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Finite Time Interval Stabilizability of Non-Linear Perturbed Continuous Descriptor Control System

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Abstract: ***The structure uncertain non-linear descriptor systems have been considered. A controller feedback and open loop stabilization of non-linear perturbed descriptor system have been developed using Lyapunov function approach and finite time stability principle. The theoretical justifications with the necessary requirements and illustration have also been presented.***

Keywords: **Perturbation system, Descriptor systems.**

I. INTRODUCTION

Singular control systems are those systems whose dynamic is governed by a mixture of differential and algebraic equations, and named as generalized system, descriptor systems, as well as semi-state systems. These systems include many real-world applications such as feedback system and robotics, chemical systems, biological systems etc. [5].

The initial conditions of these systems should be designed based on the solvability of the algebraic equations which is not characteristic for the state space system where is no algebraic equation appeared. [5], [6], [8] some results on stabilization of some classes of descriptor system with some set of sufficient conditions are given in [9], [11], [10].

The stability (robustness) of particular class of linear systems in the time domain using the Lyapunov approach is given in [13]. Decomposition of unstructured impulse free perturbations was studied in [13].For more details on robustness and Stablizability of singular (descriptor) system may be found [1],[2],[3] [7],[16]

The finite time interval stablizability of linear continuous descriptor control system with illustration and algorithms have been given in [4]

In this work based on the previous results and literature, for the non-linear perturbed descriptor systems, some results have been developed.

II. SOME BASIC CONCEPT

Lemma (2.1) [14]

Let A be a real, symmetric positive- definite matrix. and let $\lambda_{min}(A)$ and $\lambda_{max}(A)$be the smallest and largest eigen values of A. Respectively Then, for any $x \epsilon R^n$,

$$\lambda_{min}(A)\|x\|^2 \leq x^T Ax \leq \lambda_{max}(A)\|x\|^2 \qquad (1)$$

Remark (2.1):

1. f$EA = AE$ and $\aleph(E) \cap \aleph(A) = \{0\}$. Let$k = ind(E)$. If f is a k-times continuously differentiable vector valued function, then $E\dot{x} + Ax = f$ is consistent and a general solution is given by

$$X = e^{-E^D At} EE^D q + E^D e^{-E^D At} \int_a^t e^{E^D As} f(s) ds + \quad (I - EE^D) \sum_{n=0}^{k-1} (-1)^n (EA^D)^n A^D f^{(n)}(t)$$

Where q.is an arbitrary constant vector and the consistent initial condition is given by

$$x_0 = EE^D q + (I - EE^D) \sum_{n=0}^{k-1} (-1)^n (EA^D)^n A^D f^{(n)}(0)$$

For some vector q. Furthermore, the solution of consistent initial condition is unique, see [5]

2. Consider the linear singular system
$E\dot{x}(t) = Ax(t) + Bu(t)$for simplicity,$(E, A)$ is standing for this system (2)

If the following conditions hold:

(1) The matrix pair (E, A) is regular and

(2) The matrix pair (E, A) is an impulse free and stable. Then for each $Q > 0$ there exist $P > 0$ is a solution of generalized Lyapunov Equation satisfying $(A^T PE) + (E^T PA) = -E^T QE$, see [15]

III. DESCRIPTION OF THE PROBLEM I

Consider the singular non-linear control system with structure uncertainty and perturbation

$$E\dot{x}(t) = (A + \delta A)x(t) + (B + \delta B)u(t) + g(x) \qquad (3)$$

Where

$x(t) \in R^n$is state vector, $E, A \epsilon R^{nxn}, B \epsilon R^{nxm}$, $g(x): R^n \to R^n$with$\|g(x)\| \leq M\|x\|$and $u(t)$ is control input, the singular matrix E has $ind(E) = k < n$,and $\delta A, \delta B$ are parametric uncertainties matrices such that $\|\delta A\| \leq a$ and$\|\delta B\| \leq b$, for some positive constants a, b and $ind(E) = k$. The solvability of this system is assumed to be satisfied. The nominal system of (3) can be defined as the following system:-

$$Ex'(t) = Ax(t) + Bu(t) \qquad (4)$$

With $x_0 \in W_k$ for some class of consistent initial condition that will be designed later on.

the only information we have about $g(x)$ is the upper bound ;$\|g(x)\| \leq M\|x\|$, the M should be designed to determine the suitable class of uncertainty ,the solvability of this system is assume satisfying

