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# Total Unidominating Functions of a Cycle

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***Abstract:* Domination in graphs has applications to several fields such as Communication networks, Facility location problems, locating radar stations problem etc. Recently dominating functions in domination theory have received much attention. We have introduced new concepts of unidominating functions and total unidominating functions. The unidominating functions and total unidominating functions of a Path are studied by the author [5] ,[6]. In this paper some total unidominating functions of a cycle are discussed and determined its total unidomination number. Also the number of total unidominating functions with minimum weight are investigated.**

***Key words:*** **Cycle, Total unidominating function, total unidomination number.**

## I. INTRODUCTION

Graph theory has established itself as an important mathematical tool in a wide variety of subjects, ranging rom operations research and chemistry to genetics and linguistics. Domination and its properties have been extensively studied by T.W.Haynes et.al. [1], [2]. Recently dominating functions in domination theory have received much attention. Hedetniemi et.al.[ 3 ] introduced the concept of dominating function and Cockayne et.al [ 4 ]introduced the concept of total dominating function. The authors have introduced the new concept of unidominating function and studied the unidominating functions of a path[ 5 ]. The concept of total unidominating function is also introduced and studied the total unidominating functions of a path [6]. In this paper total unidominating functions of a cycle are discussed and the total unidomination number of a cycle is obtained. Also the number of total unidominating functions of a cycle with minimum weight is investigated. Further the results obtained are illustrated.

## II. TOTAL UNIDOMINATING FUNCTIONS AND TOTAL UNIDOMINATION NUMBER

In this section the concepts of total unidominating function, total unidomination number are defined as follows:

**Definition 1:** Let $G(V,E)$ be a connected graph. A function $f: V \to \{0,1\}$ is said to be a **total unidominating function if**

$$\sum_{u\in N(v)} f(u) \geq 1 \quad \forall\, v \in V \text{ and } f(v) = 1,$$

$$\sum_{u\in N(v)} f(u) = 1 \quad \forall\, v \in V \text{ and } f(v) = 0.$$

**Definition 2:** The **total unidomination number** of a connected graph $G(V,E)$ is defined as

$min\{f(V)/f$ is a total unidominating function$\}$.

It is denoted by $\gamma_{tu}(G)$.

## III. TOTAL UNIDOMINATION NUMBER OF A CYCLE

In this section we find the total unidomination number of a cycle and the number of total unidominating functions of minimum weight for a cycle.

**Theorem 3.1:** The total unidomination number of a cycle $C_n$ is

$$\gamma_{tu}(C_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil & \text{If } n \equiv 0,1 (mod\ 4), \\ \left\lceil \frac{n}{2} \right\rceil + 1 & \text{If } n \equiv 2,3 (mod\ 4). \end{cases}$$

Proof: Let $C_n$ be a cycle with vertex set $V = \{v_1, v_2, \ldots, v_n\},\ n \geq 3.$

To find the total unidomination number of a cycle $C_n$, the following four cases arise.

**Case 1:** Let $n \equiv 0(mod\ 4)$.

Define a function $f: V \to \{0,1\}$ by

$$f(v_i) = \begin{cases} 1 & \text{for } i \equiv 2,3 (mod\ 4), \\ 0 & \text{for } i \equiv 0,1 (mod\ 4). \end{cases}$$

Now we check the condition for total unidominating function at every vertex.

**Sub case 1:** Let $i \equiv 0(mod\ 4)$ and $i \neq n$. Then $f(v_i) = 0$.

$$\text{Now} \sum_{u\in N(v_i)} f(u) = f(v_{i-1}) + f(v_{i+1}) = 1 + 0 = 1.$$

$$\text{For } i = n, \sum_{u\in N(v_n)} f(u) = f(v_{n-1}) + f(v_1) = 1 + 0 = 1.$$

**Sub case 2:** Let $i \equiv 1(mod\ 4)$ and $i \neq 1$. Then $f(v_i) = 0$.

$$\text{Now} \sum_{u\in N(v_i)} f(u) = f(v_{i-1}) + f(v_{i+1}) = 0 + 1 = 1.$$

$$\text{For } i = 1, \sum_{u\in N(v_1)} f(u) = f(v_n) + f(v_2) = 0 + 1 = 1.$$

**Sub case 3:** let $i \equiv 2(mod\ 4)$. Then $f(v_i) = 1$.

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Now $\sum_{u\in N(v_i)} f(u) = f(v_{i-1}) + f(v_{i+1}) = 0 + 1 = 1.$

