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Solution of Goal Programming Problem by New Approach

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Abstract-In this paper, new alternative methods for the solution of Goal programming problem is introduced. This method is easy to solve goal programming problem. This is powerful method to get improved solution. It reduces number of iterations and save valuable time by skipping calculations of net evaluation.

***Key Words:* Goal Programming Problem, Optimal Solution, Simplex Method, Alternative Method.**

I. INTRODUCTION

Linear Programming basically is the technique applicable only when there is a single goal (objective function), such as maximizing the profit or minimizing the cost or loss. There are situations where the system may have multiple (possibly conflicting) goals. For example, a firm may have a set of goals, such as employment stability, high product quality, maximization of profit, minimizing overtime or cost, etc. in such situations, we need a different technique that seek a compromise solution based on the relative importance of each objective. This technique is known as Goal Programming. It aims at minimizing the deviations from the targets that were set by the management. In this technique. We start with the most important goal and continues until the achievement of a less important goal. Whether the goals are attainable or not, the objective function is stated in such a manner that optimization means : "as close as possible to the indicated goals".

Khobragade et al. [2, 3, 4] suggested an alternative approach to solve linear programming problem.

In this paper, an attempt has been made to solve goal programming problem (GPP) by new method which is an alternative method. This method is different from Khobragade et al. [2-4] Method.

II. ALTERNATIVE SIMPLEX METHOD FOR GOAL PROGRAMMING PROBLEM

The major steps of the simplex method for the linear goal programming problem are :

Step I. Identify the decision variables of the key decision and formulate the given problem as linear goal programming problem.

Step 2. Determine the initial basic feasible solution and set up initial simplex table.

Step 3. Select max $\sum x_{ij}$, $x_{ij}\geq 0$, for entering vector.

Step 4. Choose greatest coefficient of decision variables. (i) If greatest coefficient is unique, then element corresponding to this row and column becomes pivotal (leading) element. (ii) If greatest coefficient is not unique, then use tie breaking technique.

Step 5. Use usual simplex method for this table and go to next step.

Step 6. Ignore corresponding row and column. Proceed to step 5 for remaining elements and repeat the same procedure until an optimal solution is obtained or there is an indication for unbounded solution.

Step 7. If all rows and columns are ignored, then current solution is an optimal solution.

III. SOLVED PROBLEMS

Problem- 1

$$Min.\ z = d_1^- + d_1^+ + d_2^- + d_3^-$$

Sub to : $80x_1 + 40x_2 + d_1^- - d_1^+ = 900$

$x_1 + d_2^- = 17 \quad x_2 + d_3^- + = 15$

Solution:

c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-
1	d_1^-	900	**80**	40	1	−1	0	0
1	d_2^-	17	1	0	0	0	1	0
1	d_3^-	15	0	1	0	0	0	1
First Iteration								
0	x_1	90/8	1	½	1/80	−1/80	0	0
1	d_2^-	23/4	0	−1/2	−1/80	−1/80	1	0

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1	d_3^-	15	0	**1**	0	0	0	1
Second Iteration								
0	x_1	15/4	1	0	1/80	–1/80	0	–1/2
1	d_2^-	53/4	0	0	–1/80	1/80	1	1/2
0	x_2	15	0	1	0	0	0	1

Optimum solution is

$$x_1 = 15/4,\ x_2 = 15 \quad d_2^- = \frac{53}{4},\ d_1^- = 0,\ d_2^+ = 0$$

Problem- 2

$$Min.\ z = d_1^- + d_2^- + 0\,S_1 + 0\,S_2 + 0\,S_3 + 0\,d_1^+ + 0\,d_2^+$$

Sub to : $2x_1 + 4x_2 + S_1 = 600$

$$4x_1 + 5x_2 + S_2 = 1000,\ 5x_1 + 4x_2 + S_3 = 1200,\ 20x_1 + 32x_2 + d_1^- - d_1^+ = 5400, \quad 0.3x_1 + 0.75x_2 + d_2^- - d_2^+ = 108$$

