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Thermal Stresses of a Thick Annular Disc due to Partially Distributed Heat Supply

Ovais A. Ahmed and N. W. Khobragade and L. H. Khalsa
1,2, Department of Mathematics, MJP Educational Campus, Nagpur University, Nagpur- 440 033, India.
3, Department of Mathematics, M.G. College Armori, India

Abstract- This paper is concerned with the determination of temperature distribution, displacement function and thermal stresses of a thick annular disc occupying the space $D: a \le r \le b,\ -h \le z \le h$ ***. The governing heat conduction equation has been solved by using Marchi-Zgrablich transform and Marchi-Fasulo transform techniques.***

Key words: **Unsteady state thermo elastic problem, thermal stresses, thick annular disc, Love's function.**

I. INTRODUCTION

Khobragade et al. [6-7] have derived temperature distribution, displacement function, thermal stresses and thermal deflection of a thick and thin circular plate. Further **Khobragade et al. [1-4]** have established displacement function, temperature distribution and stresses and deflection of a triangular plate.

This paper is concerned with transient thermoelastic problem of a thick circular plate occupying the space $D: a \le r \le b,\ -h \le z \le h,$ due to heat generation with radiation type boundary conditions.

II. STATEMENT OF THE PROBLEM

Consider thick annular disc of thickness $2h$ occupying the space $D: a \le r \le b,\ -h \le z \le h,$ the material is homogenous and isotropic. The differential equation governing the displacement potential function $\phi(r,z,t)$ as Nowacki [11] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu}\right)\alpha_t T \qquad (1)$$

Where ν and a_t are Poisson's ratio and linear coefficient of thermal expansion of the material of the plate and T is the temperature of the plate satisfying the differential equation as **Noda [12]** is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2)$$

Subject to initial condition

$$M_t(T,1,0,0) = F(r,z) \quad a \le r \le b, -h \le z \le h. \qquad (3)$$

The boundary conditions are

$$\left.\begin{aligned} M_r(T,1,k_3,a) &= g_1(z, t) \\ M_r(T,1,k_4,b) &= g_2(z, t) \end{aligned}\right\}, \quad -h \le z \le h,\ t > 0 \qquad (4)$$

$$\left.\begin{aligned} M_z(T,1,k_1,h) &= f_1(r, t) \\ M_z(T,1,k_2,-h) &= \left(\frac{-Q_0}{\lambda}\right) f_2(r, t) \end{aligned}\right\}, \quad a \le r \le b,\ t > 0 \qquad (5)$$

where k is thermal diffusivity of material of the plate.

The displacement function in the cylindrical coordinate system are represented by Love's function as **Khobragade [5]** are

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 L}{\partial r \partial z} \qquad (6)$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2} \qquad (7)$$

The Love's function **[8]** must satisfy

$$\nabla^2 \nabla^2 L = 0 \qquad (8)$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$

The component of stresses are represented by the thermoelastic displacement potential ϕ and Love's function L as **Noda [12]** are

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu \nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right)\right\} \qquad (9)$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial \phi}{\partial r} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu \nabla^2 L - \frac{1}{r}\frac{\partial^2 L}{\partial r^2}\right)\right\} \qquad (10)$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial z^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left\{\left((2-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\}\right\} \qquad (11)$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left\{\left((1-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\}\right\} \qquad (12)$$

For traction free surface stress function $\sigma_z = \sigma_{r\theta} = 0$ at $z = \pm h$ for thick annular disc.

