

# Optimal control law for the concentration of carbon dioxide in a tomato greenhouse and optimal-tuning PI control as LQR for the automatic device

J. E. Moisés Gutiérrez-Arias, Nubia Iliá Ponce de León-Puig, Gabriela Pérez-Osorio, Eladio Flores-Mena, Josefina Castañeda-Camacho.

Facultad de Ciencias de la Electrónica, DFacultad de Ciencias Químicas.

Benemérita Universidad Autónoma de Puebla, Av. San Claudio y 18 sur Col. San Manuel, C.P. 72570, Puebla, México

**Abstract**— *Tomato model-space has been developed and it is called a big leaf-big fruit model, this model is formed by the mass balances: the non-structural biomass (nutrients) and the structural biomass of fruits and leaves. Also, a model that describes the behavior on carbon dioxide concentration inside greenhouse is obtained. From the two models we get a new complete crop-greenhouse model. This model allows getting an optimal control for the carbon dioxide enrichment in a tomato greenhouse which gives benefits, because it is possible to achieve a saving for energy consumption and more tomato production. The optimal control theory is applied to the crop-greenhouse integrated system, which is based on four state variables: the consumption of nutrients, the fruits and leaves growth and the carbon dioxide concentration. This work contributes with the optimal control law that gives the desired CO<sub>2</sub> concentration behavior during the growth time for the crop. This behavior will be a reference signal for the controller implementation in the electronic device that will be made in a future work. The simulations for the crop-greenhouse system are presented for a two weeks period. This paper also contributes at optimal-tuning PI control as Linear Quadratic Regulator of the electronic device. The simulation of the optimal control PI are showed here.*

**Index Terms**— **Optimal control, Optimal-tuning, Linear Quadratic Regulator, Carbon dioxide, structural biomass of leaves, structural biomass of fruit, state space.**

## I. INTRODUCTION

In past years, researches have proposed different optimal climate control methods for greenhouse systems. These efforts have not been applied in practice because it is difficult for real application [1], [2], [3], [9], [12]. The difficulty lies in that the crop growth is based in many different variables and the mathematical analysis with all variables is complicated [7], [10]. In this work, the variable that has our interest is the carbon dioxide. The carbon dioxide enrichment is practiced in the greenhouse crops in order to increase the yield and the benefit. There are studies that demonstrate the CO<sub>2</sub> enrichment improves the net photosynthesis in the plants achieving the increase of the total weight, height, and the number of leaves and branches [9]. Other research has

demonstrated that the CO<sub>2</sub> enrichment makes physic-chemical changes in the crop, like color and firmness [8]. Optimization problems with two or more objectives are very common in engineering and many other disciplines. The process of optimizing a collection of objective function is called multi-objective optimization and it is difficult because of the large number of conditions and variables involved in the system [4]. In this work the optimization problem has two objectives, first, decrease the energy consumption for carbon dioxide enrichment and second, increase the tomato production. The search process can be accomplished in two ways: deterministic and stochastic search algorithms [6]. Optimal strategies for CO<sub>2</sub> enrichment can be deduced experimentally or analytically [11]. Experimentation is not able to produce a valid result for all condition set. Mathematical analysis gives us a better option to obtain an optimal strategy because it considers all the variables involved in the system. This method is based on ventilation, photosynthesis, dry matter and production rate models. One of the main objectives is to contribute with the optimal control problem and its implementation in real time. The tomato crop has been chosen because it is one of the most important crop in our country and is the second farm product consumed in the world. To achieve the objective, we start from the tomato and greenhouse mathematical set model considering the following variable: plant and fruit dry weight, the nutrients amount and the CO<sub>2</sub> concentration. In this paper, we obtained the behavior of CO<sub>2</sub> concentration inside the greenhouse that is necessary for growth during two weeks. We start from the fact that this behavior is a reference signal that a classic controller will have to follow for each time instant. In future work, all results in this paper help us to design an electronic device that could be used in real tomato greenhouses. With the approach of optimal PI tuning for first order processes, also the provision of simultaneously and optimally finding the two parameters of a PI controller (i. e. K<sub>i</sub>, K<sub>p</sub>), development has been addressed in this paper.

## II. GENERAL FORMULATION OF THE OPTIMAL CONTROL PROBLEM

Optimal control problems appeared as essential tools in modern control theory. Several authors have proposed different basic mathematical formulations of fixed time problems [5]. The optimal control of any system has to be based on three concepts: the dynamic model of the system, a functional and the system restrictions. In matrix notation the state equation is represented as follows:

$$\dot{x} = f(x(t), u(t), t) \quad (1)$$

Where  $x(t)$  is the state vector,  $u(t)$  is the control vector and  $t$  is the time. A criterion is required to evaluate the performance of the system, normally, the functional is defined by:

$$J = \Phi(x(t_f), t_f) + \int_{t_0}^{t_f} L(x(t), u(t), t) dt \quad (2)$$

Where  $t_0$  and  $t_f$  are the initial and final time,  $\Phi$  and  $L$  are scalar functions,  $t_f$  can be fixed or free. Starting at the initial state

$x(t_0) = x_0$  and applying the control signal  $u(t)$  for  $t \in [t_0, t_f]$  it makes that system follows some trajectory of states, then the functional assigns a unique real number for each trajectory of the system. The fundamental problem of optimal control is to determine an admissible control  $u^*$  which makes that (1) follows one admissible trajectory  $x^*$  that minimize the value of the functional in (2). Then,  $u^*$  is named optimal control and  $x^*$  is an optimal trajectory.