IV. DEFINITION AND THEOREM

Based on [5], [4], the following definition is developed

Definition (4.1)

The linear non-linear perturbation singular system of the form

$$E\dot{x} = (A + \delta A)x(t) + (\dot{B} + \delta B)u(t) + g(x), x(to) = xo \in W_k$$

(Some class of consistence initial condition), Where $E, A \epsilon R^{n \times n}, g(x): R^n \to R^n, x(t) \epsilon R^n, u(t) \epsilon R^m$and $\|\delta A\| \leq a$,

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$\|Bu(t)\| \le \epsilon$ and $\|\delta Bu(t)\| \le \epsilon_1$ is called finite time stable w.r.t $\{\alpha, \beta, \epsilon, \epsilon_1, a, J, P, V(x)\}$ iff $\forall x(t_0) = x_0 \in W_k$ with $\|x_0\|^2_{E^TPE} < \alpha$ Implies that $\|x(t)\|^2_{E^TPE} < \beta, \alpha < \beta, \forall t \epsilon J = \{t: t_0 \le t \le t_0 + T\}$.

Remark (2.2):

Consider the system

$E\dot{x} = (A + \delta A)x(t) + (B + \delta B)u(t) + g(x)$ (5)

Such that the nominal system (E, A) is regular then there exist H nonsingular matrices such that

$1-\ X = HW = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ where $w_1 \in R^k, w_2 \in R^{n-k}$

$2-\ H^{-1}EH = diag(E_1, 0), H^{-1}AH = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, H^{-1}B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$

,with $|E_1| \ne 0$

$H^{-1}\delta AH = \begin{bmatrix} \delta A_1 & \delta A_2 \\ \delta A_3 & \delta A_4 \end{bmatrix}, H^{-1}\delta B = \begin{bmatrix} \delta B_1 \\ \delta B_2 \end{bmatrix}$ With appropriate dimension

$3-\ H^{-1}g(X(t)) = H^{-1}g(TW) = H^{-1}g(w_1, w_2) = \begin{bmatrix} g_1(w_1, w_2) \\ g_2(w_1, w_2) \end{bmatrix}$

Then the system (5) is transformed in to

$$E_1\dot{w}_1 = (A_1 + \delta A_1)w_1 + (A_2 + \delta A_2)w_2 + (B_1 + \delta B_1)u + g_1(w_1, w_2) \quad (6)$$

$$0 = (A_3 + \delta A_3)w_1 + (A_4 + \delta A_4)w_2 + (B_2 + \delta B_2)u + g_2(w_1, w_2) \quad (7)$$

The proof is direct:-

Since the nominal system is regular then there exist H nonsingular matrix such that

$W = H^{-1}X = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ Where $w_1 \in R^k, w_2 \in R^{n-k}$ and (5) hence

$$H^{-1}EH\dot{w} = H^{-1}(A + \delta A)HW + H^{-1}(B + \delta B)u(t) + H^{-1}g(w_1, w_2)$$

$$\begin{bmatrix} E_1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \end{bmatrix} = \begin{bmatrix} A_1 + \delta A_1 & A_2 + \delta A_2 \\ A_3 + \delta A_3 & A_4 + \delta A_4 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} B_1 + \delta B_1 \\ B_2 + \delta B_2 \end{bmatrix}u + \begin{bmatrix} g_1(w_1, w_2) \\ g_2(w_1, w_2) \end{bmatrix}$$

$$E_1\dot{w}_1 = (A_1 + \delta A_1)w_1 + (A_2 + \delta A_2)w_2 + (B_1 + \delta B_1)u + g_1(w_1, w_2) \quad (8)$$

$$0 = (A_3 + \delta A_3)w_1 + (A_4 + \delta A_4)w_2 + (B_2 + \delta B_2)u + g_2(w_1, w_2) \quad (9)$$

Theorem (4.1)

Consider the system

$E\dot{x}(t) = (A + \delta A)x(t) + (B + \delta B)u(t) + g(x)$ (10)

Where $x(t) \in R^n$, $u(t) \in R^m$ and $E, A \epsilon R^{nxn}$, $B \epsilon R^{nxm}$, $g(x): R^n \to R^n$ are constant matrix with $|E| = 0$ and $ind(E) = k < n$

Then if

$1-\ (E, A)$ the nominal is regular and solvable for $u(t) \subset [0, J]^{ind(E)-1}$, $J \triangleq \{t: t_0 \le t \le t_0 + T\}$ (11)

