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Stabilization of Uncertain Descriptor Bilinear Dynamical System Using Logarithmic Norm

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Abstract: ***This paper shows the importance of Logarithmic norm in studying stability of linear/nonlinear descriptor systems with parametric uncertainty. Some basic definitions and theorems about Logarithmic norm and the Diniderivative, have been given. Stabilization of uncertain bilinear descriptor systems via logarithmic norm approach has been proposed and proved with illustrations.***

Index Terms**: Bilinear systems, Descriptor system, Logarithmic Norm, Uncertain System.**

I. INTRODUCTION

Bilinear systems are important sub classes of nonlinear systems with many applications in engineering, biology and economics. The control of bilinear systems has been extensively studied in the 70s and at the beginning of the 80s of the last century and remarkable by [7]. In [2] GermundDahlquist introduced the logarithmic norm in order to derive error bound in initial value problem using differential inequalities that distinguished between forward and reverse time integration. Logarithmic norm plays an important role in the stability analysis of a continuous dynamical system see [12],[13] . Both [2],[5] provide tables for how to compute logarithm for some common norms as well as numerous basic properties of the logarithmic norm.

II. DESCRIPTION OF THE PROBLEM

Consider the descriptor bilinear system

$x_1' = (A+\delta A)x_1(t) + (B+\delta B)u(x_1(t))x_1(t)$...(1a)

$0 = (C+\delta C)x_1(t) + (D+\delta D)x_2(t)$...(1b)

Where $x_1(t) \in \mathbb{R}^p$, $x_2(t) \in \mathbb{R}^{n-p}$, $A \in \mathbb{R}^{p\times p}$, $B \in \mathbb{R}^{p\times 1}$, $C \in \mathbb{R}^{(n-p)\times p}$, $D \in \mathbb{R}^{(n-p)\times(n-p)}$ and δA , δB, δC, δD are constant perturbations matrices with $\|\delta A\| \le a$, $\|\delta B\| \le b$, $\|\delta C\| \le c$, $\|\delta D\| \le d$ and a, b, c, d are positive integers. With a nonlinear control$u(x_1(t))$, the solution of"(1)"exists for so-called consistent initial value $x_0$of $x(t)$. The set of the consistent initial values of (1) are denoted by W_k . One can define manifold $M_k \subseteq \mathbb{R}$ determined by "(1b)" as $M_k = \mathcal{N}([C+\delta C \quad D+\delta D])$. For the system governed by "(1)", the set W_k of the consistent initial values is equal to the manifold M_k that means $W_k = M_k$ in other words, a consistent initial value $x_0$has satisfy

$0 = (C+\delta C)x_{10} + (D+\delta D)x_{20}$

that equivalent to

$x_0 \in W_k = M_k = \mathcal{N}([C+\delta C \quad D+\delta D])$.

III. BASIC CONCEPT

In this section, a survey over different concepts and methods that one can found it in [1-10]

A. Definition

Let f be a function defined on $I = [a, b]$.

Then

$$D^+f(x) = \lim_{h\to 0^+} sup \frac{f(x+h)-f(x)}{h}$$

$$D_+f(x) = \lim_{h\to 0^+} inf \frac{f(x+h)-f(x)}{h}$$

Be right upper and lower Diniderivative respectively.

B. Remark

The properties of the Dini derivative can be found in [3],[11].

C. Definition

The Logarithmic norm is a real valued functional on operators and is derived from either an inner product or vector norm or its induced operator norm.

D. Definition

Let A be square matrix and $\|\cdot\|$ be an induced matrix norm then associated logarithmic norm μ of A is defined

$$\mu(A) = \lim_{h\to 0^+} sup \frac{\|I+hA\|-1}{h}$$

Where I is identity matrix of the same dimension of ,h is real, positive number.

E. Remark

One can find the properties of the logarithmic norm in [3].

F. Lemma

(Generalization of Gromwell's lemma)

Let $a, b, n, k \in \mathbb{R}$, $a < b$, $n > 1$ and $K > 0$, $f: [a, b] \to \mathbb{R}^+$an integral function such that $\forall\ \alpha, \beta \in [a, b]\ (\alpha < b)$: $\int_\alpha^\beta f(s)\,ds > 0$ and $x: [a, b] \to \mathbb{R}^+$ if

$x(t) \le K + \int_0^t f(s)[x(s)]^n$ds and

$1 - (n-1)K^{n-1}\int_\alpha^t f(s)\,ds > 0$, then

$$x(t) \le \frac{K}{\left[1-(n-1)K^{n-1}\int_a^t f(s)\,ds\right]^{\frac{1}{n-1}}}$$

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IV. STABILIZATIONOF UN CERTAIN DESCRPTOR BILINEAR DYNAMICALSYSTEM USING LOGARTHMIC NORM

A. *Theorem*

Consider the descriptor bilinear control system "(1a)", "(1b)" with

1- $\delta A$is chosen to be a constant perturbation such that $e^{\mu[A+\delta A]t} \le \gamma e^{-\propto t} \quad \forall t \ge 0$,α, γ are positive integers