Solution

c_B	y_B	x_B	x_1	x_2	S_1	S_2	S_3	d_1^-	d_1^+	d_2^-	d_2^+
0	S_1	600	**2**	4	1	0	0	0	0	0	0
0	S_2	1000	4	5	0	1	0	0	0	0	0
0	S_3	1200	5	4	0	0	1	0	0	0	0
1	d_1^-	5400	20	**32**	0	0	0	1	–1	0	0
1	d_2^-	108	0.3	0.75	0	0	0	0	0	1	–1
First Iteration											
0	S_1	–75	–1/2	0	1	0	0	–1/8	1/8	0	0
0	S_2	625/4	7/8	0	0	1	0	–5/32	5/32	0	0
0	S_3	525	**5/2**	0	0	0	1	–1/8	1/8	0	0
0	x_2	675/4	20/32	1	0	0	0	1/32	–1/32	0	0
1	d_2^-	$\frac{-297}{16}$	$\frac{-27}{160}$	0	0	0	0	$\frac{-3}{128}$	$\frac{3}{128}$	1	–1
Second Iteration											
0	S_1	30	0	0	1	0	1/5	–3/20	3/20	0	0
0	S_2	–55/2	0	0	0	1	–7/20	–9/80	9/80	0	0
0	x_1	2/10	1	0	0	0	2/5	–1/20	1/20	0	0
0	x_2	75/2	0	1	0	0	–1/4	1/16	–1/16	0	0
1	d_2^-	$\frac{135}{8}$	0	0	0	0	27/400	$\frac{-51}{1600}$	$\frac{51}{1600}$	1	–1
Third Iteration											
0	S_1	100/1	0	0	1	–1/5	0	–3/14	3/14	0	0
0	S_3	550/7	0	0	0	1	1	9/28	–9/28	0	0
0	x_1	1250/7	1	0	0	–2/5	0	–5/28	5/28	0	0
0	x_2	400/7	0	1	0	¼	0	1/7	–1/7	0	0
1	d_2^-	81/7	0	0	0	$\frac{-27}{400}$	0	$\frac{-3}{56}$	$\frac{3}{56}$	1	–1

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Optimum Solution is

$$x_1 = \frac{1250}{7},\ x_2 = \frac{400}{7}\ d_2^- = \frac{81}{7},\ S_2 = 0,\ S_1 = 100/7,\ S_3 = \frac{550}{7}$$

Problem- 3

$$Min.\ z = P_1 d_1^- + P_4 d_1^+ + 5\,P_3\ d_2^- + 3P_3\ d_3^- + P_2\ d_4^+$$

Sub to : $x_1 + x_2 + d_1^- - d_1^+ = 80 \quad x_1 + x_2 + d_4^- - d_4^+ = 90 \quad x_1 + d_2^- = 70 \quad x_2 + d_3^- = 45$

$$x_1, x_2 + d_1^-, d_1^+, d_2^-, d_3^-, d_4^-, d_4^+ \geq 0$$

Solution: -

			0	0	P_1	P_4	$5P_3$	$3P_3$	0	P_2
c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-	d_4^-	d_4^+
P_1	d_1^-	80	1	1	1	–1	0	0	0	0
0	d_4^-	90	1	1	1	0	0	0	1	–1
$5P_3$	d_2^-	70	1	0	0	0	1	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	0	1	0	0
First Iteration										
P_1	d_1^-	10	0	1	1	–1	–1	0	0	0
0	d_4^-	20	0	1	0	0	–1	0	1	–1
0	x_1	70	1	0	0	0	1	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	0	1	0	0
Second Iteration										
0	x_2	10	0	1	1	–1	–1	0	0	0
0	d_4^-	10	0	0	–1	1	0	0	1	–1
0	x_1	70	1	0	0	0	1	0	0	0
$3P_3$	d_3^-	35	0	0	–1	1	1	1	0	0
Third Iteration										
0	x_2	20	0	1	0	0	–1	0	1	–1
P_4	d_4^+	10	0	0	–1	1	0	0	1	–1
0	x_1	70	1	0	0	0	1	0	0	0
$3P_3$	d_3^-	25	0	0	0	0	1	1	–1	1

$\therefore$ The optimum solution is, $x_1 = 70,\ x_2 = 20,\ d_1^+ = 10,\ d_3^- = 25 \quad d_1^- = d_2^- = d_4^- = d_4^+ = 0$