**Sub case 4:** Let $i \equiv 3(mod\ 4)$. Then $f(v_i) = 1$.

Now $\sum_{u\in N(v_i)} f(u) = f(v_{i-1}) + f(v_{i+1}) = 1 + 0 = 1.$

Thus $\sum_{u\in N(v)} f(u) \geq 1 \;\; when\ f(v) = 1$

$and \;\; \sum_{u\in N(v)} f(u) = 1 \;\; when\ f(v) = 0.$

Hence it follows that $f$ is a total unidominating function.

$$\text{Now} \sum_{u\in V} f(u) = \sum_{i=1}^{n} f(v_i) = \underbrace{0+1+1+0}+\cdots+\underbrace{0+1+1+0} = 2\left(\frac{n}{4}\right) = \frac{n}{2}.$$

By the definition of total unidomination number, $\gamma_{tu}(C_n) \leq \frac{n}{2} - - - (1)$

If $f$ is a total unidominating function of $C_n$, then we can see that amongst four consecutive vertices at most two vertices can have functional value $0$ and at least two vertices must have functional value $1$. Therefore sum of the functional values of four consecutive vertices is greater than or equal to 2.

That is $\sum_{i=1}^{4} f(v_i) \geq 2, \sum_{i=5}^{8} f(v_i) \geq 2, \ldots, \sum_{i=n-3}^{n} f(v_i) \geq 2.$

Therefore

$$\sum_{u\in V} f(u) = \sum_{i=1}^{4} f(v_i) + \sum_{i=5}^{8} f(v_i) + \cdots + \sum_{i=n-3}^{n} f(v_i) \geq \underbrace{2+2+\cdots+2}_{\frac{n}{4} times} \geq \frac{n}{2}.$$

This is true for any total unidominating function.

Therefore $\min\{f(V)/f \text{ is a total unidominating function}\} \geq \frac{n}{2}$.

Thus $\gamma_{tu}(C_n) \geq \frac{n}{2} - - - (2)$

Therefore from the inequalities (1) and (2), $\gamma_{tu}(C_n) = \frac{n}{2}$.

**Case 2:** Let $n \equiv 1(mod\ 4)$.

Define a function $f: V \to \{0,1\}$ by

$$f(v_i) = \begin{cases} 1 & \text{for } i \equiv 2,3(mod\ 4), \\ 0 & \text{for } i \equiv 0,1(mod\ 4), i \neq n-1, \end{cases}$$

and $f(v_{n-1}) = 1$.

As in Case 1 it can be verified that $f$ is a total unidominating function and

$$\sum_{u\in V} f(u) = \underbrace{0+1+1+0}+\cdots+\underbrace{0+1+1+1+0} = \frac{2(n-5)}{4} + 3 = \frac{n+1}{2} = \left\lceil\frac{n}{2}\right\rceil.$$

Therefore $\gamma_{tu}(C_n) \leq \left\lceil\frac{n}{2}\right\rceil - - - (1)$

Now $n \equiv 1(mod\ 4) \Rightarrow n-5 \equiv 0(mod\ 4)$.

Let $f$ be a total unidominating function of $C_n$, $n \geq 5$. Then as in Case 1, for any $n-5$ consecutive vertices, we have

$$\sum_{i=1}^{n-5} f(v_i) \geq \frac{n-5}{2}.$$

Equality is to be taken in the above inequation to get minimum weight. The possibilities of assigning the functional values for the remaining five vertices are as follows:

1,1,0,0,1 or 1,1,1,0,0 or 0,1,1,1,0 or 0,0,1,1,1 and their functional value sum is 3.

$$\text{Now } f(V) = \sum_{u\in V} f(u) = \frac{n-5}{2} + 3 = \frac{n+1}{2} = \left\lceil\frac{n}{2}\right\rceil.$$

Thus

$\min\{f(V)/f \text{is a total unidominating function}\} \geq \left\lceil\frac{n}{2}\right\rceil$.

that is $\gamma_{tu}(C_n) \geq \left\lceil\frac{n}{2}\right\rceil - - - (2)$

Therefore from the inequalities (1) and (2), $\gamma_{tu}(C_n) = \left\lceil\frac{n}{2}\right\rceil$.