$2-\ x_0 \in W_k$ is selected such that the solution is unique(consistent initial conditions) (12)

$3-\ \delta A, \delta B$ are parametric uncertainties matrices such that $\|\delta A\| \le a$ and $\|Bu(t)\| < \epsilon, \|\delta Bu(t)\| \le \epsilon_1$, for some positive constants a, ϵ, ϵ_1 (13)

4- The candidate Lyapunov function is defined to be $V(x, t) = x^TE^TPEx$ where $E^TPE > 0$ and the matrix $P = P^T$ is the unique solution of $(A^TPE) + (E^TPA) = -E^TQE$, for a given matrix Q such that $E^TQE > 0$ (14)

$5-\ \|x_0\|^2_{E^TPE} < \alpha, \forall x_0 \in W_k$ (15)

$6-\ \|g(x)\| \le M\|x\|$ Such that $M > 0, M$ to be speufied such that the following is valid (16)

$7-\ S = \left\{x \epsilon R^n \,|\, \|x\| \ge \frac{\epsilon N + \epsilon_1 N}{(-(E^TQE) + aN + MN)\theta}\right\}$ (17)

$8-\ \ln\frac{\beta}{\alpha} > \left(\left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)}t + \frac{aN}{\lambda_{\min}(E^TPE)}t + \frac{MN}{\lambda_{\min}(E^TPE)}t\right)(1-\theta)\right), 0 < \theta < 1$

and $\|x\| \ge \frac{\epsilon N + \epsilon_1 N}{(-(E^TQE) + aN + MN)\theta}$ where $N = \|PE\| + \|E^TP\|$ (18)

Then $V(x(t)) < \beta, \alpha < \beta$ and the system is finite time stable w.r.t. $\{\alpha, \beta, \epsilon, \epsilon_1, \theta, J, M, P, V(x), S, a\}$

Proof:-

Since the Lyapunov function

$V(x(t)) \triangleq x^T(t)(E^TPE)x(t)$ Where $E^TPE > 0$

$\frac{d}{dt}V(x(t)) = \dot{V}(x(t))\Big|_{\text{along the solution of system}} \frac{dV}{dx}\cdot\frac{dx}{dt}$

$\dot{V}(x(t)) = \dot{x}^T(t)E^TPEx(t) + x^T(t)E^TPE\dot{x}(t)($ (19)

$$\dot{V}(x(t)) = ((A + \delta A)x + (B + \delta B)u + g(x))^TPE + x^TE^TP((A + \delta A)x + (B + \delta B)u + g(x))$$

$$\Rightarrow \dot{V}(x(t)) = x^T[A^TPE + E^TPA]x + u^TB^TPEx + x^TE^TPBu + (\delta Bu)^TPEx + x^TE^TP\delta Bu + x^T\delta A^TPEx + x^TE^TP\delta Ax + g(x)^TPEx + x^TE^TPg(x) \quad (20)$$

And from condition (13) one can get

$$\Rightarrow \dot{V}(x(t)) \triangleq -x^T(E^TQE)x + u^TB^TPEx + x^TE^TPBu + (\delta Bu)^TPEx + x^TE^TP\delta Bu + x^T\delta A^TPEx + x^TE^TP\delta Ax + g(x)^TPEx + x^TE^TPg(x) \quad (21)$$

$$\dot{V}(x(t)) \le -(E^TQE)\|x\|^2 + \|u^TB^TPEx\| + \|x^TE^TPBu\| + \|(\delta Bu)^TPEx\| + \|x^TE^TP\delta Bu\| + \|x^T\delta A^TPEx\| + \|x^TE^TP\delta Ax\| + \|g(x)^TPEx\| + \|x^TE^TPg(x)\|$$

$$\dot{V}(x(t)) \le (-(E^TQE) + a\|PE\| + \|E^TP\| + M\|PE\| + \|E^TP\|)\|x\|^2 + (\epsilon\|PE\| + \|E^TP\| + (\epsilon_1\|PE\| + \|E^TP\|)\|x\|$$

, where $N = \|PE\| + \|E^TP\|$ (22)

Hence by adding and subtracting some terms, the following is obtained

$$\dot{V}(x(t)) \le (-(E^TQE) + aN + MN)(1-\theta)\|x\|^2 - (-E^TQE + aN + MN)\theta\|x\|^2 + (\epsilon N + \epsilon_1 N)\|x\| \ \forall\ 0 < \theta < 1 \quad (23)$$