Problem- 4

$$Min.\ z = P_1 d_1^- + P_2 d_2^- + 2\ P_2\ d_3^- + P_3\ d_1^+$$

Sub to : $10x_1 + 10x_2 + d_1^- - d_1^+ = 400$

$x_1 + d_2^- = 40 \quad x_2 + d_3^- = 30$

$$x_1, x_2, d_1^+, d_1^-, d_2^-, d_3^- \geq 0$$

Solution:

			0	0	P_1	P_3	P_2	$2P_2$
c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-
P_1	d_1^-	400	**10**	10	1	–1	0	0

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P_2	d_2^-	40	1	0	0	0	1	0
$2P_2$	d_3^-	30	0	1	0	0	0	1
First Iteration								
P_1	x_1	40	1	1	1/10	–1/10	0	0
P_2	d_2^-	0	0	–1	–1/10	1/10	1	0
$2P_2$	d_3^-	30	0	**1**	0	0	0	1
Second Iteration								
P_1	x_1	10	1	0	1/10	–1/10	0	–1
P_2	d_2^-	30	0	0	**–1/10**	1/10	1	1
$2P_2$	x_2	30	0	1	0	0	0	1
Third Iteration								
P_1	x_1	40	1	0	0	0	1	0
P_2	d_1^+	300	0	0	–1	1	10	10
$2P_2$	x_2	30	0	1	0	0	0	1

Optimum Solution is

$x_1 = 40,\ x_2 = 30, d_1^+ = 300$, $d_1^- = d_2^- = d_3^- = 0$

Problem- 5

$Min.\ z = P_1 d_1^- + 2P_2 d_2^- + P_2\ d_3^- + P_3\ d_1^+$

Sub to : $x_1 + x_2 + d_1^- - d_1^+ = 400$

$x_1 + d_2^- = 240$

$x_2 + d_3^- = 300$

Solution:

			0	0	P_1	P_3	$2P_2$	P_2
c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-
P_1	d_1^-	400	1	1	1	–1	0	0
$2P_2$	d_2^-	240	**1**	0	0	0	1	0
P_3	d_3^-	300	0	1	0	0	0	1
First Iteration								
P_1	d_1^-	160	0	**1**	1	–1	–1	0
0	x_1	240	1	0	0	0	1	0
P_2	d_3^-	300	0	1	0	0	0	1
Second Iteration								
0	x_2	160	0	1	1	–1	–1	0
0	x_1	240	1	0	0	0	1	1
P_2	d_3^-	140	0	0	–1	**1**	1	1
Third Iteration								
0	x_2	300	0	1	0	0	0	1
0	x_1	240	1	0	0	0	1	0
P_3	d_1^+	140	0	0	–1	1	1	1

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Optimum Solution is

$x_1 = 240,\ x_2 = 300,\ d_1^+ = 140,\ d_2^- = d_3^- = d_1^- = 0$

IV. CONCLUSION

An alternative simplex method have been derived to obtain the solution of Goal programming problem. The proposed algorithm has simplicity and ease of understanding. This reduces number of iterations and improves the optimum solutions in most of the cases. This method saves valuable time as there is no need to calculate the net evaluation Zj-Cj.

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Dr. N.W. Khobragade for being M.Sc in statistics and Maths, he attained Ph.D in both subjects. He has been teaching since 1986 for 28 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities.

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APPENDIX: AN ALTERNATIVE ALGORITHM FOR SIMPLEX METHOD:

To find optimal solution of any LPP by an alternative method for simplex method, algorithm is given as follows:

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Step 1. Check objective function of LPP is of maximization or minimization type. If it is to be minimization type then convert it into a maximization type by using the result:

Min. Z = - Max.$(-Z)$.

Step 2. Check whether all b_i (RHS) are non-negative. If any b_i is negative then multiply the corresponding equation of the constraints by(-1).

Step 3. Express the given LPP in standard form then obtain initial basic feasible solution.

Step 4. Select $\max \sum x_{ij}$, $x_{ij} \geq 0$, for entering vector.

Step 5. Choose greatest coefficient of decision variables.

(i) If greatest coefficient is unique, then element corresponding to this row and column becomes pivotal (leading) element.

(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step 6. Use usual simplex method for this table and go to next step.

Step 7. Ignore corresponding row and column. Proceed to step 5 for remaining elements and repeat the same procedure until an optimal solution is obtained or there is an indication for unbounded solution.

Step 8. If all rows and columns are ignored, then current solution is an optimal solution.