2- The sub matrix D with $|D| \neq 0$ and $\|D^{-1}\delta D\| < 1$

3- A nonlinear control $u(x_1$ (t)) given by $u(x_1(t)) = \frac{r(x_1(t))}{\|x_1(0)\|^2}$, $\forall x_1(t) \in W_k$such that$x_1(0) \neq 0$

$r(x_1(t))$is a vector function satisfy

$\|r(x_1(t))\| \le f(t)\|x_1(t)\|^2$

Where

$f: \mathbb{R}^+ \to \mathbb{R}^+and\propto, \gamma$ positive integer such that $\forall t \ge 0$

$$1 - 2\gamma^2(\|B\| + b)\int_0^t e^{-2\propto s}\|f(s)\|ds$$

is positive and bounded.

Then the system "(1)" is asymptotically stable

Proof:

Let $x(0) = [x_1(0) \quad x_2(0)]^T$ and the space of consistent initial condition (W_k) can be obtained by substitute the condition (3) in to equation "(1b) "

$W_k = \{(x_1, x_2)|x_2(0) = -\frac{(C+\delta C)}{(D+\delta D)}x_1(0)$ that's mean $x(0) \in \mathcal{N}([C + \delta CD + \delta D])$with$D + \delta D \neq 0$ and $x_1(0) \neq 0\}$

From "(1a)"

$$x_1' = (A + \delta A)x_1(t) + (B + \delta B)u(x_1(x))x_1(t)$$

Using Dini derivative

$$D_t^+\|x_1(t)\| \le \lim_{h\to 0^+} sup \frac{\|x_1(t+h)\| - \|x_1(t)\|}{h}$$

$$= \lim_{h\to 0^+} \frac{\|x_1(t) + hx_1'(t)\| - \|x_1(t)\|}{h}$$

$$\le \lim_{h\to 0^+} \frac{\|x_1(t) + h(A+\delta A)x_1(t)\| - \|x_1(t)\|}{h} + \|(B+\delta B)(u(x_1(t))x_1(t)\|$$

$$\le \lim_{h\to 0^+} \frac{\|I + h(A+\delta A)\| - 1}{h}\|x_1(t)\| + \|(B+\delta B)(u(x_1(t))x_1(t)\|$$

$$= \mu[A+\delta A]\|x_1(t)\| + \|(B+\delta B)(u(x_1(t))x_1(t)\|$$

Where

$$\mu[A+\delta A] = \lim_{h\to 0^+} \frac{\|I + h(A+\delta A)\| - 1}{h} \triangleq \bar{\mu}$$

Now

$$D_t^+\|x_1(t)\| \le \bar{\mu}\|x_1(t)\| + \|(B+\delta B)(u(x_1(t))x_1(t)\|$$

Multiply both sides of the resulting inequality by $e^{-\bar{\mu}t}$ to get

$$\left.\begin{matrix} D_t^+\|x_1(t)\|e^{-\bar{\mu}t} - \bar{\mu}e^{-\bar{\mu}t}\|x_1(t)\| \\ \le \|(B+\delta B)u(x_1(t))x_1(t)\|e^{-\bar{\mu}t} \end{matrix}\right\} \ldots(2)$$

And by integrating both sides of (2) one can get

$$\left.\begin{matrix} \|x_1(t)\|e^{-\bar{\mu}t} - \|x_1(0)\| \\ \le \int_0^t \|(B+\delta B)u(x_1(s))x_1(s)\|e^{-\bar{\mu}s} \end{matrix}\right\} \ldots(3)$$

Substitute $u(x_1(t))$ in (3) to get

$$\|x_1(t)\| \le e^{\bar{\mu}t}\|x_1(0)\| + \int_0^t e^{\bar{\mu}(t-s)}\|B + \delta B\| \frac{\|f(s)\|\|x_1(s)\|^2}{\|x_1(0)\|^2}\|x_1(s)\|ds.$$

$$\left.\begin{matrix} \|x_1(t)\| \le e^{\bar{\mu}t}\|x_1(0)\| \\ \left[1 + \int_0^t e^{-\bar{\mu}s}\|B+\delta B\|\|f(s)\|\|x_1(s)\|^3\|x_1(0)\|^{-3}ds\right] \end{matrix}\right\} \ldots(4)$$

Now divide both sides of (4) by $e^{\bar{\mu}t}\|x_1(0)\|$

$$\frac{\|x_1(t)\|}{e^{\bar{\mu}t}\|x_1(0)\|} \le 1 + \int_0^t e^{2\bar{\mu}s}\|B + \delta B\|\,\|f(s)\|\left[\frac{\|x_1(s)\|}{e^{\bar{\mu}s}\|x_1(0)\|}\right]^3 ds$$