### A. Necessary Conditions for a Solution

Restriction (1) are added to the functional (2) with a Lagrange multipliers vector time variant  $\Psi(t)$  and the functional is rewritten as follows:

$$J = \Phi(x(t_f)) + \int_{t_0}^{t_f} [L(x(t), u(t), t) - \Psi^T f(x(t), u(t), t) - \dot{x}] dt \quad (3)$$

The Hamiltonian scalar function is defined and depends on the variable state vector, the control signal and the new vector  $\Psi(t)$ .

$$H = (x(t), u(t), \Psi(t), t) = L(x(t), u(t), t) + \Psi(t) f(x(t), u(t), t) \quad (4)$$

Then, it is possible to write an auxiliary system starting form a new auxiliary vector that depends on time  $\Psi(t)$ . The new system is formed form Hamiltonian function as follow:

$$\dot{\Psi}^T = -\frac{\partial H}{\partial x} = -\frac{\partial L}{\partial x} - \Psi^T \frac{\partial f}{\partial x} \quad (5)$$

The auxiliary system allows to know the final conditions of the general system, which can be written as follows:

$$\Psi^T(t_f) = \frac{\partial \Phi}{\partial x}(t_f) \quad (6)$$

An infinitesimal variation in  $u(t)$  denominated  $\delta u(t)$  produces a variation in the functional  $J$  like  $\delta J$ . For a stationary solution it is required that arbitrary variation is equal to zero,  $\delta J = 0$ . This is true when

$$\frac{\partial H}{\partial u} = \frac{\partial L}{\partial u} + \Psi^T \frac{\partial f}{\partial u} = 0 \quad (7)$$

From the Hamiltonian function (4) it is possible to get the control form. Then, to find the vector function of control  $u(t)$  that produces a stationary value of the functional we must to solve the following differential equation system:

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), t) \\ \dot{\Psi}(t) &= -\frac{\partial H^T}{\partial x} \end{aligned} \quad (8)$$

The boundary conditions for this differential equations are separated, it means that some of them are defined in  $t=t_0$  and the others in  $t=t_f$ . This is a problem with boundary values of two points. Note the equations that describe the states  $x(t)$  and the auxiliary states  $\Psi(t)$  in (8) are coupled. For this reason  $u(t)$  depends on  $\Psi(t)$  through the stationary condition and the auxiliary states depend on  $x(t)$  and  $u(t)$ . The first system in (8) has the initial conditions of the system while the last system in (8) has the final condition of the system.

## III. CROP AND GREENHOUSE DYNAMIC MODELS

### A. Dynamic model of the Crop

The model in space states of the tomato crop has three principles states (van Straten et al., 2011) [13]:

- Non-structural Biomass (Nutrients).
- Leaves Structural Biomass.
- Fruit Structural Biomass

#### 1) Biomass Balance of nutrients

Nutrients are being produced by photosynthesis. The gross canopy photosynthesis rate in dry matter per unit area is  $P$ . Nutrients are converted to leaf and fruits, this is known as growth. Leaf and fruits have a demand for nutrients, which will be honored if there are sufficient nutrients available. We denote  $W_B$  as the total nutrients in the plant and it is expressed as dry weight per area unit. The biomass balance equation of nutrients is the following [13]:

$$\frac{dW_B}{dt} = P - h\{\cdot\} \left( \frac{(1 + \theta_V)}{z} G_L^{dism} - (1 + \theta_F) G_F^{dism} \right) - h\{\cdot\} \left( \frac{R_L}{z} + R_F \right) \quad (9)$$

The biomass balance equation of nutrients (9) can take two values depending on the nutrients abundance  $h\{\cdot\}$ , where the first expression is taken when  $h\{\cdot\}=1$  (abundance of nutrients) and the second one is taken when  $h\{\cdot\}=0$  (lack of nutrients).

$$\frac{dW_B}{dt} = P - \frac{(1 + \theta_V)}{z} G_L^{dism} - (1 + \theta_F) G_F^{dism} - \frac{R_L}{z} - R_F, \quad (10)$$

$R_F$  - Respiration needs of fruits.

$R_L$  - Respiration needs of leaves.

$\theta_V$  - Additional amount of nutrients needs for one of unit of structural vegetative parts.

$G_L^{dism}$  - Unit area growth demand of leaves.

$\theta_F$  - Additional amount of nutrients needs for one unit of structural fruit parts.

$G_F^{dem}$  - Unit area growth demand of fruit.

$z$  - Total vegetative parts.

$h\{\cdot\}$  - Nutrients abundance.

### 2) Biomass balance of leaves

The leaf growth is equal to the amount of nutrients converted to structural leaf biomass in the plant and it is given by  $h\{\cdot\}G_L^{dem}$ . The model does not incorporate an extra state for stem and roots, but the factor  $z$  assumes that each increment in leaf will be accompanied by an increment in stem and roots. If there are no sufficient assimilates (nutrients), growth stops, normally the nutrients are used for the maintenance, but in lack of nutrients, maintenance in the model goes at the expense of structural parts (leaves and fruit). The biomass balance of leaves is expressed in the form [13]:

$$\frac{dW_L}{dt} = h\{\cdot\}G_L^{dem} - (1 - h\{\cdot\})R_L - H_L \quad (11)$$

Depending on the abundance of nutrients  $h\{\cdot\}$ , the biomass leaf balance equation (11) can take two values:

$$\frac{dW_L}{dt} = \begin{matrix} G_L^{dem} - H_L \\ -R_L - H_L \end{matrix} \quad (12)$$

Where

$H_L$  is the leaf picking rate.

### 3) Biomass balance of fruit

Similarly to the biomass of leaf case, the growth of fruits in the plant from the nutrients is given by  $h\{\cdot\}G_F^{dem}$ . The term  $G_F^{dem}$  depends principally on the pivotal temperature, cultivation temperature level and the reference temperature [13].

$$\frac{dW_F}{dt} = h\{\cdot\}G_F^{dem} - (1 - h\{\cdot\})R_F - H_F \quad (13)$$

Finally, (13) can take two different values depending on nutrients abundance  $h\{\cdot\}$ , where  $H_F$  is the fruit harvest rate:

$$\frac{dW_L}{dt} = \begin{matrix} G_F^{dem} - H_F \\ -R_F - H_F \end{matrix} \quad (14)$$

## B. Dynamic Model of the Greenhouse

### 1) Balance of CO2 energy in the greenhouse

The balance of carbon dioxide energy within greenhouse is given the equation [13]:

$$\frac{V_g}{A_g} \frac{dC_{CO_2}}{dt} = -\eta CO_2 \frac{P}{\Delta w} + \eta CO_2 \frac{R}{\Delta w} - \varphi_{CO_2, g_0}^{vent} + uCO_2 \quad (15)$$

Then each term is described.