**Case 3:** Let $n \equiv 2(mod\ 4)$.

Define a function $f: V \to \{0,1\}$ by

$$f(v_i) = \begin{cases} 1 & \text{for } i \equiv 2,3(mod\ 4), i \neq n, \\ 0 & \text{for } i \equiv 0,1(mod\ 4), i \neq n-2, n-1. \end{cases}$$

and $f(v_{n-2}) = f(v_{n-1}) = 1, f(v_n) = 0$.

Again in similar lines to Case 1 we can verify that the function $f$ defined in this case is also a total unidominating function.

$$\text{Now} \sum_{u\in V} f(u) = \underbrace{0+1+1+0}+\cdots+\underbrace{0+1+1+1+1+0} = 2\left(\frac{n-6}{4}\right) + 4 = \frac{n+2}{2} = \frac{n}{2} + 1.$$

Therefore $\gamma_{tu}(C_n) \leq \frac{n}{2} + 1 - - - (1)$

Now $n \equiv 2(mod\ 4) \Rightarrow n-6 \equiv 0(mod\ 4)$. Then as in Case 2, for any total unidominating function $f$ of $C_n$, $n \geq 10$, with minimum weight, we have

$$\sum_{i=1}^{n-6} f(v_i) = \frac{n-6}{2}.$$

The possibilities of assigning the functional values for the remaining six vertices are as follows:

1,1,0,0,1,1, or 1,1,1,0,0,1 or 0,1,1,1,1,0 or 1,0,0,1,1,1 or 1,1,1,1,0,0 or 0,0,1,1,1,1.

We can see that for any other assignment of functional values to these vertices can no more make $f$ a total unidominating function with minimum weight.

$$\text{Now} f(V) = \sum_{i=1}^{n-6} f(v_i) + \sum_{i=n-5}^{n} f(v_i) = \frac{n-6}{2} + 4 = \frac{n}{2} + 1$$

Thus

$\min\{f(V)/f \text{is a total unidominating function}\} \geq \frac{n}{2} + 1$.

that is $\gamma_{tu}(C_n) \geq \frac{n}{2} + 1 - - - (2)$

Therefore from the inequalities (1) and (2), $\gamma_{tu}(C_n) = \frac{n}{2} + 1 = \left\lceil\frac{n}{2}\right\rceil + 1$.

**Case 4:** Let $n \equiv 3(mod\ 4)$.

**Sub case 1:** Let $n = 3$.

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Then clearly a function $f$ defined by $f(v_1) = 1, f(v_2) = 1, f(v_3) = 1$ is the only total unidominating function and $\gamma_{tu}(C_3) = \left\lceil \frac{3}{2} \right\rceil + 1 = 3$.

**Sub case 2:** Let $n \geq 7$.

Define a function $f : V \to \{0,1\}$ by

$$f(v_i) = \begin{cases} 1 & \text{for } i \equiv 2,3 (mod\ 4), i \neq n, \\ 0 & \text{for } i \equiv 0,1 (mod\ 4), i \neq n-3, n-2. \end{cases}$$

and $f(v_{n-3}) = 1, f(v_{n-2}) = 1, f(v_n) = 0$.

Again in similar lines to Case 1 we can verify that the function $f$ defined here is also a total unidominating function.

$$\text{Now } \sum_{u \in V} f(u) = \underbrace{0+1+1+0}_{} + \cdots + \underbrace{0+1+1+0}_{} + \underbrace{0+1+1+1+1+1+0}_{}$$

$$= 2\left(\frac{n-7}{4}\right) + 5 = \frac{n+3}{2} = \left\lceil \frac{n}{2} \right\rceil + 1.$$

$$\text{Therefore } \gamma_{tu}(C_n) \leq \left\lceil \frac{n}{2} \right\rceil + 1 \quad ---(1)$$

Now $n \equiv 3 (mod\ 4) \Rightarrow n - 3 \equiv 0 (mod\ 4)$. Then as in Case 2, for any total unidominating function $f$ of $C_n$, $n \geq 11$ with minimum weight we have

$$\sum_{i=1}^{n-7} f(v_i) = \frac{n-7}{2}.$$

Since $f$ is a total unidominating function with minimum weight, the possibilities of assigning functional values to the remaining 7 vertices are as follows:

