

Optimum Solution of QPP by Wolfe's Method: New Approach

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Abstract- In this paper, new alternative methods for the solution of Quadratic Programming Problem QPP is introduced. This method is easy to solve QPP. This is powerful method to get improved solution. It reduces number of iterations and save valuable time by skipping calculations of net evaluation.

Key words: Quadratic programming problem, optimal solution, Wolfe's method, alternative method.

I. INTRODUCTION

Integer programming problem is a special class of L.P.P. where all or some variables are constrained to assume non – negative integer values. This type of problem is of particular importance in business and industry where discrete nature of the variables is involved in many decision – making situations.

Khobragade et al. [2, 3, 4] suggested an alternative approach to solve linear programming problem.

In this paper, an attempt has been made to solve linear programming problem (LPP) by new method which is an alternative for simplex method. This method is different from Khobragade et al. [2-4] Method.

II. WOLFE'S ALGORITHM FOR QPP

Let the QPP be

Maximize $Z = f(x_1, \dots, x_n)$

$$= \sum_{j=1}^n C_j x_j + \frac{1}{2} \sum_{j=1}^n \sum_{k=1}^n C_{jk} x_j x_k$$

Subject to the constraints:

$$\sum_{j=1}^n a_{ij} x_j \leq b_i, \quad i = 1, 2, \dots, m \quad x_j \geq 0, \quad j = 1, 2, \dots, n$$

Let the quadratic form $\sum_j \sum_k C_{jk} x_j x_k$

be negative semi-definite.

The iterative procedure of Wolfe's modified simplex algorithm to solve the above QPP is stated below :

Step (1). Convert the inequality constraints into equations by introducing the slack variables S_i^2 in the i th constraints

$i = 1, 2, \dots, m$ and the slack variables S_{m+j}^2 in the j th non – negative constraint, $j = 1, 2, \dots, n$.

Step (2). Construct the Lagrangian function as

$$L(x, S, \lambda) = f(x) - \sum_{i=1}^m \lambda_i \left[\sum_{j=1}^n a_{ij} x_j - b_i + S_i^2 \right] - \sum_{j=1}^n \lambda_{m+j} [-x_j + S_{m+j}^2]$$

Where,

$$x = (x_1, \dots, x_n), S = (S_1, \dots, S_{m+n}), \lambda = (\lambda_1, \dots, \lambda_{m+n}).$$

Differentiating $L(x, S, \lambda)$ partially with respect to the components of x, S, λ and equate the first order partial derivatives equal to zero.

Derive the Kuhn – Tucker conditions from resulting equations.

Step (3). Introduce the non – negative artificial variables $\omega_j, j = 1, 2, \dots, n$ in the Kuhn – Tucker conditions

$$C_j + \sum_{k=1}^n C_{jk} x_k - \sum_{i=1}^m \lambda_i a_{ij} + \lambda_{m+j} = 0$$

and construct an objective function

$$Z = \omega_1 + \omega_2 + \dots + \omega_n$$

Step (4). Obtain an initial basic feasible solution to the LPP

Minimize $Z = \omega_1 + \omega_2 + \dots + \omega_n$

Subject to the constraints:

$$\sum_{k=1}^n C_{jk} x_k - \sum_{i=1}^m \lambda_i a_{ij} + \lambda_{m+j} + \omega_j = -C_j, \quad j = 1, 2, \dots, n$$

$$\sum_{j=1}^n a_{ij} x_j + x_{n+i} = b_i, \quad i = 1, 2, \dots, m$$

$$\omega_j, \lambda_i, \lambda_{m+j}, x_j \geq 0 \quad i = 1, 2, \dots, m, j = 1, 2, \dots, n$$

Where $x_{n+i} = S_i^2$ and satisfying the complementary slackness condition:

$$\sum_{j=1}^n \lambda_{m+j} x_j + \sum_{i=1}^m x_{n+i} \lambda_i = 0.$$

Step (5). Use simplex method [15] to obtain an optimum solution to the LPP of step (4), the solution satisfying the complementary slackness condition.

Step (6). The optimum solution obtained in step (5) is an Optimum solution to the given QPP also.

III. SOLVED PROBLEMS

Problem- 1

Maximize $Z = 2x_1 + 3x_2 - 2x_1^2$ (1)

Subject to the constraints:

$x_1 + 4x_2 \leq 4,$

$x_1 + x_2 \leq 2, \quad x_1, x_2 \geq 0$ and are integers.