- Carbon dioxide returned to the greenhouse air for plan photosynthesis:

$$\eta CO_2 \frac{P}{\Delta w}$$

- Carbon dioxide returned to the greenhouse air for plan photosynthesis:

$$\eta CO_2 \frac{R}{\Delta w}$$

The term  $\frac{V_g}{A_g}$  is the reason of the volume of greenhouse per unit area.

$R$  is the total respiration plant per unit of time.

- Lost of carbon dioxide mass by ventilation:

$$\varphi_{CO_2, g_0}^{vent} = uv (C_{CO_2} - C_{CO_2, 0})$$

Where  $uv$  is the ventilation flow rate per unit of area.

$C_{CO_2}$  ( $kgm^{-3}$ ) is the carbon dioxide concentration within greenhouse.

$C_{CO_2, 0}$  ( $kgm^{-3}$ ) is the carbon dioxide concentration on the outside greenhouse.

- Carbon dioxide supply:

$$uCO_2 = u_{CO_2}^{vp} \varphi_{CO_2, in, g_0}^{max}$$

$u_{CO_2}^{vp}$  is the opening supply valve and.

$\varphi_{CO_2, in, g_0}^{max}$  is the maximum flow rate of carbon dioxide.

In this greenhouse model, the position of the carbon dioxide supply valve is the control input. For this reason, the valve relates directly to the actuator that is present on a physical way in the greenhouse.

### C. Integrated Model Crop-Greenhouse

From previous description of greenhouse and crop models is possible to get a complete system formed by three crop equations and greenhouse equation. This new equation system describes the complete system behavior and it is important to note that all of the equations are related principally by the P element and the state variables of the crop. It is important to say that the three equations related to the crop are taken with the assumption that there is an abundance of nutrients ( $h\{\cdot\}=1$ ). Therefore, general system is as follows:

$$\begin{aligned} \dot{W}_L(t) &= G_L^{dem} - H_L \\ \dot{W}_F(t) &= G_F^{dem} - H_F \\ \dot{W}_B(t) &= P - \frac{(1 + \theta_V)}{z} G_L^{dem} - (1 + \theta_F) G_F^{dem} - \frac{R_L}{z} - R_F \\ \frac{V_g}{A_g} \frac{dC_{CO_2}}{dt} &= -\eta CO_2 \frac{P}{\Delta w} + \eta CO_2 \frac{R}{\Delta w} - \varphi_{CO_2, g_0}^{vent} + uCO_2 \end{aligned} \quad (16)$$

System (16) is the representation of the complete differential equation system which is related to each other by the variables crop and greenhouse. For the next step the system will be solved using MatLab to know the behavior of all variables with initial and final conditions known.

IV. SYNTHESIS OF OPTIMAL CONTROL

We consider the system (16). The terms for the equation system are substituted using the equation table of the mathematical model (Table I) and the values are substituted using the table of physical parameters (Table II). The resulting model is:

$$\begin{aligned} \dot{W}_L &= 2.29996 \times 10^{-6} W_L(t) \\ \dot{W}_F &= 4.3925 \times 10^{-6} W_F(t) \\ \dot{W}_B(t) &= P(t) - 5.39 \times 10^{-6} W_L(t) - 5.92 \times 10^{-6} W_F(t) \\ 3C_{CO_2}(t) &= 1.0266(R(t) - P(t)) + 0.1555 \times 10^{-10} u_{CO_2} \end{aligned} \quad (17)$$

P and R are the following:

$$P(t) = \frac{3.7192 \times 10^{-11} W_L^{2.511}(t)}{1.6353 \times 10^{-9} + 4.0439 \times 10^{-5} W_L^{2.511}(t)}$$

$$R(t) = 1.5942 \times 10^{-6} W_F(t) + 0.4856 \times 10^{-6} W_L(t)$$

A. Solution Method Description

The Hamiltonian scalar function is obtained considering the relation (18) with the Lagrange multipliers and the functional (17).

$$H = (x, u, \Psi, t) =$$

$$\begin{aligned} &\frac{1}{2} [W_L^2(t_f) + W_F^2(t_f) + W_B^2(t_f) + C_{CO_2}(t_f) + (u_{CO_2}^{vp})^2(t)] \\ &+ 2.29996 \times 10^{-6} W_L(t) \Psi_1(t) + 4.3925 \times 10^{-6} W_F(t) \Psi_2(t) + \\ &[P - 5.39 \times 10^{-6} W_L(t) - 5.92 \times 10^{-6} W_F(t)] \Psi_3(t) + \\ &\frac{1}{3} [1.0266(R - P) + 1.1554 \times 10^{-10} (u_{CO_2}^{vp})] \Psi_4(t) \end{aligned} \quad (19)$$

The system of auxiliary variables is formed using (5) and has the following form:

$$\begin{aligned} \dot{\Psi}_1 &= W_L + 2.2996 \times 10^{-6} \Psi_1 + \frac{\partial P}{\partial W_L} \Psi_3 - \\ &- 5.39 \times 10^{-6} \Psi_3 + \frac{1}{3} \frac{\partial (R - P)}{\partial W_L} \Psi_4 (1.0266) \\ \dot{\Psi}_2 &= W_F + 4.3925 \times 10^{-6} \Psi_2 - 5.92 \times 10^{-6} \Psi_3 + \\ &\frac{1}{3} \frac{\partial R}{\partial W_F} \Psi_4 (1.0266) \\ \dot{\Psi}_3 &= W_B \\ \dot{\Psi}_4 &= C_{CO_2} \end{aligned} \quad (20)$$