1,1,1,1,1,0,0 or 0,1,1,1,1,1,0 or 0,0,1,1,1,1,1 or 1,1,0,0,1,1,1 or 1,1,1,0,0,1,1. The functional value sum of these vertices is 5.

$$\text{So } f(V) = \sum_{i=1}^{n} f(v_i) = \frac{n-7}{2} + 5 = \frac{n+3}{2} = \left\lceil \frac{n}{2} \right\rceil + 1.$$

Therefore for any total unidominating function $f$,

$$\sum_{u \in V} f(u) \geq \left\lceil \frac{n}{2} \right\rceil + 1.$$

Thus

$$\min\{f(V) / f \text{ is a total unidominating function}\} \geq \left\lceil \frac{n}{2} \right\rceil + 1.$$

That is $\gamma_{tu}(C_n) \geq \left\lceil \frac{n}{2} \right\rceil + 1 \quad ---(2)$

Therefore from the inequalities (1) and (2), $\gamma_{tu}(C_n) = \left\lceil \frac{n}{2} \right\rceil + 1$. ■

**Theorem 3.2:** The number of total unidominating functions with minimum weight of a cycle $C_n$ is

$$\begin{cases} 4 & \text{when } n \equiv 0 (mod\ 4), \\ n & \text{when } n \equiv 1 (mod\ 4), \\ n\left(1 + \left\lfloor \frac{n}{8} \right\rfloor\right) & \text{when } n \equiv 2 (mod\ 4), n \neq 10, \\ 15 & \text{when } n = 10, \\ n + n\left(\left\lfloor \frac{n}{4} \right\rfloor - 1\right) + n\left[\frac{1}{6}\left(\left\lfloor \frac{n}{4} \right\rfloor - 1\right)\left(\left\lfloor \frac{n}{4} \right\rfloor - 2\right)\right] & \text{when } n \equiv 3 (mod\ 4), n \neq 3, 15, \\ 1 & \text{when } n = 3, \\ 50 & \text{when } n = 15. \end{cases}$$

**Proof:** Let $C_n$ be a cycle with vertex set $V = \{v_1, v_2, \ldots, v_n\}$.

Now the number of total unidominating functions with minimum is found in the following four cases.

**Case 1:** Let $n \equiv 0 (mod\ 4)$.

Let $f$ be the total unidominating function defined in Case 1 of Theorem 3.1. Then the functional values of $f$ are $01100110 --- 0110$ circularly.

Take $a - 0110$. Then the functional values of $f$ are in the pattern of $aa \ldots a$ circularly (here there are $\frac{n}{4}$ $a's$). These letters can be arranged circularly in one and only one way. Therefore there is only one total unidominating function with minimum weight $\left\lceil \frac{n}{2} \right\rceil$.

By rotating the functional values of $f$ circularly we get the possible total unidominating functions. Here for three circular rotations we get three other such total unidominating functions respectively and the fourth circular rotation coincides with the given total unidominating function $f$.

Therefore there are four total unidominating functions with minimum weight.

**Case 2:** Let $n \equiv 1 (mod\ 4)$.

Let $f$ be the total unidominating function given in Case 2 of Theorem 3.1. Then the functional values of $f$ are $01100110 --- 011001110$ circularly.

By taking $a - 0110$, $b - 01110$, the functional values of $f$ are in the pattern of $aa \ldots ab$ circularly (here there are $\frac{n-5}{4}$ $a's$ ). We see that these $\frac{n-5}{4}$ $a's$ and one $b$ can be arranged circularly in one and only one way.

Therefore there is only one total unidominating function with minimum weight $\left\lceil \frac{n}{2} \right\rceil$.

By rotating the functional values of $f$, here we get $n$ such total unidominating functions.

**Case 3:** Let $n \equiv 2 (mod\ 4)$.

Let $f$ be the total unidominating function given in Case 3 of Theorem 3.1. Then the functional values of $f$ are $01100110 --- 0110011110$ circularly.