Solution:

$x_1 + 4x_2 + S_1^2 = 4,$

$x_1 + x_2 + S_2^2 = 2$

$-x_1 + S_3^2 = 0$

$-x_2 + S_4^2 = 0$

Construct the Lagrangian function

$L = L(x, S, \lambda)$

$= (2x_1 + 3x_2 - 2x_1^2) - \lambda_1 (x_1 + 4x_2 + S_1^2 - 4)$

$- \lambda_2 (x_1 + x_2 + S_2^2 - 2)$

$- \lambda_3 (-x_1 + S_3^2) - \lambda_4 (-x_2 + S_4^2)$

$\frac{\partial L}{\partial x_1} = 2 - \lambda_1 - \lambda_2 + \lambda_3 - 4x_1 = 0 \Rightarrow$

$4x_1 + \lambda_1 + \lambda_2 - \lambda_3 = 2$

$\frac{\partial L}{\partial x_2} = 3 - 4\lambda_1 - \lambda_2 + \lambda_4 = 0$

$\Rightarrow 4\lambda_1 + \lambda_2 - \lambda_4 = 3$

$\frac{\partial L}{\partial \lambda_1} = -(x_1 + 4x_2 + S_1^2 - 4) = 0$

$\Rightarrow x_1 + 4x_2 + S_1^2 = 4$

$\frac{\partial L}{\partial \lambda_2} = -(x_1 + x_2 + S_2^2 - 2) = 0$

$\Rightarrow x_1 + x_2 + S_2^2 = 2$

$\frac{\partial L}{\partial \lambda_3} = -(-x_1 + S_3^2 - 2) = 0$

$\Rightarrow -x_1 + S_3^2 = 0$

$\frac{\partial L}{\partial \lambda_4} = -(x_2 + S_4^2 - 2) = 0$

$\Rightarrow -x_2 + S_4^2 = 0$

Upon Simplification & necessary manipulation these yield:

Equation (1) \Rightarrow Max. $Z = A_1 + A_2$

Subject to:

$4x_1 + \lambda_1 + \lambda_2 - \lambda_3 + A_1 = 2$

$4\lambda_1 + \lambda_2 - \lambda_4 + A_2 = 3$

$x_1 + 4x_2 + x_3 = 4$

$x_1 + x_2 + x_4 = 2$

$x_1, x_2, x_3, x_4 \geq 0$

$A_1, A_2, \lambda_1 \geq 0$

Initial Iteration:

c_B	y_B	x_B	x_1	x_2	x_3	x_4	λ_1	λ_2	λ_3	λ_4	A_1	A_2
1	A_1	2	4	0	0	0	1	1	-1	1	1	0
1	A_2	3	0	0	0	0	4	1	0	0	0	1
0	x_3	4	1	4	1	0	0	0	0	0	0	0
0	x_4	2	1	1	0	1	0	0	0	0	0	0
First Iteration												
0	x_1	1/2	1	0	0	0	1/4	1/4	-1/4	0	1/4	0
1	A_2	3	0	0	0	0	4	1	0	-1	0	1
0	x_3	7/2	0	4	1	0	-1/4	-1/4	-1/4	0	-1/4	0
0	x_4	3/2	0	1	0	1	-1/4	-1/4	-1/4	0	-1/4	0
Second Iteration												
0	x_1	1/2	1	0	0	0	1/4	1/4	-1/4	0	1/4	1
1	A_2	3	0	0	0	0	4	1	0	-1	0	0
0	x_2	7/8	0	1	1/4	0	-1/16	1/16	1/16	0	-1/16	0
0	x_4	5/8	0	0	-1/4	1	-3/16	3/16	3/16	0	-3/16	0
Third Iteration												
0	x_1	5/16	1	0	0	0	0	3/16	-1/4	1/16	1/4	-1/16
0	λ_1	3/4	0	0	0	0	1	1/4	0	-1/4	0	1/4

0	x_2	59/64	0	1	1/4	0	0	-3/64	1/16	-1/16	-1/16	1/64
0	x_4	49/64	0	0	-1/4	1	0	-9/64	3/16	-3/16	-3/16	3/64

Therefore the optimum solution is: $x_1 = 5/16$,

$x_2 = 59/64$ And Maximum $Z = 409/128 = 3.19$

Problem- 2

Maximize $Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$ (2)

Subject to the constraint:

$$x_1 + 2x_2 \leq 2,$$

$x_1, x_2 \geq 0$ and are integers.