The stationary condition give us the following control form, which was obtained from (7) and depends on fourth appended state:

$$u_{CO_2}^{vp} = -\frac{1}{3} 0.1554 \times 10^{-10} \Psi_4(t) \quad (21)$$

It is necessary to solve the equation systems (17) and (20), in this way we can know the  $\Psi_4$  value and finally we will get the control form. The system (17) has initial condition and the system (20) has final conditions. The systems are coupled because the control form (21) has been substituted. To solve the complete system like a system with initial conditions, the auxiliary equations are considered in reverse time, then the behavior of the auxiliary variables is returned to the direct time. When we solve the appended equation system in reverse time the system becomes a system with initial conditions. It is important to note that (21) depends on the fourth state but this state depends on the other three states. Using MatLab tools we solve the equation systems (17) and (20).

$$+1.668 \times 10^{-7}$$

It is important to note that the terms P(t) and R(t) have involved two of the three state variables of the crop and they are time dependent functions, so the entire system is connected and it can be solved simultaneously.

We consider the following functional, which has the same form shown in (3)

$$J = 1/2[W_L^2(t_f) + W_F^2(t_f) + W_B^2(t_f) + C_{CO_2}(t_f) + \int_{t_0}^{t_f} W_L^2(t_f) + W_F^2(t_f) + W_B^2(t_f) + C_{CO_2}(t_f) + (u_{CO_2}^{vp})^2(t)] dt \quad (18)$$

The first term involves the three first variables at the end time, they are related to the final production and the nutrients, and the integral contains the control input in order to avoid the risk for big control inputs. The idea is to minimize the functional (18) related with the equations system (17).

Table 1 Greenhouse and crop mathematical model equations.

Term	Description
$P = P^{max} \left( \frac{I^{PAR}}{I^{PAR} + K_I} \right) \left( \frac{C_{CO_2}}{C_{CO_2} + K_C} \right) fm\{\cdot\}$	Production of assimilates by photosynthesis.
$R = h\{\cdot\} \left( \frac{\theta_V}{z} G_L^{dem} + \theta_F G_F^{dem} \right) + \frac{R_L}{z} + R_F$	Total amount breathed plant per unit of time.
$I^{PAR} = f_{PAR/I} \tau_r I_0$	The PAR light intensity at the crop level.
$fm\{\cdot\} = \frac{(w_L/p_m)^m}{1 + (w_L/p_m)^m}$	Maturity factor.
$G_L^{dem} = f_L(T) k_{GF}^{ref} f_{TG}(T) f_D\{\cdot\} W_L$	Growth leaves demand.
$G_F^{dem} = k_{GF}^{ref} f_{TG}(T) f_D\{\cdot\} W_F$	Growth fruit demand.
$f_L(T) = f_{L/F}^{ref} e^{v_2 \left( \frac{T - T_L^{ref}}{F} \right)}$	Temperature dependent ratio.
$f_{TG}(T) = Q_{10R}^{\left( \frac{T - T_L^{ref}}{F} \right)}$	Temperature dependent with Q <sub>10G</sub> relation.
$f_{TR}(T) = Q_{10R}^{\left( \frac{T - T_L^{ref}}{F} \right)}$	Function of temperature with Q <sub>10G</sub> relation.
$f_D\{\cdot\} = \frac{c_{f1} - c_{f2} D}{c_{f1} - c_{f2}}$	Correction factor for the fruit growth rate.
$R_L = k_{RL}^{ref} f_{TR}(T) W_L$	Respiration demand of leaves.
$R_F = k_{RF}^{ref} f_{TR}(T) W_F$	Respiration demand of fruits.
$H_L = k_{HL} W_L$	Leaf picking rate.
$H_F = k_{HF} W_F$	Harvest rate.
$k_{HL} = C_{yL} k_H$	Coefficient of harvest.
$k_{HF} = C_{yF} k_H$	Coefficient of harvest.
$k_H = Cd1 + Cd2 \ln \left( \frac{T}{Cd3} \right) - Cd3 - Cd4 e_D$	Harvest rate.
$uv = \left( \frac{pv1 u_v^{Aplsd}}{1 + pv2 u_v^{Aplsd}} + pv3 + pv4 u_v^{Apwsd} \right) u + pv5$	Ventilation flow rate.

**V. SIMULATION OF CONTROL LAW**

The MatLab tools were used to elaborate the program that solve the differential equations system formed by (17) and (20). The simulation period is for two weeks. The results obtained are described below. It is important to know that in the following results, the temperature and solar radiation are time varying, in order to make the simulation more real. Figures (1) and (2) show the temperature and solar radiation varying in a time of two weeks.

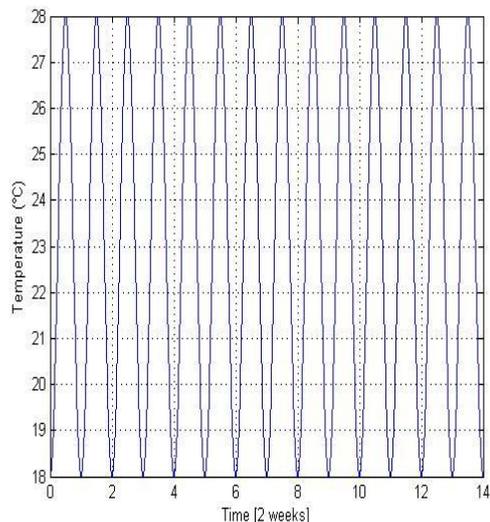


Fig 1. Time varying temperature for two weeks.

Table 2 Physic parameters.