By taking $a - 0110$, $c - 011110$, the functional values of $f$ are in the pattern of $aa \ldots ac$ circularly (here there are $\frac{n-6}{4}$ $a's$ ). As in similar lines of Case 2, it can be seen that there is only one total unidominating function with minimum weight $\left\lceil \frac{n}{2} \right\rceil + 1$. By rotating the functional values of $f$ circularly as in Case 2 we get $n$ total unidominating functions with minimum weight.

For any total unidominating function $f$ of $C_n$ at least two consecutive vertices and at most five consecutive vertices can have functional value 1. Based on this point we find all possible total unidominating functions with minimum weight. We further investigate some more total unidominating functions of $C_n$ of same weight in the following way.

Define a function $f_1 : V \to \{0,1\}$ by

$f_1(v_i) = f(v_i) \forall v_i \in V$ for $i \neq n-4, n-6$

and $f_1(v_{n-4}) = 0, f_1(v_{n-6}) = 1$, $n \geq 10$.

It can be verified that this function is a total unidominating function and

$$\sum_{u \in V} f_1(u) = \frac{n}{2} + 1.$$

The functional values of $f_1$ are $01100110 --- 0111001110$ circularly.

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Take $a-$ 0110, $b-$ 01110. Then the functional values of $f_1$ are in the pattern of $aa\ldots abb$ circularly (here there are $\frac{n-10}{4}$ $a's$). Therefore these $\frac{(n-10)}{4}$ $a's$ and two $b's$ can be arranged circularly in $\frac{1}{2}\left(\frac{n-10}{4}+2\right)=\frac{n-2}{8}=\left\lfloor\frac{n}{8}\right\rfloor$ ways if $\frac{n-2}{4}$ is even and if $\frac{n-2}{4}$ is odd then the number of arrangements are

$\frac{1}{2}\left(\frac{n-2}{4}-1\right)=\frac{n-6}{8}=\left\lfloor\frac{n}{8}\right\rfloor$. Therefore there are $1+\left\lfloor\frac{n}{8}\right\rfloor$ total unidominating functions with minimum weight $\frac{n}{2}+1$.

Therefore there are $1+\left\lfloor\frac{n}{8}\right\rfloor$ total unidominating functions with minimum weight $\frac{n}{2}+1$.

By rotating the functional values of $f_1$ circularly, we get $n$ total unidominating functions when $n \neq 10$.

If $n=10$ then the functional values of $f_1$ are 0111001110 circularly so that after four circular rotations the same function $f_1$ is repeated. Hence when $n=10$ there are five total unidominating functions with minimum weight.

Therefore the number of total unidominating functions with minimum weight $\frac{n}{2}+1$ is

$n\left[1+\left\lfloor\frac{n}{8}\right\rfloor\right]$ When $n \neq 10$ and $10+5=15$ when $n=10$.

**Case 4:** Let $n \equiv 3(mod\ 4)$.

Let $f$ be the total unidominating function given in Case 4 of Theorem 3.1. Then the functional values of $f$ are $01100110---01100111110$ circularly.

By taking $a-$ 0110, $d-$ 0111110, the functional values of $f$ are in the pattern of $aa\ldots ad$ circularly (here there are $\frac{n-7}{4}$ $a's$). We see that these $\frac{n-7}{4}$ $a's$ and one $d$ can be arranged circularly in only one way. Then as in Case 3, there are $n$ total unidominating functions after rotating these functional values.

Define another function $f_1 : V \to \{0,1\}$ by

$f_1(v_i)=f(v_i)\ \forall v_i \in V\ for\ i \neq n-5, n-7$

and $f_1(v_{n-7})=1, f_1(v_{n-5})=0, n \geq 11$.

We can easily verify that $f_1$ is a total unidominating function.

Now

$$\sum_{u\in V} f_1(u)=\underbrace{0+1+1+0}+\cdots+\underbrace{0+1+1+0}$$
$$+\underbrace{0+1+1+1+0}+\underbrace{0+1+1+1+1+0}$$
$$=\frac{2(n-11)}{4}+7=\frac{n+3}{2}=\left\lceil\frac{n}{2}\right\rceil+1.$$

The functional values of $f_1$ are $01100110---011001110011110$.

