5 and (d) N = 6 uniform linear arrays.**

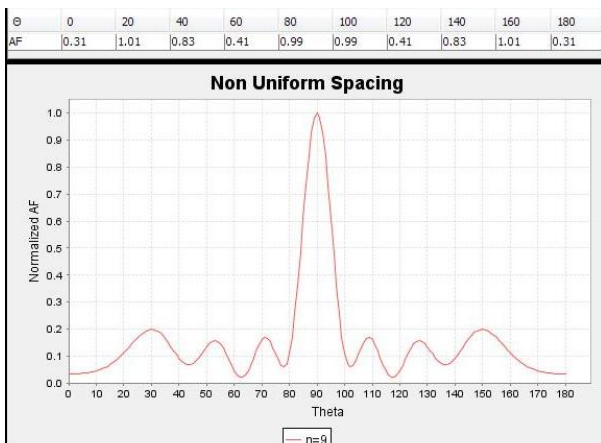
Another benefit of adopting the non uniform array with un equal spacing is that different radiation pattern with reduced side lobes can be achieved for the same number of antenna element. This approach creates opportunity for patterns with the lowest side lobe to be achieved. It is a cost effective approach when implemented. This is because the same number of antenna can be used to obtain a desired radiation pattern. This is unlike the uniform linear array whose radiation pattern changes with increase in the number of antenna elements. The uniform linear array may result to more cost implications.

**Table1: Array spacing showing side lobe level for the same number of antennas (8).**

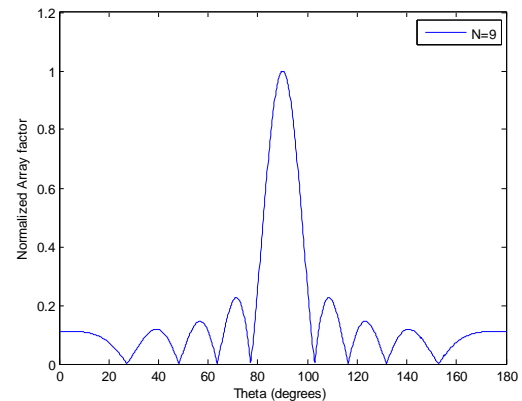
S/No	Antenna Spacing ( $\lambda$ )	SLL	SLL(dB)
1	0,0,0.2,0.5,0.2,0.4,0.2,0.5,0.7	0.231	-12.73
2	0,0,0.1,0.4,0.1,0.4,0.2,0.4,0.5	0.211	-13.51
3	0,0,0.5,0.2,0.4,0.1,0.4,0.2,0.5	0.051	-25.84
4	0,0,0.3,0.1,0.4,0.2,0.5,0.2,0.5	0.189	-14.47
5	0,0,0.4,0.1,0.4,0.2,0.4,0.2,0.4	0.146	-16.71
6	0,0,0.5,0.2,0.4,0.1,0.4,0.2,0.4	0.093	-20.63
7	0,0,0.4,0.1,0.3,0.1,0.4,0.2,0.5	0.019	-34.42
8	0,0,0.4,0.1,0.4,0.3,0.5,0.3,0.6	0.111	-19.09
9.	0,0,0.5,0.2,0.4,0.2,0.5,0.3,0.6	0.061	-24.29
10	0,0,0.3,0.1,0.3,0.1,0.3,0.1,0.4	0.001	-60.00

For point-point communication where array antennas find more application, it is desirable to obtain radiation pattern with small beam width and few radiations in other direction so that an effective communication process is established.

Consider a large number of antennas (N= 9), the radiation pattern generated with non-uniform array (figure.3) has well reduced number of side lobes and reduced beam width when compared to that of the uniformly spaced array.



(a)



(b)

**Fig 3. Radiation plot for 9 element array. (a) non uniformly spaced linear array (b) uniformly spaced linear array.**

#### IV. CONCLUSION

The numerical method for analysis of non-uniformly spaced linear array with the corresponding element excitation amplitude is considered. The approach is robust and reliable since the parameter considered, are determined using the right method and not mere arbitrary values. It is illustrated in this paper that, it is possible to generate radiation pattern with enhanced directivity (reduced beam width) with low side lobes using few number of antennas using the method adopted in this work. The outcome is better than the uniform linear array antenna with equal spacing of the same number of antennas. It is also seen that reduced side lobe radiation can be achieved for the same number of antennas due to the random spacing ability of the non-uniformly spaced array model. It is recommended in future that optimization of this approach be obtained for array analysis and compared to its synthesis counterpart

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