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Application of First Order differential Equations to Heat Transfer Analysis in solid

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Abstract— **Differential equations are fundamental importance in engineering mathematics because any physical laws and relations appear mathematically in the form of such equations. In this paper we discussed about first order linear homogeneous equations, first order linear non homogeneous equations and the application of first order differential equation to heat transfer analysis particularly in heat conduction in solids.**

Index Terms — **Differential Equations, Heat Transfer Analysis, Heat conduction in solid, Radiation of heat in space**

I. INTRODUCTION

In "real-world," there are many physical quantities that can be represented by functions. Involving only one of the four variables e.g., (x, y, z, t).

Equations involving highest order derivatives of order one = 1st order differential equations.

Examples:

Function σ(x) = the stress in a uni – axial stretched tapered metal rod (Or)

Function v(x) = the velocity of fluid flowing a straight channel with varying cross-section

SOLUTION METHOD OF FIRST ORDER ODES

Solution of Linear (Homogeneous equation)

Typical form of the equation:

$$\frac{du(x)}{dx}+p(x)u(x)=0 \qquad -------- (1)$$

The solution $u(x)$) in Equation (1) is

$$u(x)=\frac{K}{F(x)} \qquad -------- (2)$$

Where K = constant to be determined by given condition and the function $F(x)$ has the form:

$$F(x)=e^{\int p(x)dx} \qquad -------- (3)$$

In which the function $p(x)$ is given in the differential equation in Equation

Solution of linear (Non-homogeneous equations)

Typical form of the differential equation:

$$\frac{du(x)}{dx}+p(x)u(x)=g(x) \qquad -------- (4)$$

The appearance of function $g(x)$ in Equation (4) makes the DE Non-homogeneous.

The solution of ODE in Equation (4) is similar by a little more complex than that for the homogeneous equation in (1):

$$u(x)=\frac{1}{F(x)}\int F(x)g(x)dx+\frac{K}{F(x)} \qquad ---- (5)$$

Where function $F(x)$ can be obtained from Equation (3) as: $F(x)=e^{\int p(x)dx}$

Example

Solve the following differential equation $\frac{du(x)}{dx}-(\sin x)u(x)=0$ → (a), with condition $u(0)=2$

Solution:

By comparing terms in Equation (a) and (4), we have:

$$p(x)=-\sin x \;\&\; g(x)=0$$

Thus by using Equation (5), we have the solution

$$u(x)=\frac{K}{F(x)}$$

Where the function F(x) is: $F(x)=e^{\int p(x)dx}=e^{\cos x}$ leading to the solution

$$u(x)=Ke^{-\cos x}$$

Since the given condition is u (0) = 2, we have:

$$2=Ke^{-\cos(0)}=K\left(e^{-1}\right)=\frac{K}{e}=\frac{K}{2.7183}$$

(Or) K = 5.4366.

Hence the solution of Equation (a) is

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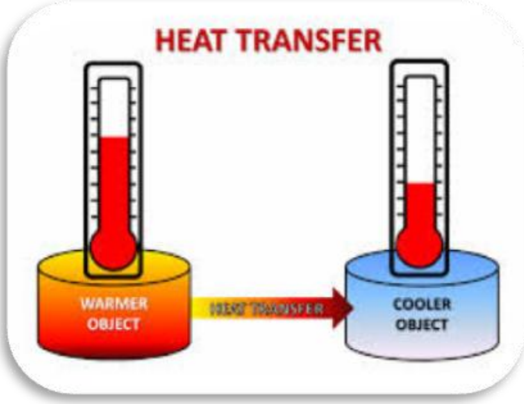
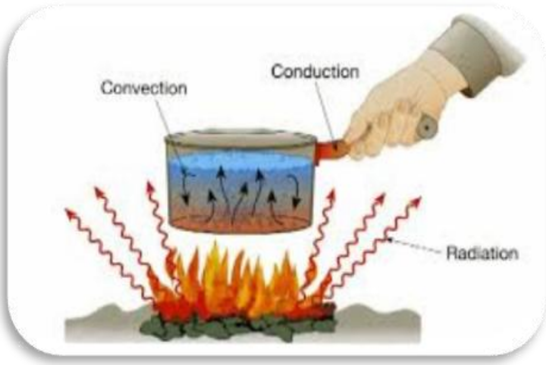
$$u(x) = 5.4366e^{-\cos x}$$

II. APPLICATION OF FIRST ORDER DIFFERENTIAL EQUATIONS TO HEAT TRANSFER ANALYSIS (HEAT CONDUCTION IN SOLID)

Heat transfer analysis

Heat transfer describes the exchange of thermal energy, between physical systems depending on the temperature and pressure, by dissipating heat. The fundamental modes of heat transfer are conduction or diffusion, convection and radiation.





The Three Modes of Heat Transmission:

- Heat conduction in solids
- Heat convection in fluids
- Radiation of heat in space

Fourier Law for Heat Conduction in Solids:

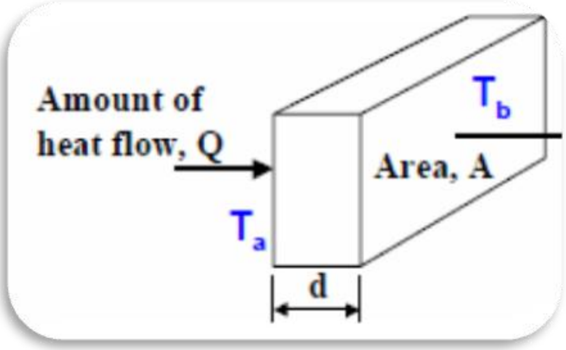
Heat flows in SOLIDS by conduction. Heat flows from the part of solid at high temperature to the part of low temperature- a situation similar to water flow from higher elevation to low elevation. Thus, there is definite relationship between heat flow (Q) and the temperature difference (ΔT) in the solid. Relating the Q and ΔT is what the Fourier law of heat conduction is all about.