Take $a-$ 0110, $b-$ 01110, $c-$ 011110. Then the functional values of $f_1$ are in the pattern of $aa\ldots abc$ circularly (here there are $\frac{n-11}{4}$ $a's$). As there are are $\frac{(n-11)}{4}+1+1=\frac{n-3}{4}$ letters $a's, b's$ and $c's$, they can be arranged circularly in $\frac{\left(\frac{n-3}{4}-1\right)!}{\left(\frac{n-11}{4}\right)!}=\frac{\left(\frac{n-7}{4}\right)!}{\left(\frac{n-11}{4}\right)!}=\frac{n-7}{4}=\left\lfloor\frac{n}{4}\right\rfloor-1$ ways. Therefore there exists $\left\lfloor\frac{n}{4}\right\rfloor-1$ total unidominating functions. By the rotation of functional values of $f_1$ there exists $n\left(\left\lfloor\frac{n}{4}\right\rfloor-1\right)$ total unidominating functions with minimum weight.

Define another function $f_2 : V \to \{0,1\}$ by

$f_2(v_i)=f_1(v_i)\forall v_i \in V$ for $i \neq n-11, n-9, n-6, n-4$,

and $f_2(v_{n-4})=0, f_2(v_{n-6})=1, f_2(v_{n-9})=0$,

$f_2(v_{n-11})=1,\ n \geq 15$.

We can see that $f_2$ is a total unidominating function.

Now

$$\sum_{u\in V} f_2(u)=\underbrace{0+1+1+0}+\cdots+\underbrace{0+1+1+0}+\underbrace{0+1+1+1+0}$$
$$+\underbrace{0+1+1+1+0}+\underbrace{0+1+1+1+0}$$
$$=\frac{2(n-15)}{4}+9=\frac{n+3}{2}=\left\lceil\frac{n}{2}\right\rceil+1.$$

The functional values of $f_2$ are $0110---0110011100111001110$ circularly.

Take $a-$ 0110, $b-$ 01110. Then the functional values of $f_2$ are in the pattern of $aa\ldots abbb$ (here there are $\frac{n-15}{4}$ $a'$sand 3 $b's$). These $\frac{n-15}{4}+3=\frac{n-3}{4}$ letters can be arranged in

$\left[\frac{\left(\frac{n-7}{4}\right)!}{\left(\frac{n-15}{4}\right)!3!}\right]=\left[\frac{1}{6}\left(\frac{n-7}{4}\right)\left(\frac{n-11}{4}\right)\right]=\left[\frac{1}{6}\left(\left\lfloor\frac{n}{4}\right\rfloor-1\right)\left(\left\lfloor\frac{n}{4}\right\rfloor-2\right)\right]$.

By the rotation of functional values of $f_2$ there exists $n\left[\frac{1}{6}\left(\left\lfloor\frac{n}{4}\right\rfloor-1\right)\left(\left\lfloor\frac{n}{4}\right\rfloor-2\right)\right]$ functions when $n \neq 15$.

Thus there are $n+n\left(\left\lfloor\frac{n}{4}\right\rfloor-1\right)+n\left[\frac{1}{6}\left(\left\lfloor\frac{n}{4}\right\rfloor-1\right)\left(\left\lfloor\frac{n}{4}\right\rfloor-2\right)\right]$ total unidominating functions with minimum weight $\left\lceil\frac{n}{2}\right\rceil+1$ when $n \neq 15$.

If $n=15$ then as in as in Case 3 there will be only five total unidominating functions from the functional values pattern of $f_2$. Therefore when $n=15$ the number of total unidominating functions with minimum weight is

$15+15\left(\left\lfloor\frac{15}{4}\right\rfloor-1\right)+5\left[\frac{1}{6}\left(\left\lfloor\frac{15}{4}\right\rfloor-1\right)\left(\left\lfloor\frac{15}{4}\right\rfloor-2\right)\right]$

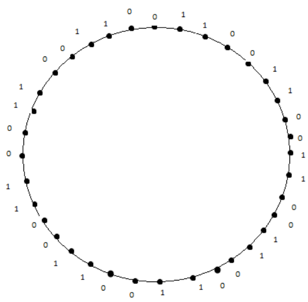
$=15+30+5=50.$ ∎

## IV. ILLUSTRATIONS

**Example 4.1:** Let $n=36$.

Clearly $36 \equiv 0(mod\ 4)$.