Solution:

$$x_1 + 2x_2 + S_1^2 = 2,$$

$$-x_1 + S_2^2 = 0$$

$$-x_2 + S_3^2 = 0$$

Construct the Lagrangian function

$$L = L(x, S, \lambda)$$

$$= (4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2)$$

$$- \lambda_1 (x_1 + 2x_2 + S_1^2 - 2) - \lambda_2 (-x_1 + S_2^2)$$

$$- \lambda_3 (-x_2 + S_3^2)$$

$$\frac{\partial L}{\partial x_1} = 4 - 4x_1 - 2x_2 - \lambda_1 + \lambda_2 = 0 \Rightarrow$$

$$4x_1 + 2x_2 + \lambda_1 - \lambda_2 = 4$$

$$\frac{\partial L}{\partial x_2} = 6 - 2x_1 - 4x_2 - 2\lambda_1 + \lambda_3 = 0$$

$$\Rightarrow 2x_1 + 4x_2 + 2\lambda_1 - \lambda_3 = 6$$

$$\frac{\partial L}{\partial \lambda_1} = -(x_1 + 2x_2 + S_1^2 - 2) = 0$$

$$\Rightarrow x_1 + 2x_2 + S_1^2 = 2$$

$$\frac{\partial L}{\partial \lambda_2} = -(-x_1 + S_2^2) = 0$$

$$\Rightarrow -x_1 + S_2^2 = 0$$

$$\frac{\partial L}{\partial \lambda_3} = -(-x_2 + S_3^2) = 0$$

$$\Rightarrow -x_2 + S_3^2 = 0$$

Upon Simplification & necessary manipulation these yield:

Equation (2) \Rightarrow Max. $Z = A_1 + A_2$

Subject to:

$$4x_1 + 2x_2 - \lambda_1 - \lambda_2 + A_1 = 4$$

$$2x_1 + 4x_2 + 2\lambda_1 - \lambda_3 + A_2 = 6$$

$$x_1 + 2x_2 + x_3 = 2$$

$$x_1, x_2, x_3 \geq 0$$

$$A_1, A_2, \lambda_1 \geq 0$$

Initial Iteration:

c_B	y_B	x_B	x_1	x_2	x_3	λ_1	λ_2	λ_3	A_1	A_2
1	A_1	4	4	2	0	1	-1	0	1	0
1	A_2	6	2	4	0	2	0	-1	0	1
0	x_3	2	1	2	1	0	0	0	0	0
First Iteration										
1	A_1	1	3	0	0	0	-1	1/2	1	-1/2
0	x_2	3/2	1/2	1	0	1/2	0	-1/4	0	1/4
0	x_3	-1	0	0	1	-1	0	1/2	0	-1/2
Second Iteration										
1	A_1	1	3	0	0	0	-1	1/2	1	-1/2
0	x_2	1	1/2	1	1/2	0	0	0	0	0
0	λ_1	1	0	0	-1	1	0	-1/2	0	1/2
Third Iteration										
0	x_1	1/3	1	0	0	0	-1/3	+1/6	1/3	-1/6

0	x_2	5/6	0	1	1/2	0	1/6	-1/12	-1/2	1/4
0	λ_1	1	0	0	-1	1	0	-1/2	0	1/2

∴ The optimum solution is, $x_1 = 1/3, x_2 = 5/3$

∴ $M_{zx}. z = 277/13 = 21.31$

Problem 3

Max. $z = 8x_1 + 10x_2 - x_1^2 - x_2^2$

Sub to: $3x_1 + 2x_2 \leq 6, x_1, x_2 \geq 0$

Solution: we have,

$3x_1 + 2x_2 + S_1^2 = 6$

$-x_1 + S_2^2 = 0$

$-x_2 + S_3^2 = 0$

Construct the Lagrangian function :

$L = (8x_1 + 10x_2 - x_1^2 - x_2^2) - \lambda_1(3x_1 + 2x_2 + S_1^2 - 6) - \lambda_2(-x_1 + S_2^2) - \lambda_3(-x_2 + S_3^2)$

$\frac{\partial L}{\partial x_1} = 8 - 2x_1 - 3\lambda_1 + \lambda_2 = 0 \Rightarrow 2x_1 + 3\lambda_1 - \lambda_2 = 8$

$\frac{\partial L}{\partial x_2} = 10 - 2x_2 - 2\lambda_1 + \lambda_3 = 0 \Rightarrow 2x_2 + 2\lambda_1 - \lambda_3 = 10$