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$\Leftrightarrow \dot{V}(x(t)) \leq (-(E^TQE) + aN + MN)(1-\theta)\|x\|^2, \forall\|x\| \geq \frac{\epsilon N+\epsilon_1 N}{(-E^TQE+aN+MN)\theta}$

(24)

And from Lemma (see 2.1) and (22) one get

$$\dot{V}(x(t)) \leq \left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)} + \frac{aN}{\lambda_{\min}(E^TPE)} + \frac{MN}{\lambda_{\min}(E^TPE)}\right)(1-\theta)V(x(t)), 0 < \theta < 1$$

And the monotonicity of integration $\forall\, t\epsilon[0, J]$

$$\int_0^t \frac{\dot{V}(x(t))}{V(x(t))}dt \leq \left(\int_0^t \left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)} + \frac{aN}{\lambda_{\min}(E^TPE)} + \frac{MN}{\lambda_{\min}(E^TPE)}\right)(1-\theta)dt\right)$$

$$\Leftrightarrow \ln\frac{V(x(t))}{V(x(0))} \leq \left(\left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)}t + \frac{aN}{\lambda_{\min}(E^TPE)}t + \frac{MN}{\lambda_{\min}(E^TPE)}t\right)(1-\theta)\right)$$

$$V(x(t)) \leq \left[e^{\left(\left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)}t + \frac{aN}{\lambda_{\min}(E^TPE)}t + \frac{MN}{\lambda_{\min}(E^TPE)}t\right)(1-\theta)\right)}\right]V(x(0)) \;, 0 < \theta < 1, t\epsilon J$$

(25)

From (15) we have that

$$V(x(t)) \triangleq \|x\|^2_{E^TPE} \leq \left[e^{\left(\left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)}t + \frac{aN}{\lambda_{\min}(E^TPE)}t + \frac{MN}{\lambda_{\min}(E^TPE)}t\right)(1-\theta)\right)}\right]\|x_0\|^2_{E^TPE}, x_0 \in W_k$$

$$\|x(t)\|^2_{E^TPE} \leq \left[e^{\left(\left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)}t + \frac{aN}{\lambda_{\min}(E^TPE)}t + \frac{MN}{\lambda_{\min}(E^TPE)}t\right)(1-\theta)\right)}\right]\alpha$$

(26)

On setting M and α , β such that

$$\left[e^{\left(\left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)}t + \frac{aN}{\lambda_{\min}(E^TPE)}t + \frac{MN}{\lambda_{\min}(E^TPE)}t\right)(1-\theta)\right)}\right], \alpha < \beta$$

$$\ln\frac{\beta}{\alpha} > \left(\left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)}t + \frac{aN}{\lambda_{\min}(E^TPE)}t + \frac{MN}{\lambda_{\min}(E^TPE)}t\right)(1-\theta)\right), \alpha < \beta$$

(27)

Then we have $\|x(t)\|^2{}_{E^TPE} \leq \beta$ for all $\|x_0\|^2{}_{E^TPE} < \alpha$, and hence the system is finite time stable w.r.t $\{\alpha, \beta, \epsilon, \epsilon_1, \theta, M, P, V(x), J, a, S\}$, $\alpha < \beta, t \in J, 0 < \theta < 1$ and the class of uncertain $\{g(x): R^n \to R^n | \|g(x)\| \leq M$, where M is selected such that (27) is satisfied for $t \in J, 0 < \theta < 1\}$.

Feedback Action

Definition (4.2)

The linear non-linear perturbation singular system of the form

$E\dot{x} = (A + \delta A)x(t) + (B + \delta B)u(t) + g(x), x(to) = xo \in W_k$

(Some class of consistence initial condition), Where $E, A\epsilon R^{n\times n}, g(x): R^n \to R^n, x(t)\epsilon R^n, u(t)\epsilon R^m$ and $\|\delta A\| \leq a$, $\|Bu(t)\| \leq K\|X\|$ and $\|\delta Bu(t)\| \leq \delta K\|X\|$ is called finite time stable w.r.t $\{\alpha, \beta, K, \delta K, a, J, P, V(x)\}$ iff $\forall x(t_0) = x_0 \in W_k$ with $\|x_0\|^2{}_{E^TPE} < \alpha$ Implies that $\|x(t)\|^2{}_{E^TPE} < \beta$, $\alpha < \beta, \forall t\epsilon J = \{t: t_0 \leq t \leq t_0 + T\}$