Variable	Value	Description
$z$	0.6081	Fraction leaf of total vegetative mass.
$\theta_V$	0.23	Surplus assimilate requirement factor per unit fruit increment.
$\theta_F$	0.2	Surplus assimilate requirement factor per unit vegetative increment.
$p_h$	$2.7 \times 10^{-3}$	Parameter of switching function [ $m^2 kg^{-1}$ ].
$p_m$	$1.8 \times 10^{-2}$	Parameter in maturity factor.
$m$	2.511	Maximum gross canopy photosynthesis rate [ $kg m^{-2} s^{-1}$ ].
$p^m$	$2.2 \times 10^{-6}$	Monod constant for PAR [ $W m^{-2}$ ].
$K_1$	577	Monod constant for CO2 [ $kg m^{-3}$ ].
$K_C$	0.211	PAR fraction of global radiation.
$f_{PAR/I}$	0.475	Transmittance of the roof.
$\tau_r$	0.7	Reference fruit growth rate coefficient [ $s^{-1}$ ].
$k_{GF}^{ref}$	$3.8 \times 10^{-6}$	Reference temperature [ $^{\circ}C$ ].
$T_{GF}^{ref}$	20	Temperature function parameter growth.
$Q_{10G}$	1.6	Reference leaf-fruit partitioning factor.
$f_{L/f}^{ref}$	1.38	Parameter partitioning fruit-leaf [ $K^{-1}$ ].
$v_2$	0.168	Fruit-leaf partitioning reference temperature [ $^{\circ}C$ ].
$T_{L/F}^{ref}$	19	Maintenance respiration coefficient leaf [ $s^{-1}$ ].
$k_{RL}^{ref}$	$2.9 \times 10^{-7}$	Temperature function parameter respiration.
$Q_{10R}$	2	Reference temperature for respiration.
$T_R^{ref}$	25	Reference temperature for respiration [ $^{\circ}C$ ].
$k_{RF}^{ref}$	$1.2 \times 10^{-7}$	Maintenance respiration coefficient leaf [ $s^{-1}$ ].
$\eta$	0.7	Absorbed in relation to the total energy of the net radiation heat received.
$Cd1$	$2.13 \times 10^{-7}$	Parameter in development rate function [ $s^{-1}$ ].
$Cd2$	$2.47 \times 10^{-7}$	Parameter in development rate function [ $s^{-1}$ ].
$Cd3$	$7.50 \times 10^{-11}$	Parameter in development rate function [ $s^{-1}$ ].
$C_Y L$	1.636	Parameter in harvest function (fruit).
$C_Y F$	0.4805	Parameter in harvest function (leaf).
$C_{CO_2,0}$	1.6637	
$C_{CO_2/dw}$	1.4667	Ratio CO2 per unit dry weight.
$C_{CO_2,tnq}$	$2.10 \times 10^{-6}$	Ratio CO2 per unit dry weight.
$V_g$	3	Volume per unit greenhouse area.
$A_g$		
$pv1$	$7.17 \times 10^{-5}$	Parameter.
$pv2$	0.0156	Parameter.
$pv3$	$2.71 \times 10^{-5}$	Parameter.
$pv4$	$6.32 \times 10^{-5}$	Parameter.
$pv5$	$7.40 \times 10^{-5}$	Parameter.

### A. Analysis with a Step Input

In a first simulation we use a step function as control input ( $u_{CO_2}^{sp} = 1$ ). We solved the equation system and later we got the graphics that represent the behavior of most important variables of the system. Figure (3) shows the behavior of three state variables. The red line (x) shows the fruit behavior which has acceptable growth just like the leaves, which is the green line (o). The black line (+) represents the nutrients behavior which is acceptable because it is observed that the crop is consuming the available nutrients during the growth time.

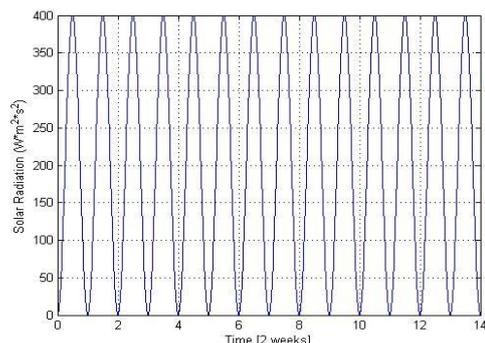


Fig 2. Time varying solar radiation for two weeks.

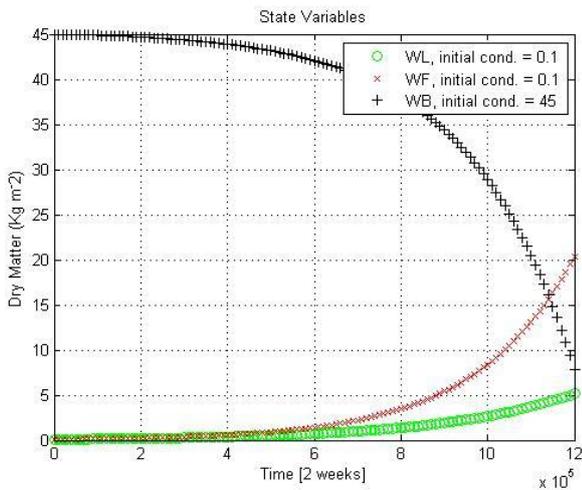


Fig 3. Behavior of fruits leaves and nutrients dry matter with an input step.

Figure (4) shows the carbon dioxide behavior. Note the CO<sub>2</sub> concentration increases up to 9000 ppm. This is very high and it means so much energy consumption.

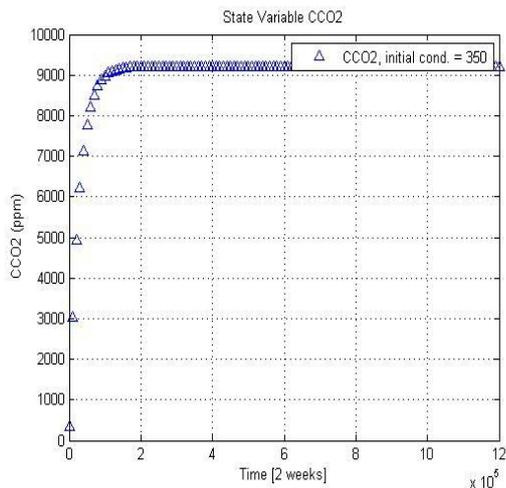


Fig 4. Behavior of CO<sub>2</sub> concentration with a step as input control.