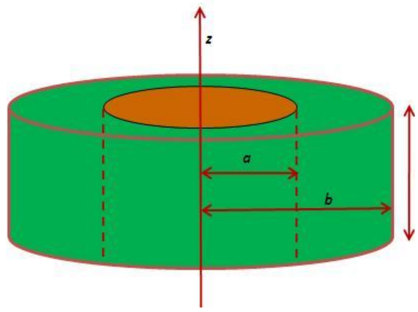

Fig. 1. The geometry of the problem

Equations (1) to (12) constitute the mathematical formulation of the problem under consideration

III. SOLUTION OF THE PROBLEM

Applying Marchi-Zgrablich transform to the equation (2) we get

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$$\frac{b}{k_4}S_0(k_3,k_4,\mu_m b)g_2-\frac{a}{k_3}S_0(k_3,k_4,\mu_m a)g_1$$

$$-\mu_m^2\overline{T}(\mu_m,z,t)+\frac{d^2\overline{T}(\mu_m,z,t)}{dz^2}+\frac{\overline{g}(\mu_m,z,t)}{k}=\frac{1}{\alpha}\frac{d\overline{T}}{dt} \quad (13)$$

Again applying Marchi-Fasulo transform to above equation, we obtain

$$\frac{b}{k_4}S_0(k_3,k_4,\mu_m b)g_2{}^*-\frac{a}{k_3}S_0(k_3,k_4,\mu_m a)g_1{}^*$$

$$-\mu_m^2\overline{T}^*(\mu_m,n,t)+\frac{P_n(h)}{k_1}\overline{f}_1-\frac{P_n(-h)}{k_2}\left(\frac{-Q_0}{\lambda}\right)\overline{f}_2$$

$$-\lambda_n^2\overline{T}^*(\mu_m,n,t)+\frac{\overline{g}^*(\mu_m,n,t)}{k}=\frac{1}{\alpha}\frac{d\overline{T}^*}{dt}$$

This can be written as

$$\frac{d\overline{T}^*}{dt}+\alpha p^2\overline{T}^*=\Pi \quad (14)$$

where

$$p^2=\mu_m^2+\lambda_n^2$$

$$\Pi=\alpha\left[\frac{b}{k_4}S_0(k_3,k_4,\mu_m b)g_2{}^*\right.$$

$$-\frac{a}{k_3}S_0(k_3,k_4,\mu_m a)g_1{}^*$$

$$\left.+\frac{P_n(h)}{k_1}\overline{f}_1+\frac{P_n(-h)}{k_2}\left(\frac{Q_0}{\lambda}\right)\overline{f}_2\right]$$

Solution of equation (14) is given by

$$\overline{T}^*=e^{-\alpha p^2 t}\left[\overline{F}^*+\int_0^t \Pi e^{\alpha p^2 t'}dt'\right] \quad (15)$$

Applying inversion of Marchi-Fasulo transform we get

$$\overline{T}(\xi_m,n,t)=\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}e^{-\alpha p^2 t}\left[\overline{F}^*+\int_0^t \Pi e^{\alpha p^2 t'}dt'\right] \quad (16)$$

Applying inversion of Marchi-Zgrablich transform to the differential equation (16), we get

$$T(r,z,t)=\sum_{m,n=1}^{\infty}\frac{S_0(k_3,k_4,\mu_m r)}{\mu_m}\frac{P_n(z)}{\lambda_n}\Omega \quad (17)$$

where

$$\Omega=e^{-\alpha p^2 t}\left[\int_0^t \Pi e^{\alpha p^2 t'}dt'+\overline{F}^*(m,n)\right]$$

Equation (17) is the desired solution of the given problem.

Let us assume Love's function L, which satisfy condition (8) as

$$L=\sum_{m,n=1}^{\infty}\frac{S_0(k_3,k_4,\mu_m r)}{\mu_m}\frac{P_n(z)}{\lambda_n} \quad (18)$$

