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Pythagorean Triangle With (One Leg of Right Triangle p^2-q^2) $-2\frac{area}{perimeter}=\alpha\beta$

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Abstract -
Patterns of Pythagorean Triangle for each of which, the ratio (One leg of right triangle $p^2-q^2)-2\frac{area}{perimeter}=\alpha\beta$ may be expressed as the product of two integers. A few interesting relations are also given.

INTRODUCTION

It is well known that Pythagoras triangle is a treasure house which contains many interesting results for an extensive review of literature. The method of obtaining three non-zero integers x, y and z under certain relations satisfying the equation $x^2+y^2=z^2$ has been a matter of interest to various mathematicians. One may refer [1-7]. In [8-11] special Pythagoras problems are studies. In this communication we present yet another interest Pythagorean Triangles, where in each of which, $(One\ leg\ of\ right\ trinagle\ (p^2-q^2))-2\left(\frac{Area}{Perimeter}\right)$ may be expressed as the product of two integers. A few interesting relations are also given. Also, the recurrence relations for the sides of the triangle are presented.

NOTATION

$$I_{m,n}=n\left[1+\frac{(n-1)(m-2)}{2}\right]$$

=polygonal number of rank n with sides m

Problem:

(One leg of right triangle $p^2-q^2)-2\frac{area}{perimeter}=\alpha\beta$

Method of Analysis:

Choosing $p=\alpha$

$p-q=\beta$

Solving we get $p=\alpha$ and $q=\alpha-\beta$. In which follows, we obtain the values of x , y , z

$x(\alpha,\beta)=2pq \Rightarrow x=2\alpha^2-2\alpha\beta$
$y(\alpha,\beta)=p^2-q^2 \Rightarrow y=2\alpha\beta-\beta^2$
$z(\alpha,\beta)=p^2+q^2 \Rightarrow z=2\alpha^2+\beta^2-2\alpha\beta$,

where $\alpha>\beta$

Few examples are given

α	β	p	q	x	y	z
5	2	5	3	30	16	34
2	1	2	1	4	3	5
7	4	7	3	42	4	58
3	2	3	1	6	8	10
1	6	10	4	80	84	116

Recurrence relation:

1. $x(\alpha+1, \beta+1) - 2x(\alpha,\beta) + x(\alpha-1, \beta-1) = 0$
2. $y(\alpha+1, \beta+1) - 2y(\alpha,\beta) + y(\alpha-1, \beta-1) = 2$
3. $z(\alpha+1, \beta+1) - 2z(\alpha,\beta) + z(\alpha-1, \beta-1) = 2$

Properties:

1. $z - x$ = a perfect square
2. $x - (\alpha^2 - \beta^2)$ = a perfect square
3. $x(\alpha,\beta) - (\alpha - \beta)^2$ = a perfect square
4. $y(\alpha,\beta) + (\alpha + \beta)^2$ = a perfect square
5. $z(\alpha,\beta) - (\alpha - \beta)^2$ = a perfect square
6. $y(\alpha,1) - 1$ = a perfect square
7. $x(\alpha, \beta) - 2T_{4,\alpha} + 2\alpha\beta = 0$
8. $x(\alpha,\alpha) = 0$
9. $y(\alpha,\alpha)$ = a perfect square
10. $z(\alpha,\alpha)$ = a perfect square
11. $y(\alpha,\alpha) - z(\alpha,\alpha) = 0$
12. $x(\alpha,1) - z(\alpha,1) + 1 = 0$
13. $T_{8,\alpha} - x(\alpha, 1)$ = a perfect square
14. $y(\alpha,1) - 2T_{2,\alpha} - (\alpha - 1) = 0$

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