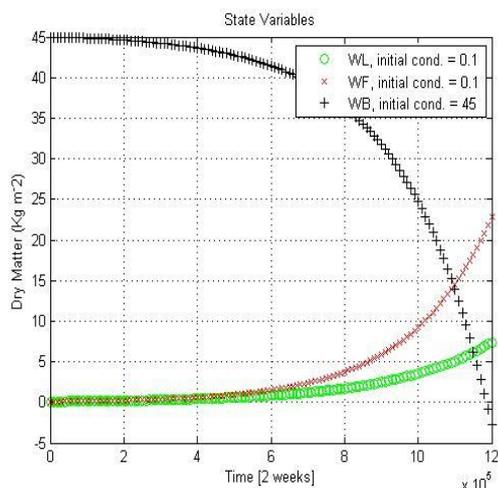


Fig 5. Behavior of fruits, leaves and nutrients dry matter with the control law deduced.

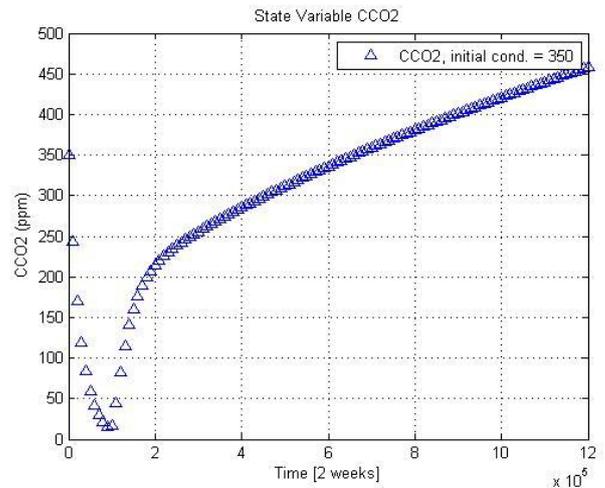


Fig 6. Behavior of CO<sub>2</sub> concentration with the control law deduced.

### V. AUTOMATIC CONTROL SYSTEM FOR THE CONCENTRATION OF CARBON DIOXIDE INSIDE THE GREENHOUSE

Figure (7) shows the complete system which will control the carbon dioxide concentration. The valve and supply tank are the pneumatic system. And the CO<sub>2</sub> sensor will make the reading of current carbon dioxide inside de greenhouse.

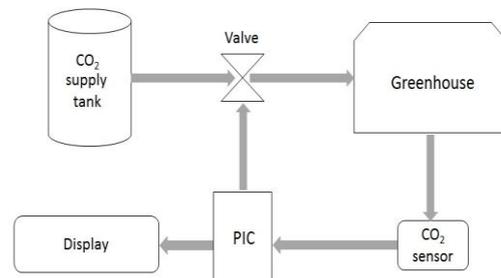


Fig 7. Automatic control system or CO<sub>2</sub> concentration.

#### A. Mathematical Model of Pneumatic Pressure System

The pressure system that will do the carbon dioxide enrichment is formed by a valve and a storage tank, Figure (8). In it, the flux through the restriction is a function of the difference of pressure. This kind of system is characterized in terms of a resistance and a capacitance.

The resistance is defined like the change in the differential pressure to make a change in the mass flux [15]:

$$R = \frac{d(\Delta P)}{dq} \tag{22}$$

Where  $\Delta P$  a small change in the pressure of the gas and  $dq$  is a small change in the flux of the gas.

By the other hand, the capacitance is defined like:

$$C = \frac{dm}{dp} = V \frac{dp}{dp} \tag{23}$$

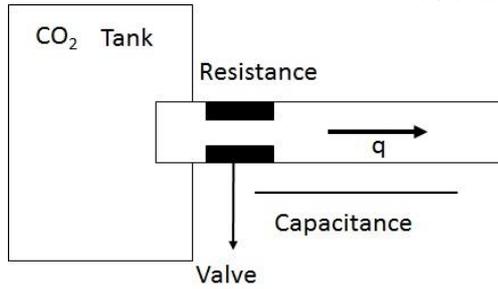


Fig 8. Pressure system with tank and restriction.

Where

$C$ = Capacitance,  $lbft^2/lb$

$m$ =Gas mass in the tank,  $lb$

$p$ =Pressure of gas,  $lb/ft^2$

$V$ = Volume of tank

$\rho$ =Density,  $lb/ft^3$

From the general law of gases, the capacitance is expressed as [15]:

$$p \left( \frac{V}{m} \right)^n = \frac{p}{\rho - n} = \text{constant} \quad (24)$$

Where  $n$  is the polytropic exponent. The expression for ideal gases is:

$$pv = \frac{p}{\rho} = \frac{\bar{R}}{M}T = R_{gas}T \quad (25)$$

Where:

$p$ =Absolute pressure,  $lb/ft^2$

$\bar{v}$ =Volume occupied by one mol of gas,  $ft^3/lbmol$

$\bar{R}$ =Universal constant gas,  $ftlb/lbmolR$

$v$ =Specific volume of the gas,  $ft^3/lb$

$M$ =Molecular weight of the gas,  $lb/lbmol$

$R_{gas}$ =Constant of the gas.

Therefore, the capacitance is obtained as:

$$C = \frac{V}{nR_{gas}T} \quad (26)$$

For the system in this paper, the capacitance was got from (26) using the parameters in Table (III). The calculated value was  $1.3447e^{-3}Kgm^2/N$ .

Table 3 Values to calculate the capacitance of the system.

Description	Value	Units
$V$ - Tank volume	0.17657	$ft^3$
$n$ -Polytropic exponent	1	
$R_{gas}$ - Gas constant	0.243844	$ftlb/lbR$
$\bar{R}$ -Universal constant of gases	10.73158	$ftlb/lbmol$
$M$ - Molecular weight CO <sub>2</sub>	44.01	$g/mol$
$T$ -Absolut temperature	538.47	$R$

The formula to calculate the resistance is

$$R = \frac{8\eta L}{\pi r^4} \quad (27)$$

Where

$\eta$  is the viscosity of the gas,  $N/m^2$ .