Corollary

Consider the system

$E\dot{x}(t) = (A + \delta A)x(t) + (B + \delta B)u(t) + g(x)$

Where $x(t) \in R^n$, $u(t) \in R^m$ and $E, A\epsilon R^{nxn}$, $B\epsilon\epsilon R^{nxm}, g(x): R^n \to R^n$ are constant matrix with $|E| = 0$ and $ind(E) = k < n$

Then if

1- $\delta A, \delta B$ are parametric uncertainties matrices such that $\|\delta A\| \leq a$ and $\|Bu\| < K\|X\|$ $\|\delta Bu\| \leq \delta K\|X\|$, for some positive constants (28)

2- The candidate Lyapunov function is defined to be $V(x,t) = x^TE^TPEx$ where $E^TPE > 0$ and the matrix $P = P^T$ is the unique solution of $(A^TPE) + (E^TPA) = -E^TQE$, for a given matrix Q such that $E^TQE > 0$ (29)

3 – $\|x_0\|^2{}_{E^TPE} < \alpha, \alpha < \beta$ (30)

4 – $\|g(x)\| \leq M\|x\|$ Such that $M > 0$ (31)

5 – $\ln\frac{\beta}{\alpha} > \frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)}t + \frac{KN}{\lambda_{\min}(E^TPE)}t + \frac{\delta KN}{\lambda_{\min}(E^TPE)}t + \frac{aN}{\lambda_{\min}(E^TPE)}t + \frac{MN}{\lambda_{\min}(E^TPE)}t$

2. Where $N = \|PE\| + \|E^TP\|$ (32)

Then $V(x(t)) < \beta$ and the system is finite time stable w.r.t. $\{\alpha, \beta, K, \Delta k, M, P, V(x), a\}$, $\alpha < \beta$

Proof:-

From define Lyapunov function

$V(x(t)) \triangleq x^T(t)(E^TPE)x(t)$ Where $E^TPE > 0$

$\frac{d}{dt}V(x(t)) = \dot{V}(x(t))|_{\text{along the solution of system}} \frac{dx}{dt}, \frac{dx}{dt}$ (33)

$\dot{V}(x(t)) = ((A+\delta A)x + (B+\delta B)u + g(x))^T PE + x^TE^TP((A+\delta A)x + (B+\delta B)u + g(x))$

And from condition (17) one can get

$\Rightarrow \dot{V}(x(t)) \triangleq -x^T(E^TQE)x + u^TB^TPEx + x^TE^TPBu + (\delta Bu)^TPEx + x^TE^TP\delta Bu + x^T\delta A^TPEx + x^TE^TP\delta Ax + g(x)^TPEx + x^TE^TPg(x)$

(34)

$\dot{V}(x(t)) \leq -(-x^T(E^TQE)x + KN + \delta KN + aN + MN)\|x\|^2$

Where $N = \|PE\| + \|E^TP\|$

And from Lemma (see 2.1) and similarly theorem (4.1) one get and hence

$$\dot{V}(x(t)) \leq \left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)} + \frac{KN}{\lambda_{\min}(E^TPE)} + \frac{\delta KN}{\lambda_{\min}(E^TPE)} + \frac{aN}{\lambda_{\min}(E^TPE)} + \frac{MN}{\lambda_{\min}(E^TPE)}\right)V(x(t))$$

And the monotonicity of integration $\forall\, t\epsilon[0, J]$

$$\int_0^t \frac{\dot{V}(x(t))}{V(x(t))}dt \leq \int_0^t \left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)} + \frac{KN}{\lambda_{\min}(E^TPE)} + \frac{\delta KN}{\lambda_{\min}(E^TPE)} + \frac{aN}{\lambda_{\min}(E^TPE)} + \frac{MN}{\lambda_{\min}(E^TPE)}\right)dt$$

$$V(x(t)) \leq \left[e^{\left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)}t + \frac{KN}{\lambda_{\min}(E^TPE)}t + \frac{\delta KN}{\lambda_{\min}(E^TPE)}t + \frac{aN}{\lambda_{\min}(E^TPE)}t + \frac{MN}{\lambda_{\min}(E^TPE)}t\right)}\right]V(x(0))$$