$\frac{\partial L}{\partial \lambda_1} = -(3x_1 + 2x_2 + S_1^2 - 6) = 0 \Rightarrow 3x_1 + 2x_2 + S_1^2 = 6$

$\frac{\partial L}{\partial \lambda_2} = -x_1 + S_2^2 = 0$

$\frac{\partial L}{\partial \lambda_3} = -x_2 + S_3^2 = 0$

Satisfied equation (1)

∴ $Max.z = A_1 + A_2$

Sub. to: $2x_1 + 3\lambda_1 - \lambda_2 = 8 \Rightarrow 2x_1 + 3\lambda_1 - \lambda_2 + A_1 = 8$

$2x_2 + 2\lambda_1 - \lambda_3 = 10 \Rightarrow 2x_2 + 2\lambda_1 - \lambda_3 + A_2 = 10$

$3x_1 + 2x_2 + x_3 = 6 \Rightarrow 3x_1 + 2\lambda_2 + x_3 = 6$

$x_1, x_2, x_3 \geq 0 \quad A_1, A_2, \lambda_i \geq 0$

Initial Iteration:

c_B	y_B	x_B	x_1	x_2	x_3	λ_1	λ_2	λ_3	A_1	A_2
1	A_1	8	2	0	0	3	-1	0	1	0
1	A_2	10	0	2	0	2	0	-1	0	1
0	x_3	6	3	2	1	0	0	0	0	0
First Iteration										
1	A_1	4	0	-4/3	-2/3	3	-1	0	1	0
0	A_2	10	0	2	0	2	0	-1	0	1
0	x_1	2	1	2/3	1/3	0	0	0	0	0
Second Iteration										
0	λ_1	4/3	0	-4/9	-2/9	1	-1/3	0	1/3	0
1	A_2	22/3	0	26/9	4/9	0	2/3	-1	-2	1
0	x_1	2	1	2/3	1/3	0	0	0	0	0
Third Iteration										
0	λ_1	32/13	0	0	-2/13	1	-3/13	-2/13	25/39	2/13
0	x_2	33/13	0	1	2/13	0	3/13	-9/26	9/13	9/26
0	x_1	4/13	1	0	3/13	0	-2/13	3/13	-6/13	3/13

∴ The optimum solution is, $x_1 = 4/13, x_2 = 33/13$

∴ $M_{zx}. z = 277/13 = 21.31$

Problem 4

Max. $z = 6x_1 + 3x_2 - 4x_1x_2 - 2x_1^2 - 3x_2^2$

(4)

Sub to: $x_1 + x_2 \leq 1$

$2x_1 + 3x_2 \leq 4, x_1, x_2 \geq 0$

Solution: - We have,

$2x_1 + 3x_2 + S_2^2 = 4$

$-x_1 + S_3^2 = 0$

$-x_1 + S_4^2 = 0$

$-x_2 + S_4^2 = 0$

Construct the Lagrangian function:

$$L = (6x_1 + 3x_2 - 4x_1x_2 - 2x_1^2 - 3x_2^2) - \lambda_1(x_1 + x_2 + S_1^2 - 1)$$

$$\frac{\partial L}{\partial \lambda_3} = -x_1 + S_3^2 = 0$$

$$- \lambda_2(2x_1 + 3x_2 + S_2^2 - 4) - \lambda_3(-x_1 + S_3^2) - \lambda_4(-x_2 + S_4^2)$$

$$\frac{\partial L}{\partial \lambda_4} = -x_2 + S_4^2 = 0$$

$$\frac{\partial L}{\partial x_1} = 6 - 4x_2 - 4x_1 - \lambda_1 - 2\lambda_2 + \lambda_3 = 0 = 4x_1 + 4x_2 + \lambda_1$$

$$+ 2\lambda_2 - \lambda_3 = 6$$

Then Equation (4) becomes

$$Max.z = A_1 + A_2$$

$$\frac{\partial L}{\partial x_2} = 3 - 4x_1 - 6x_2 - \lambda_1 - 3\lambda_2 + \lambda_4 = 0 = 4x_1 + 6x_2 + \lambda_1$$

$$Sub. to : 4x_1 + 4x_2 + \lambda_1 + 2\lambda_2 - \lambda_3 = 6 + A_1$$

$$+ 3\lambda_2 - \lambda_4 = 3$$

$$4x_1 + 6x_2 + \lambda_1 + 3\lambda_2 - \lambda_4 = 3 + A_2$$

$$\frac{\partial L}{\partial \lambda_1} = -(x_1 + x_2 + S_1^2 - 1) = 0 \Rightarrow x_1 + x_2 + S_1^2 = 1$$

$$x_1 + x_2 + x_3 = 1$$

$$2x_1 + 3x_2 + x_4 = 4$$

$$\frac{\partial L}{\partial \lambda_2} = -(2x_1 + 3x_2 + S_2^2 - 4) = 0 \Rightarrow 2x_1 + 3x_2 + S_2^2 = 4$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$A_1, A_2, \lambda_i \geq 0$$