Since δA and $f(t)$ be chosen such that condition (1), (3) satisfied and applying lemma (3.6)

$$\frac{\|x_1(t)\|}{e^{\bar{\mu}t}\|x_1(0)\|} \le \frac{1}{\sqrt{1 - 2\gamma^2(\|B\| + b)\int_0^t e^{-2\propto s}\|f(s)\|ds}}$$

$$\|x_1(t)\| \le \frac{\gamma e^{-\propto t}\|x_1(0)\|}{\sqrt{1 - 2\gamma^2(\|B\| + b)\int_0^t e^{-2\propto s}\|f(s)\|ds}}$$

Hence $\lim_{t\to\infty}\|x_1(t)\| \to 0$

And

$$\|u(t)\| \le f(t)\frac{\gamma^2 e^{-2\propto t}}{1 - 2\gamma^2(\|B\| + b)\int_0^t e^{-2\propto s}\|r(s)\|ds}$$

Now for "(1 b)"

Since $\|D^{-1}\delta D\| < 1$ and $D + \delta D = D(I + D^{-1}\delta D)$

Then

$$x_2(t) = -D^{-1}(I + D^{-1}\delta D)^{-1}(C + \delta C)x_1(t)$$

$$\lim_{t\to\infty}\|x_2(t)\| \le \frac{\|D^{-1}\|}{1 - \|D^{-1}\delta D\|}(\|C\| + c)\lim_{t\to\infty}\|x_1(t)\| \to 0$$

B. *Example*

For the uncertain bilinear descriptor system "(1a)", "(1b)" assume that

$A = \begin{pmatrix} 0 & 0 \\ 0.5 & 0 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$.

$\delta A = \begin{pmatrix} -0.3 & 0 \\ -0.7 & -0.2 \end{pmatrix}$, $\delta B = \begin{pmatrix} 0.2 & 0 \\ 0 & 0 \end{pmatrix}$, $\delta C = \begin{pmatrix} 0.2 & 0 \\ 0 & 0 \end{pmatrix}$ and $\delta D = \begin{pmatrix} 0 & 0 \\ 0.3 & 0 \end{pmatrix}$.

Step (1):$\|\delta A\| \le 1$, $\|\delta B\| \le 1$,$\|\delta C\| \le 1$, $\|\delta D\| \le 1$

Step (2): Set $f(t) = \sin(t)$,

$$\|u(x_1(t))\| \le \frac{sin(t)\|(x_1(t))\|^2}{\|x_1(0)\|^2}$$

Step (3): To find the consistent initial condition space where

$x(0) \in W_k = \mathcal{N}([C + \delta C \quad D + \delta D])$

$W_k = \{(x_{10}, x_{20}, x_{30}, x_{40})|(x_{10}, x_{20}, x_{30}, x_{40}) = (a_1, -0.8a_1 - 0.7a_3, a_3, 0.8a_1)\}$

Set $a_1 = 0, a_3 = 1$one can get (0, -0.7,1,0)

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Step (4): $D^{-1}\delta D = \begin{bmatrix} -0.3 & 0 \\ 0 & 0 \end{bmatrix}$ then $\|D^{-1}\delta D\| < 1$

Step (5): For $\gamma = 1, \propto = 2$

$$1 - 2(1+1)\int_0^t e^{-4s}\|sin(s)\|ds$$

$= 1 - 4(-1/5e^{-2s}\cos(s) - 2/5e^{-2s}\sin(s))|_0^t$,

is positive and bounded

Step (6): Using Dini derivative to find logarithmic norm as

$$\mu[A+\delta A] = \lim_{h\to 0^+}\frac{\left\|I + \begin{bmatrix} -0.3 & 0 \\ -0.2 & -0.2 \end{bmatrix} h\right\| - 1}{h} = \bar{\mu}$$

Step (7): Using theorem (4.1) one can find

$$\|x_1(t)\| \le \frac{\gamma e^{-\propto t}\|x_1(0)\|}{\sqrt{1 - 2\gamma^2(\|B\|+1)\int_0^t e^{-2\propto s}\|sin(s)\|ds}}$$

One can get $\|x_1(t)\| \le \frac{0.7e^{-2t}}{\sqrt{1-2(1+1)\int_0^t e^{-4s}\|sin(s)\|ds}}$

$\lim_{t\to\infty}\|x_1(t)\| \to 0.$

Step (8):

$x_2(t) = -D^{-1}(I + D^{-1}\delta D)^{-1}(C + \delta c)x_1(t)$

$= -\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1.4286 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -0.8 & 0 \\ 0 & 1 \end{bmatrix} x_1(t)$

$= \begin{bmatrix} 1.2286 & -1 \\ -1.4286 & 0 \end{bmatrix} x_1(t)$

$\Rightarrow \lim_{t\to\infty}\|x_2(t)\| \le \begin{bmatrix} 1.2286 & -1 \\ -1.4286 & 0 \end{bmatrix}\lim_{t\to\infty}\|x_1(t)\| \to 0$

Then the system “(1)” is asymptotically stable.

V. CONCLUSION

The aim is to develop the modern formulism of the logarithmic norm and use it to stabilizing the bilinear uncertain system and give a simple example to illustrate the method.

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