$R$  is the inner radius of the pipe in meters.

$L$  is the length of the pipe expressed in meters.

For our particular system we use the values in table (IV) and the result is 1.3402.7370 pneumatic ohmns.

Table 4 Values to calculate the resistance for the system.

Description	Value	Units
Viscosity of CO <sub>2</sub>	$1.3711 e^{-5}$	$N/m^2$
Inner of pipe	0.0127	$m$
Length of pipe	1	$m$

To obtain the mathematical model of the system small variations are considered and the system is considered linear. The following terms are defined:

$\bar{P}$ = Pressure of the gas in stable state,  $lb/ft^2$

$pi$ = small change in the gas that enter  $lb/ft^2$

$po$ =small change in the pressure gas in the tank  $lb/ft^2$

$V$ = volume of the tank  $lb$

$m$ = mass of the gas in the tank

$q$ = flux gas,  $lb/sec$

$\rho$ = density gas,  $lb/ft^3$

The resistance is  $R=(pi-po)/q$  and the capacitance is obtained form (24), in this way:

$$dpo = qdt \quad \text{or} \\ Cpo \frac{pi - po}{R} dt$$

Which is written like  $RC \frac{dpo}{dt} + po = pi$ .

If  $pi$  and  $po$  are the input and output respectively, the transfer function is:

$$\frac{Po(s)}{Pi(s)} = \frac{1}{RCs + 1} \quad (28)$$

And, with the values of resistance and capacitance calculated previously, the transfer function can be written as follows:

$$\frac{Po(s)}{Pi(s)} = \frac{1}{18.022s + 1} \quad (29)$$

Now we will working with the transfer function to get a PI controller which will have an action over the actuator.

## VI. OPTIMAL-TUNNING PI CONTROL AS LINEAR QUADRATIC REGULATOR

From analysis controllability we know the system is completely controllable, it means every control will achieve stabilize the system. But the question is what control is the best to stabilize the system in less time. In this section a comparison between classical tuning for PI controller and optimal control theory for PI controller is made. PID

controllers are most common in process industries due its simplicity. In this research we will work with a PI controller because of the characteristics of the pneumatic system. The optimal control problem is reduced to the Algebraic Riccati Equation (CARE), which is solved to calculate the feedback gains for a set of matrices. These matrices regulate the penalties on the deviation in the trajectories of the state variables and the control signal. Combining the tuning philosophy of PID controllers with the concept of Linear Quadratic Regulators (LQR) allows getting an optimal control for the actuator in our automatic system [16].

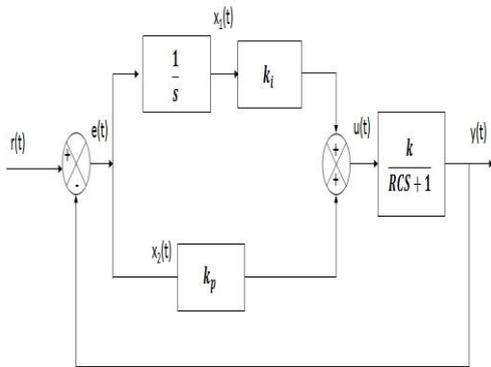


Fig 9. Block diagram of PI controller.

In figure (9) a PI controller in parallel form has been considered to control the system represented in (28). From the general equation for PI controller and supposing the feedback control system is excited with an external input  $r(t)$  to get a control signal  $u(t)$  and output  $y(t)$ , we define the state variables as:

$$x_1 = \int e(t)dt, \quad x_2 = e(t) \tag{30}$$

From the block diagram presented in figure (9) and the transfer function (28), it is clear that

$$\frac{Y(s)}{U(s)} = \frac{k}{RCs + 1} = \frac{-E(s)}{U(s)} \tag{31}$$

And then,

$$(RCs + 1)E(s) = -kU(s) \tag{32}$$

And finally, we can re-write (33) using the selection of state variables in (30).

$$RC \dot{e}(t) + e(t) = -ku(t) \tag{33}$$

State space representation for (34) is:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{x_2}{RC} - \frac{k}{RC} u(t) \end{aligned} \tag{35}$$

Using (35), the state space formulation becomes to:

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ 0 & -\frac{1}{RC} \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ -\frac{k}{RC} \end{pmatrix} u(t) \tag{36}$$

From the development of the transfer function which describes the behavior of the pneumatic system we know the R and C values. And, with the representation in (36) we can analyze the controllability of the system using the controllability matrix. The determinant of matrix controllability is different to zero, the system is completely controllable.

$$|B \ AB| = \begin{vmatrix} 0 & -\frac{1}{18.022} \\ 0 & 1/(18.022)^2 \end{vmatrix} \neq 0 \tag{37}$$

Now, in order to have an optimal formulation with the system (36), the following quadratic function (J) is selected [16],

$$J = \int_0^{\infty} (x^T Q x(t) + u^T(t) R u(t)) dt \tag{38}$$

Minimization of cost function gives the state feedback control law as:

$$u(t) = -R^{-1} B^T P x(t) = -F x(t) \tag{39}$$

Where P is the solution of the Algebraic Riccati Equation given by:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{40}$$

We know the matrix A and B, Q is a symmetric positive semi-definite matrix and R is a positive number. Both Q and R are selected by the designer. The matrix Q is chosen to be as positive semi-definite diagonal matrix. Matrix P and matrix Q have the following form:

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{pmatrix} \quad Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix} \tag{41}$$

The solution for Riccati equation (39) was obtained from care function in MatLab, this function returns the unique solution P of the algebraic Riccati equation; also it returns the gain matrix.