Derivation of Fourier Law of Heat Conduction (A solid slab)

With the left surface maintained at temperature T_a and the right surface at T_b
Heat will flow from the left to the right surface if $T_a > T_b$

By observations, we can formulate the total amount of heat flow (**Q**). Through the thickness of the slab as:

$$Q \infty \frac{A(T_a - T_b)t}{d}$$



Q

Where A = the area to which heat flows; t = time allowing heat flow; and d = the distance of heat flow. Replacing the ∝sign in the above expression by an = sign and a constant k, leads to:

$$Q = k\frac{A(T_a - T_b)t}{d} \quad ----- \ (6)$$

The constant k in Equation (6) is "thermal conductivity" – treated as a property of the solid material with aUnit : Btu/in-s-℉ of W/m-℃

The amount of total heat flow in a solid as expressed in Equation (6) is useful, but make less engineering sense without specifying the area A and time t in the heat transfer process.

Consequently, the "Heat flux" (q) – a sense of the intensity of heat conduction is used more frequently in engineering analyses. From Equation (6), we may define the heat flux as:

$$q = \frac{Q}{At} = k\frac{(T_a - T_b)}{d} \quad ----- \ (7)$$

with a unit of: Btu/in2-s, or W/cm2

We realize Equation (7) is derived from a situation of heat flow through a thickness of a slab with distinct temperatures at both surfaces.

In a situation the temperature variation in the solid is CONTINUOUS, by function **T(x),** as illustrated below:

By following the expression in Equation (6), we will have:

$$q = k\frac{T(x) - T(x+\Delta x)}{\Delta x} = -k\frac{T(x+\Delta x) - T(x)}{\Delta x} \quad ----- \ (8)$$

If function T(x) is a CONTINUOUS varying function w.r.t variable x, (meaning Δx→0), we will have the following from Equation (8):

$$q(x) = \lim_{\Delta x \to 0}\left[-k\frac{T(x+\Delta x) - T(x)}{\Delta x}\right] = -k\frac{dT(X)}{dx} \quad ----- \ (9)$$

Equation (9) is the mathematical expression of Fourier Law of Heat Conduction in the x-direction.

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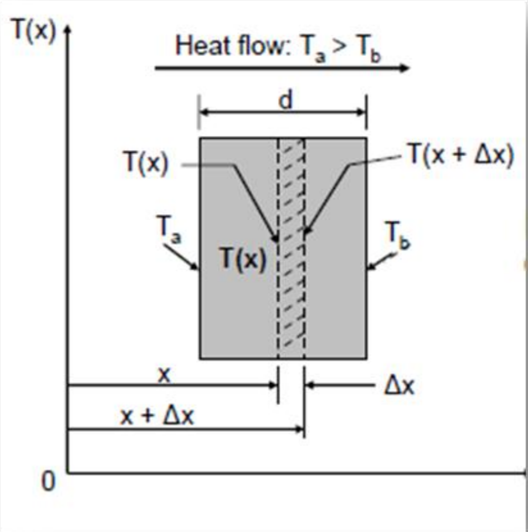
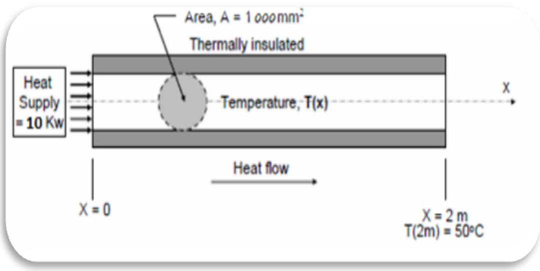


Example:

A metal rod has a cross-sectional area 1000 mm^2 and 2m in length. It is thermally insulated in its circumference, with one end being in contact with a heat source supplying heat at 10 kW, and the other end maintained at 50°C. Determine the temperature distribution in the rod, if the thermal conductivity of the rod material is k = 100 kW/m-°C.



Solution:

The total heat flow Q per unit time t (Q/t) in the rod is given by the heat source to the left end, i.e. 10 kW.
Because heat flux is q = Q/(At) as shown in Equation (7), we have (Q/t) = qA = 10 kW . But the Fourier Law of heat conduction requires

$$q(x) = -k\frac{dT(X)}{dx} \quad \text{as the equation (9)}$$

$$Q = qA = -kA\frac{dT(X)}{dx}$$

$$\frac{dT(X)}{dx} = \frac{Q}{-kA}$$

$$= \frac{10}{100\left(1200\times 10^{-6}\right)} = -83.33\,{}^{o}C/m$$

Expression in (a) is a 1st order differential equation, and its solution is:

$$T(x) = -83.33x + c$$

If we use the condition: T (2) = 50^{o}C, we will find c = 216.67, which leads to the complete solution

$$T(x) = 216.67 - 83.33x$$

IV. CONCLUSION

Finding the temperature distribution in the rod is $T(x) = 216.67 - 83.33x\ {}^{o}C/m$ by the method of solution of first order ordinary differential equation. This same procedure is often utilized in Heat convection in fluids and Radiation of heat in space. This application is useful for solving several different types of Fluid Mechanics Analysis. Fundamentally, it consists of finding optimal solution of first order ordinary linear homogeneous equations and first order ordinary linear non homogeneous equations.

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