Using (1) and (17), we get displacement potential ϕ as

$$\phi=A\sum_{m,n=1}^{\infty}\frac{S_0(k_3,k_4,\mu_m r)}{\mu_m}\frac{P_n(z)}{\lambda_n}\Omega \quad (19)$$

where $A=\left(\frac{1+\nu}{1-\nu}\right)a_t$

IV. DETERMINATION OF DISPLACEMENT FUNCTION

Substituting equations (18)and (19)in equation (6), (7) we get

$$u_r=A\sum_{m,n=1}^{\infty}\frac{S_0{}'(k_3,k_4,\mu_m r)}{\mu_m}\frac{P_n(z)}{\lambda_n}\Omega$$

$$-\sum_{m,n=1}^{\infty}\frac{S_0{}'(k_3,k_4,\mu_m r)}{\mu_m}\frac{P_n{}'(z)}{\lambda_n} \quad (20)$$

$$u_z=A\sum_{m,n=1}^{\infty}\frac{S_0(k_3,k_4,\mu_m r)}{\mu_m}\frac{P_n{}'(z)}{\lambda_n}\Omega$$

$$+2(1-\nu)\sum_{m,n=1}^{\infty}\frac{1}{\mu_m\lambda_n}\left[S_0{}''(k_3,k_4,\mu_m r)P_n(z)\right.$$

$$\left.+\frac{1}{r}S_0{}'(k_3,k_4,\mu_m r)P_n(z)+S_0P_n{}''(z)\right]$$

$$-\sum_{m,n=1}^{\infty}\frac{S_0{}'(k_3,k_4,\mu_m r)}{\mu_m}\frac{P_n{}''(z)}{\lambda_n} \quad (21)$$

V. DETERMINATION OF STRESS FUNCTIONS

Substituting equations (18)and (19)in equations (9) to (12), we obtain

$$\sigma_{rr}=2G\left\{A\sum_{m,n=1}^{\infty}\Omega\left(\frac{\mu_m S_0\,P_n(z)}{\lambda_n}-\frac{\mu_m S_0{}''P_n(z)}{\lambda_n}\right.\right.$$

$$\left.-\frac{1}{r}\frac{S_0{}'P_n(z)}{\lambda_n}-\frac{S_0\,P_n{}'(z)}{\mu_m\lambda_n}\right)$$

$$\left.+\nu\left(\frac{\mu_m S_0{}''P_n{}'(z)}{\lambda_n}+\frac{1}{r}\frac{S_0{}'P_n{}'(z)}{\lambda_n}-\frac{S_0\,P_n{}'''(z)}{\mu_m\lambda_n}\right)\frac{\mu_m S_0{}''P_n(z)}{\lambda_n}\right\} \quad (22)$$

$$\sigma_{\theta\theta}=2G\left\{\frac{1}{r}A\sum_{m,n=1}^{\infty}\frac{1}{\lambda_n}\left(S_0{}'P_n(z)-\mu_m S_0{}''P_n(z)\right.\right.$$

$$\left.\left.-\frac{1}{r}S_0{}'P_n(z)+\frac{S_0P_n(z)}{\mu_m}\right)\Omega\right.$$

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$$+\nu\left(\frac{S_0'' P_n'(z)}{\lambda_n}+\frac{1}{r}\frac{S_0' P_n'(z)}{\lambda_n}-\frac{S_0 P_n'''(z)}{\mu_m\lambda_n}\right)$$

$$\left.-\frac{1}{r}\sum_{m,n=1}^{\infty}\frac{\mu_m S_0'' P_n(z)}{\lambda_n}\right\} \qquad (23)$$

$$\sigma_{zz}=2G\left\{A\sum_{m,n=1}^{\infty}\left(\frac{S_0\, P_n'(z)}{\mu_m\lambda_n}-\frac{\mu_m S_0'' P_n(z)}{\lambda_n}\right.\right.$$

$$\left.+\frac{1}{r}\frac{S_0' P_n(z)}{\lambda_n}-\frac{S_0\, P_n'(z)}{\mu_m\lambda_n}\right)\Omega$$

$$+(2-\nu)\left(\frac{\mu_m S_0'' P_n(z)}{\lambda_n}+\frac{1}{r}\frac{S_0' P_n(z)}{\lambda_n}+\frac{S_0 P_0''(z)}{\mu_m\lambda_n}\right)$$