Initial Iteration :

c_B	y_B	x_B	x_1	x_2	x_3	x_4	λ_1	λ_2	λ_3	λ_4	A_1	A_2
1	A_1	6	4	4	0	0	1	2	-1	0	1	0
1	A_2	3	4	6	0	0	1	3	0	-1	0	1
0	x_3	1	1	1	1	0	0	0	0	0	0	0
0	x_4	4	2	3	0	1	0	0	0	0	0	0
First Iteration												
0	x_1	4	4/3	0	0	0	1/3	0	-1	2/3	1	-2/3
1	A_2	1/2	2/3	1	0	0	1/6	1/2	0	-1/6	0	1/6
0	x_3	1/2	1/3	0	1	0	-1/6	-1/2	0	1/6	0	-1/6
0	x_4	5/2	1	0	0	1	-1/2	-3/2	0	2/2	0	2/3
Second Iteration												
0	x_1	3	1	0	0	0	1/4	0	-3/4	1/2	3/4	-1/2
0	A_2	-3/2	0	1	0	0	0	1/2	1/2	-1/2	-1/2	1/2
0	x_2	-1/2	0	0	1	0	-1/4	-1/2	1/4	0	-1/4	0
0	x_4	-1/2	0	0	0	1	-3/4	-3/2	3/4	0	-3/4	7/6
Third Iteration												
0	x_1	3/2	1	0	0	0	1/4	1/2	-1/4	0	1/4	0
0	λ_4	3	0	-2	0	0	0	-1	-1	1	1	-1
0	x_3	-1/2	0	0	1	0	-1/4	-1/2	1/4	0	-1/4	0
0	x_4	-1/2	0	0	0	1	-3/4	-3/2	3/4	0	-3/4	7/6
Four Iteration												
0	x_1	1	1	0	1	0	0	-1/2	0	0	0	0
0	λ_4	3	0	-2	0	0	0	-1	-1	1	1	-1
0	λ_1	2	0	0	-4	0	1	2	-1	0	1	0
0	x_4	7/4	0	0	-3	1	0	0	0	0	0	7/6

∴ The optimum solution is, $x_1 = 1, x_2 = 0$

∴ $Max.z = 6 - 2 = 4$

IV. CONCLUSION

An alternative approach for Wolfe's method have been derived and the improved solution of quadratic programming problem have been obtained. The proposed algorithm has simplicity and ease of understanding. This reduces number of iterations and improves the optimum solutions in most of the cases. This method saves valuable time as there is no need to calculate the net evaluation $Z_j - C_j$.

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APPENDIX: AN ALTERNATIVE ALGORITHM FOR SIMPLEX METHOD

To find optimal solution of any LPP by an alternative method for simplex method, algorithm is given as follows:

Step 1. Check objective function of LPP is of maximization or minimization type. If it is to be minimization type then convert it into a maximization type by using the result:

$$\text{Min. } Z = - \text{Max. } (-Z).$$

Step 2. Check whether all b_i (RHS) are non-negative. If any b_i is negative then multiply the corresponding equation of the constraints by(-1).

Step 3. Express the given LPP in standard form then obtain initial basic feasible solution.

Step 4. Select $\max \sum x_{ij}$, $x_{ij} \geq 0$, for entering vector.

Step 5. Choose greatest coefficient of decision variables.

(i) If greatest coefficient is unique, then element corresponding to this row and column becomes pivotal (leading) element.

(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step 6. Use usual simplex method for this table and go to next step.

Step 7. Ignore corresponding row and column. Proceed to step 5 for remaining elements and repeat the same procedure until an optimal solution is obtained or there is an indication for unbounded solution.

Step 8. If all rows and columns are ignored, then current solution is an optimal solution.

AUTHOR BIOGRAPHY



Dr. N.W. Khobragade for being M.Sc in statistics and Maths, he attained Ph.D in both subjects. He has been teaching since 1986 for 28 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities.

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