The functional values of a total unidominating function $f$ defined in Case 1 of Theorem 3.1 are given at the corresponding vertices of $C_{36}$.



Total unidomination number of $C_{36}$ is $\gamma_{tu}(C_{36})=\left\lceil\frac{36}{2}\right\rceil=18$.

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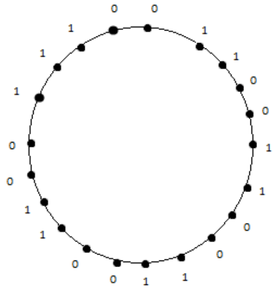
There are four total unidominating functions with minimum weight $18$. ■

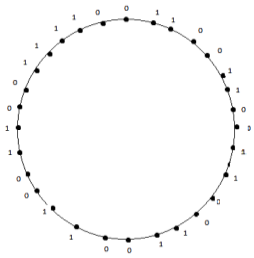
**Example 4.2:** Let $n = 21$.
Obviously $21 \equiv 1 (mod\ 4)$.
The functional values of a total unidominating function $f$ defined in Case 2 of Theorem 3.1 are given at the corresponding vertices of $C_{21}$.



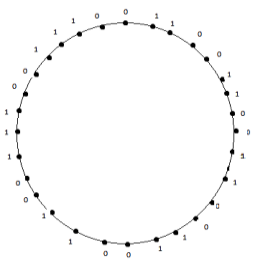
Total unidomination number of $C_{21}$ is $\gamma_{tu}(C_{21}) = \left\lceil \frac{21}{2} \right\rceil = 11$.

There are 21 total unidominating functions with minimum weight $11$. ■

**Example 4.3:** Let $n = 30$.
Clearly $30 \equiv 2 (mod\ 4)$.
The functional values of two total unidominating functions $f$ and $f_1$ in which $f$ is defined in Case 3 of Theorem 3.1 and $f_1$ is defined in Case 3 of Theorem 3.2 are given at the corresponding vertices of $C_{30}$.



$f$

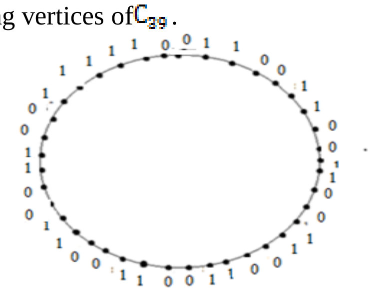


$f_1$

Total unidomination number of $C_{30}$ is $\gamma_{tu}(C_{30}) = \left\lceil \frac{30}{2} \right\rceil + 1 = 16$.

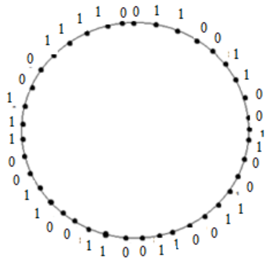
There are $30\left(1 + \left\lfloor \frac{30}{8} \right\rfloor\right) = 30(1 + 3) = 120$ total unidominating functions with minimum weight $16$. ■

**Example 4.4:** Let $n = 39$.
Clearly $39 \equiv 3 (mod\ 4)$.
The functional values of a total unidominating function defined in Case 4 of Theorem 3.1 are given at the corresponding vertices of $C_{39}$.

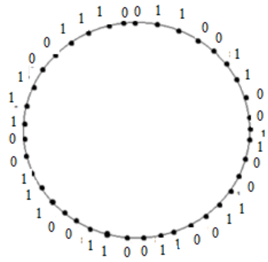


Total unidomination number of $C_{39}$ is $\gamma_{tu}(C_{39}) = \left\lceil \frac{39}{2} \right\rceil + 1 = 21$.

The functional values of another total unidominating functions $f_1$ and $f_2$ with minimum weight 21, defined in Case 4 of Theorem 3.2 are given at the corresponding vertices.



$f_1$



$f_2$

There are $39 + 39\left(\left\lceil \frac{39}{4} \right\rceil - 1\right) + (39 \times 10) = 741$ total unidominating functions with minimum weight $21$. ■

## V. CONCLUSION

The new concepts of unidominating functions and total unidominating functions may play a vital role in the theory of domination. It is interesting to discuss this aspect for cycles and investigating the total unidomination number of a cycle.

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