$$\left.-\sum_{m,n=1}^{\infty}\frac{S_0 P_n''(z)}{\mu_m\lambda_n}\right\} \qquad (24)$$

$$\sigma_{rz}=2G\left\{A\sum_{m,n=1}^{\infty}S_0'\frac{P_n'(z)}{\mu_m}\Omega+\left((1-\nu)\left(\frac{\mu_m S_0'' P_n(z)}{\lambda_n}\right.\right.\right.$$

$$\left.+\frac{1}{r}\frac{S_0' P_n(z)}{\lambda_n}-\frac{S_0\, P_n''(z)}{\mu_m\lambda_n}\right)$$

$$\left.\left.-\sum_{m,n=1}^{\infty}\frac{S_0 P_n''(z)}{\mu_m\lambda_n}\right)\right\} \qquad (25)$$

VI. SPECIAL CASE

Set $F(r,z)=z^2(1-r^2)$ (26)

Applying Marchi-Fasulo transform, are obtain

$$\overline{F}(r,n)=(1-r^2)\int_{-h}^{h}z^2P_n(z)dz$$

$$\overline{F}(r,n)=(1-r^2)\Phi_n\left[\frac{2h^2\sin(a_nh)}{a_n}+\frac{4h\cos(a_nh)}{a_n{}^2}-\frac{4\sin(a_nh)}{a_n{}^3}\right] \qquad (27)$$

Where

$$P_n(z)=Q_n\cos(a_nz)-W_n\sin(a_nz)\ ,$$

$$Q_n=a_n(\alpha_1+\alpha_2)\cos(a_nh)+(\beta_1-\beta_2)\sin(a_nh)$$

$$W_n=(\beta_1-\beta_2)\cos(a_nh)+a_n(\alpha_1-\alpha_2)\sin(a_nh)$$

Again on applying Hankel transform, we obtain

$$\overline{F}^{*}(m,n)=\Pi_n\left[\begin{array}{l}\frac{a}{\xi_m}J_1(a\xi_m)-\frac{a(a^2\xi_m^2-4)}{\xi_m^3}J_1(a\xi_m)\\ -\frac{2a^2}{\xi_m^2}J_0(a\xi_m)\end{array}\right] \qquad (28)$$

Where

$$\Pi_n=\Phi_n\left[\frac{2h^2\sin(a_nh)}{a_n}+\frac{4h\cos(a_nh)}{a_n{}^2}-\frac{4\sin(a_nh)}{a_n{}^3}\right]$$

And

$$\Phi_n=a_n(\alpha_1+\alpha_2)\cos(a_nh)+(\beta_1-\beta_2)\sin(a_nh).$$

Using equation (28) in equation (17) one obtains

$$T(r,z,t)=\frac{2}{a^2}\sum_m\sum_n\frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n(z)}{\lambda_n}e^{-kp^2t}$$

$$\times\left[\int_0^t\Pi e^{kp^2t}\,dt^1+\Pi_n\right]$$

$$\times\left(\frac{a}{\xi_m}J_1(a\xi_m)-\frac{a(a^2\xi_m^2-4)}{\xi_m{}^3}J_1(a\xi_m)-\frac{2a^2}{\xi_m^2}J_0(a\xi_m)\right) \qquad (29)$$

VII. NUMERICAL RESULTS

Set $a=2, k=15.9\times10^6, t=1$ Second in equation (29) we get

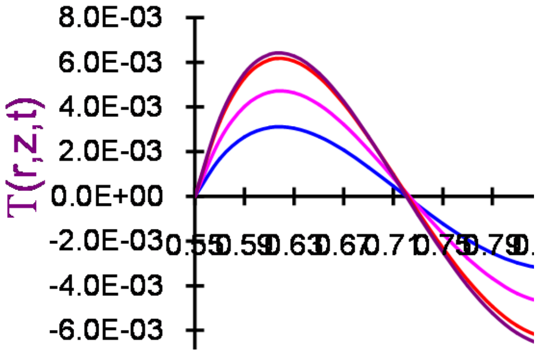
$$T(r,z,t)=(0.5)\sum_m\sum_n\frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n(z)}{\lambda_n}$$