From (14) we have that

$$\|x\|^2_{E^TPE} \leq \left(\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)}t + \frac{KN}{\lambda_{\min}(E^TPE)}t + \frac{\delta KN}{\lambda_{\min}(E^TPE)}t + \frac{aN}{\lambda_{\min}(E^TPE)}t + \frac{MN}{\lambda_{\min}(E^TPE)}t\right)\|x_0\|^2_{E^TPE}$$

$$\|x(t)\|^2_{E^TPE} \leq \left[e^{\frac{-\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)}t + \frac{KN}{\lambda_{\min}(E^TPE)}t + \frac{\delta KN}{\lambda_{\min}(E^TPE)}t + \frac{aN}{\lambda_{\min}(E^TPE)}t + \frac{MN}{\lambda_{\min}(E^TPE)}t}\right]\alpha$$

(36)

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On setting M and α, β such that

$$\left[e^{\frac{-\lambda_{\min}(E^T QE)}{\lambda_{\max}(E^T PE)}t+\frac{KN}{\lambda_{\min}(E^T PE)}t+\frac{\delta KN}{\lambda_{\min}(E^T PE)}t+\frac{aN}{\lambda_{\min}(E^T PE)}t+\frac{MN}{\lambda_{\min}(E^T PE)}t}\right]\alpha < \beta$$

$$\ln\frac{\beta}{\alpha} > \frac{-\lambda_{\min}(E^T QE)}{\lambda_{\max}(E^T PE)}t + \frac{KN}{\lambda_{\min}(E^T PE)}t + \frac{\delta KN}{\lambda_{\min}(E^T PE)}t + \frac{aN}{\lambda_{\min}(E^T PE)}t + \frac{MN}{\lambda_{\min}(E^T PE)}t$$

(37)

Then we have $\|x(t)\|^2{}_{E^T PE} \le \beta$ for all $\|x_0\|^2{}_{E^T PE} < \alpha$, and hence the system is finite time stable w.r.t $\{\alpha, \beta, K, \delta K, M, P, V(x), a\}$, $\alpha < \beta$

Illustrations

Consider the singular linear control system:

$$E\dot{x}(t) = (A + \delta A)x(t) + (B + \delta B)u(t) + g(x)$$

Where

$$E = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 2 \end{bmatrix}, \delta A = \begin{bmatrix} 0.1 & 0 & 0 \\ -0.2 & 0 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

$$\delta B = \begin{bmatrix} 0.1 \\ 0 \\ 0.2 \end{bmatrix}, u = cost, g(x) = \begin{bmatrix} e^{-2t} \\ e^{-3t} \\ 0 \end{bmatrix}$$

STEP (1)

$|E| = 0$; $rank(E) = 2$ and $rank(E^2) = 2$ so $ind(E) = 2$

STEP (2)

Compute the $\delta A, \delta B$ are parametric uncertainties matrices such That $\|\delta A\| \le a$ and $\|\delta B\| \le b$, $\|Bu(t)\| < \epsilon$ for some positive Constants a, b

$$\delta A = \begin{bmatrix} 0.1 & 0 & 0 \\ -0.2 & 0 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \delta Bu = \begin{bmatrix} 0.1 \\ 0 \\ 0.2 \end{bmatrix} cost, Bu = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} cost$$

The following norms of matrices $\delta A, \delta B$ are as follows:

$$\|\delta A\| = \max_j \textstyle\sum_i |a_{ij}| = 0.1 < 1,$$

and $\|\delta Bu\| = \max_j \sum_i |a_{ij}| = 0.3 < 2$

$$\|Bu\| = \max_j \textstyle\sum_i |Bu_{ij}| = 1 < 2$$

STEP (3)

Since $EA = AE$ and $\aleph(E) \cap \aleph(A) = \{0\}$ then there exist T nonsingular matrices such that

$X = TW = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ Where $w_1 \in R^k, w_2 \in R^{n-k}$ and

$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ hence

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\dot{w}_1 = \left(\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ -0.2 & 0 \end{bmatrix}\right)w_1 + \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}\right)cost + \left[e\right.$$

$$0 = (2 + 0.1)w_2 + (0 + 0.2)cost$$

Step (4)

