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A different approach on a Pythagorean triangle which satisfies $a(Hypotonuse) - 4a\dfrac{(Area)}{(Perimeter)} = \alpha^2$

Dr.P.SHANMUGANANDHAM, Department of Mathematics,
National College, Trichy-620 001, (Tamil Nadu)

***Abstract:** We obtain non-trivial integral values for the sides f the Pythagorean triangle such that its* $a(Hypotonuse) - 4a\dfrac{(Area)}{(Perimeter)} = \alpha^2$. *A few interesting relations between the sides of the Pythagorean triangle are presented.*

Key words: **Integral solutions, Pythagorean triangles, MSC classification number 11D09.**

I. INTRODUCTION

One well known set of solutions of the Pythagorean equation $x^2 + y^2 = z^2$ are $x = 2ab,\ y = a^2 - b^2 \ and\ z = a^2 + b^2$. Many mathematicians has been used this set of solutions to obtain the non-zero integral values for x,y and z [1-3]. As a new approach, in this paper we introduce another set of solutions

$$x = 2A+1,\ \ y = 2A^2 + 2A \ and\ z = 2A^2 + 2A + 1$$

for the equation $x^2 + y^2 = z^2$. By using this solution we obtain three non-zero integers x,y and z under certain relations satisfying the equation $x^2 + y^2 = z^2$ [4-6]. In this communication, we present yet another interesting Pythagorean triangle where in each of which the ratio $a(Hypotonuse) - 4a\dfrac{(Area)}{(Perimeter)}$ may be expressed as a perfect square.

II. METHOD OF ANALYSIS

Taking A>0 to be the generators of the Pythagorean triangle (x,y,z), the assumption that $a(Hypotonuse) - 4a\dfrac{(Area)}{(Perimeter)} = \alpha^2$ leads to the Pellian equation $Y^2 = DX^2 + a$ where D = 2a, not a perfect square and A = X.

For the clear understanding we consider the following two cases:

i) a = 3 (odd number) so that D = 6

ii) a = 4 (even number) so that D = 8

Case (i):
When a = 3 the equation

$$Y^2 = DX^2 + a \qquad (1)$$

becomes

$$Y^2 = 6X^2 + 3 \qquad (2)$$

Let $(x_0, y_0) = (1,3)$ be the initial solution of (2). Consider the Pellian

$$Y^2 = 6X^2 + 1 \qquad (3)$$

Let $(\tilde{x}_0, \tilde{y}_0) = (2,5)$ be a solution of equation (3).

n	A_{n+1}	Y_{n+1}
-1	1	3
0	11	27
1	109	267
2	1079	2643
3	10681	26163

Using Brahmagupta lemma the general solution $(\tilde{x}_n, \tilde{y}_n)$ of equation (3) is given by

$$\tilde{y}_n + \sqrt{6}\tilde{x}_n = \left[5 + 2\sqrt{6}\right]$$

Where n = 0,1,2,3….. (4)
Since irrational roots occur in pairs

$$\tilde{y}_n - \sqrt{6}\tilde{x}_n = \left[5 - 2\sqrt{6}\right]$$

Where n = 0,1,2,3… (5)
is also a solution.
From equations (4) and (5), we obtain

$$\tilde{y}_n = \frac{1}{2}\left[\left(5+2\sqrt{6}\right)^{n+1} + \left(5-2\sqrt{6}\right)^{n+1}\right] \qquad (6)$$

and

$$\tilde{x}_n = \frac{1}{2\sqrt{6}}\left[\left(5+2\sqrt{6}\right)^{n+1} - \left(5-2\sqrt{6}\right)^{n+1}\right]$$

(7)

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Using the equations (6) and (7), the solutions of equation (2) is given by

$$A_{n+1} = X_{n+1} = \frac{1}{2\sqrt{6}}\left[\left(3+\sqrt{6}\right)\left(5+2\sqrt{6}\right)^{n+1} - \left(3-\sqrt{6}\right)\left(5-2\sqrt{6}\right)^{n+1}\right]$$

n = -1,0,1,2,3…

$$Y_{n+1} = \frac{1}{2}\left[\left(3+\sqrt{6}\right)\left(5+2\sqrt{6}\right)^{n+1} - \left(3-\sqrt{6}\right)\left(5-2\sqrt{6}\right)^{n+1}\right]$$

n = -1,0,1,2,3…

III. NUMERICAL EXAMPLES

Observations:

1. The Recurrence relations for X and Y are $X_{n+3} - 10X_{n+2} + X_{n+1} = 0$ and $Y_{n+3} - 10Y_{n+2} + Y_{n+1} = 0$.
2. For all values of n, both X and Y are odd.
3. For all values of n, Y_{n+1} is divisible by 3.
4. $24X_{n+1}Y_{n+1}$ is difference of two squares.
5. $X_{n+3} + X_{n+1} \equiv 0\left(\bmod 10\right)$ and $Y_{n+3} + Y_{n+1} \equiv 0\left(\bmod 10\right)$

Case (ii):

When a = 4, the equation (1) leads to

$$Y^2 = 8X^2 + 4 \qquad (8)$$

Let $\left(x_0, y_0\right) = \left(2, 6\right)$ be initial solution of equation (8).

To obtain the general solution of (8) consider the Pellian

$$Y^2 = 8X^2 + 1 \qquad (9)$$

Let $\left(\tilde{x}_0, \tilde{y}_0\right) = \left(1, 3\right)$ be the initial solution of equation (9)

Then the general solution of equation (9) is given by

$$\tilde{y}_n = \frac{1}{2}\left[\left(3+\sqrt{8}\right)^{n+1} + \left(3-\sqrt{8}\right)^{n+1}\right]$$

and

$$\tilde{x}_n = \frac{1}{2\sqrt{8}}\left[\left(3+\sqrt{8}\right)^{n+1} - \left(3-\sqrt{8}\right)^{n+1}\right]$$

n = 0,1,2,3,…

Therefore, the general solution of equation (8) is

$$A_{n+1} = X_{n+1} = \frac{1}{\sqrt{8}}\left[\left(3+\sqrt{8}\right)^{n+1} - \left(3-\sqrt{8}\right)^{n+1}\right]$$

$$Y_{n+1} = \left[\left(3+\sqrt{8}\right)^{n+1} + \left(3-\sqrt{8}\right)^{n+1}\right]$$

n = 0,1,2,3…

Numerical Examples

n	A_{n+1}	Y_{n+1}
0	2	6
1	12	34
2	70	198
3	408	1154
4	2378	6726

Observations:

1. The recurrence relations for X and Y are $X_{n+3} - 6X_{n+2} + X_{n+1} = 0$ and $Y_{n+3} - 6Y_{n+2} + Y_{n+1} = 0$
2. For all values of n, both X_{n+1} and Y_{n+1} are even.
3. For all values of n, Y_{n+1} is divisible by 3.
4. $X_{n+3} + X_{n+1} \equiv 0\left(\bmod 6\right)$ and $Y_{n+3} + Y_{n+1} \equiv 0\left(\bmod 6\right)$

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