$$e^{-(15.9\times10^6)p^2}\times\int_0^1\Pi e^{(15.9\times10^6)p^2t^1}\,dt^1$$

$$+\Pi_n\left(\frac{2}{\xi_m}J_1(2\xi_m)-\frac{2(4\xi_m^2-4)}{\xi_m{}^3}J_1(2\xi_m)-\frac{2}{\xi_m^2}J_0(2\xi_m)\right) \qquad (30)$$

VIII. CONCLUSION

In this paper, the temperature distribution, displacement and thermal stresses of a thick annular disc are investigated with known boundary conditions. Finite integral transform techniques are used to obtain numerical results. The results are obtained in terms of Bessel's function in the form of infinite series. Any particular cases of special interest can be assigned to the parameters and functions in expressions. The results that are obtained can be useful to the design of structure or machines in engineering applications.



Graph 1: Temperature distribution versus r

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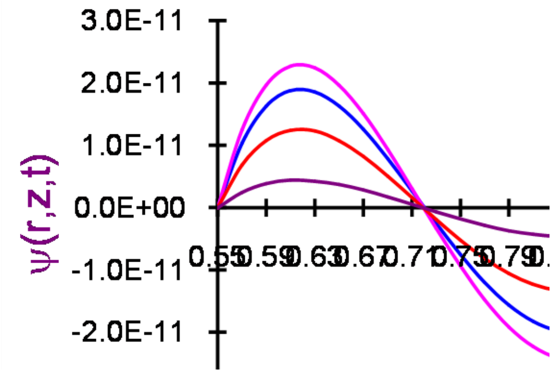
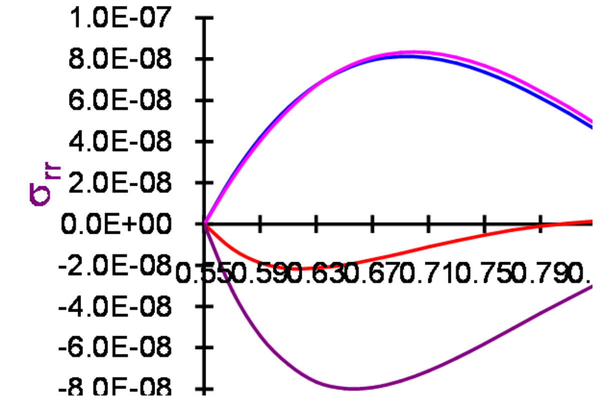
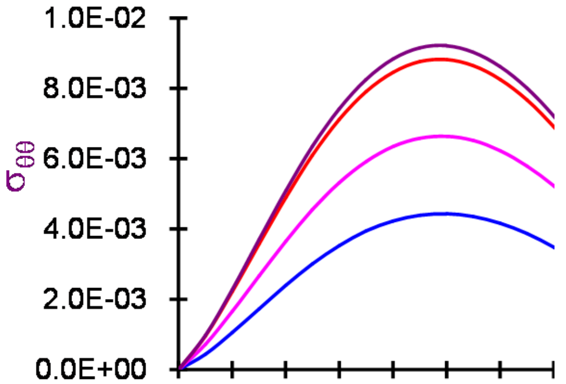


Graph 2: Displacement function versus r



Graph 3: Stress function versus r



Graph 4: Stress function versus r

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AUTHOR BIOGRAPHY

Dr. N.W. Khobragade For being M.Sc in statistics and Maths he attained Ph.D. He has been teaching since 1986 for 30 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities. At present he is working as Professor. Achieved excellent experiences in Research for 17 years in the area of Boundary value problems and its application. Published more than210 research papers in reputed journals. Seventeen students awarded Ph.D Degree and FIVE students submitted their thesis in University for award of Ph.D Degree under their guidance.