Calculate the consistent initial condition

$$w_2 = -(A_4 + \delta A_4)^{-1}\sum_{i=0}^{k-1} N^i(B_2 + \delta B_2)u^i$$

From equation (2.59) we get

$w_2 = 0.4762 * (-0.2)cost \Rightarrow w_2(0) = -0.09524, r = 1, s = -1$

$W_k = \{(w_1(0), w_2(0), w_3(0)) | w_2(0) = -0.09524,$

$W_k = \{x_0(t) \in R^n | (w_1(0), w_2(0), w_3(0) = (1, -0.09524, -1)\}$

Step (5)

Solve algebraic Ricatti equation we

$(A^T PE) + (E^T PA) = E^T QE$ For a given matrix Q s.t $E^T QE \ge 0$

$$Q = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 4 \\ -3 & 4 & 1 \end{bmatrix}, E^T QE = \begin{bmatrix} 8 & -2 & -8 \\ -2 & 3 & 2 \\ -8 & 2 & 8 \end{bmatrix} \text{And}$$

$$eig(E^T QE) = \begin{bmatrix} 2.3465 \\ 2.4113 \\ 16.5887 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 2 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} +$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}\begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -2 & -8 \\ -2 & 3 & 2 \\ -8 & 2 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \text{and}$$

$$eig(R) = \begin{bmatrix} 0.5858 \\ 2 \\ 3.4142 \end{bmatrix}$$

$$E^T PE = \begin{bmatrix} 4 & 0 & -4 \\ 0 & 2 & 0 \\ -4 & 0 & 4 \end{bmatrix}, eig(E^T PE) = \begin{bmatrix} 0 \\ 2 \\ 8 \end{bmatrix}$$

Step (6):

Find α such that $\|x_0\|^2_{E^T PE} = x_0{}^T \hat{E}^T P \hat{E} x_0 \triangleq V(x_0(t)) < \alpha$, $\forall x_0(t) = x_0 \epsilon W_k$

$\|x_0\|^2_{E^T PE} = 16.0181 < \alpha$ Choose $\alpha = 17$

Step (7)

Design the parameters α, β are constant such that if

$$\ln\frac{\beta}{\alpha} > \left(\left(\frac{-\lambda_{\min}(E^T QE)}{\lambda_{\max}(E^T PE)}t + \text{aNt} + \text{MNt}\right)(1-\theta)\right), \alpha < \beta,$$

Where

$N = (\|PE\| + \|E^T P\|) = 32$ and $\lambda_{max}(E^T PE) = 8$ and $\lambda_{min}(E^T QE) = 2.3465$ and Choose $\alpha = 17$ and $\theta = 0.5$

$M = 2, a = 1$

t_0	$t_0 \le t \le t_0 + T$	$e^{\left(\frac{-\lambda_{\min}(E^T QE)}{\lambda_{\max}(E^T PE)}t + \text{aNt} + \text{MNt}\right)(1-\theta)}$	β
0	$0 \le t \le 5$	17	25
0.2	$0.2 \le t \le 5.2$	24.3746	25
0.4	$0.4 \le t \le 5.4$	34.9486	25
0.6	$0.6 \le t \le 5.6$	50.1109	25
0.8	$1.6 \le t \le 6.6$	3.9395	25
1	$2 \le t \le 7$	3.712	25

The value of T is then selected to be $T = 5$ for $, t \epsilon J \triangleq \{t : 0 \le t \le 9\}$

Step (8)

Define $V(x(t)) = x^T(t)E^T PEx(t) \triangleq \|x(t)\|^2_{E^T PE}$ where $E^T PE > 0$ s.t

$\|x\|^2_{E^T PE} = V(x(t)) < \beta$

$$V(x(t)) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) \end{bmatrix}\begin{bmatrix} 4 & 0 & -4 \\ 0 & 2 & 0 \\ -4 & 0 & 4 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

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$$V(x(t)) = x_1(t)(4x_1(t) - 4x_3(t)) + x_2(t)(2x_2(t)) + x_3(t)(-4x_1(t) + 4x_3(t))$$

$$\Rightarrow V(x(t)) = 4x_1^2(t) - 4x_1(t)x_3(t) + 2\,x_2^2(t) - 4x_3(t)x_1(t) + 4\,x_3^2(t)$$

Hence

$$V\|x_0\|^2_{E^T PE} = 16.0181 < 17 \,\forall x_o(t) = x_0 \epsilon W_k$$

V. CONCLUSION

Based on the previses results a finite time stablizability of forced control system is guaranteed interval stabilizability of non-linear perturbed continuous

A computational algorithm have been developed for computing the perturbations gain matrix and Lyapunov function for this propose with illustration

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