The solution P using the arbitrary values for matrix Q and parameter R is:

$$P = \begin{pmatrix} 6.2485 & 18.022 \\ 18.022 & 94.588 \end{pmatrix} \tag{42}$$

Finally, we can substitute all known parameters in control law equation (38).

Solving the substitution, we got a new matrix which has the proportional and integrate gain for the PI controller.

$$u(t) = (k_i \ k_p) \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \tag{43}$$

Control form can be written in this way:

$$u(t) = k_i \int e(t) dt + k_p e(t) \tag{44}$$

As we know this form is the representation of general PI controller.

$$k_0 = \frac{y_{\infty} - y_0}{u_{\infty} - u_0}, \quad \tau_0 = t_1 - t_0, \quad \gamma_0 = t_2 - t_1$$

Using MatLab a program was made to know the parameters and finally we got the gains for the PI controller using the value table which Ziegler and Nichols proposed.

	$K_p$	$T_r$	$T_d$
P	$\frac{\gamma_0}{K_0 \tau_0}$		
PI	$\frac{0.9 \gamma_0}{K_0 \tau_0}$	$3 \tau_0$	
PID	$\frac{1.2 \gamma_0}{K_0 \tau_0}$	$2 \tau_0$	$0.5 \tau_0$

**A. PI Controller Simulation**

A program was made using MatLab to solve the Riccati equation and to make variations in matrix Q.

The Figure (10) represents the results for different variations of Q with a step input. From graphic we can chose de best behavior for the system response.

With the variation in elements of matrix Q, the overshoot slight increases with gradual fall in rise time.

In Figure (10) it is possible to see the fifth variation of Q is the best response because it has an overshoot less than two percent and the system stabilized about five seconds.

From the program in MatLab we can know the gains for PI controller that corresponds to the values of Q.

Figure (11) shows the best response for the optimal controller PI tuning, we can note the time response is about five seconds, which is a good result for the system.

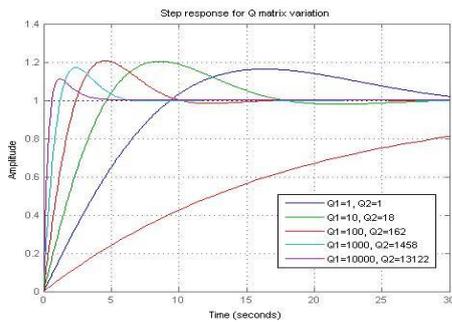


Fig 10. Step response with variations of matrix Q.

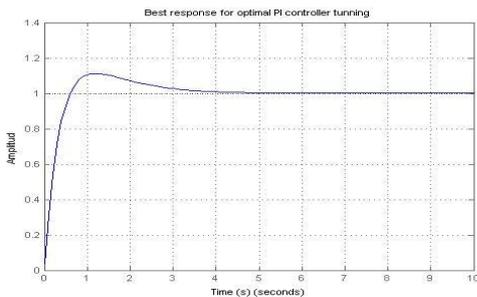


Fig 11. Best response for the optimal controller PI.

**B. Method of Ziegler-Nichols for tuning a PI controller**

It is important to make a comparison with other method tuning for PI controller, to make sure the optimal control tuning is the best way to have a control over the actuator. The proportional component of controller PI will make the system reaches the reference signal in less time and the integral component will minimize the stable state error. The Ziegler-Nichols method is based in the analysis response of the system with a step input in open-loop, the response is a reaction curve, called 'S' curve. From the analysis we get the parameters of the model with the equations:

In figure (12) we use the classical control tuning using the Ziegler-Nichols method. The result is the system is stabilized in about 10 seconds, which means the PI controller is working because it reduces the stabilizing time, But studying the results before where the optimal theory was applied for getting the PI controller we can note the best response was when the system is stabilized in five seconds.

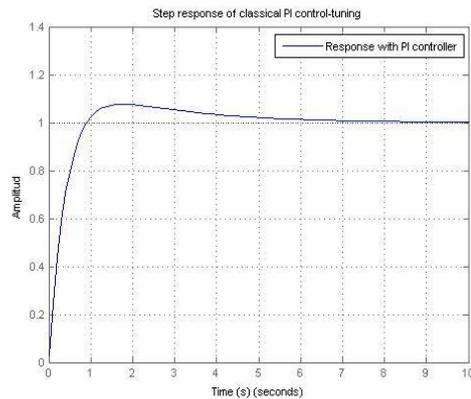


Fig 12. Response of the system with a classical PI control tuning.

From all results presented before, we can conclude that classical way for PI tuning is working very well but the best result is getting when we applied the optimal tuning to the system to get a good control over the valve in the pneumatic system because the system is stabilized in less time.

**VII. CONCLUSION**

From the integrated tomato-greenhouse model it is possible to do the optimization of all variables. In this case, the most important variable is the carbon dioxide concentration. In a first step, the optimal control theory was used to determine the desired behavior of the variables over a prescribed time period. In a second step, a PI control was designed form optimal control theory to make change in the behavior of the actuator. All the results obtained in this paper are the base to create an electronic device able to make the automatic control of carbon dioxide concentration inside a real tomato greenhouse. And for the future we want to test the automatic electronic device in real greenhouses to obtain good results in terms of crop production is concerned.

## REFERENCES

- [1] Aikman D. P., 1996. A procedure for optimizing carbon dioxide enrichment of a glasshouse tomato crop. *Journal of agricultural engineering research* 63, No. 2, 171--183.
- [2] Ioslovich I., Gutman P. and Linker R., 2009. Hamilton-Jacobi-Bellman formalism for optimal climate control of greenhouse crop. *Automatic*, 1227--1231.
- [3] Ioslovich I. and Seginer I., 1995. Normalized co-state variable for seasonal optimization of greenhouse tomato production. *Acta horticulturae* 417, 87--94.
- [4] Alexander Zemiliak, Tatiana Markina (2013). Preliminary Structure of Quasi Optimal Algorithm of Optimization of Analog Circuits. *WSEAS Transactions on CIRCUITS and SYSTEMS ISSUE 7*, volume 12.
- [5] Hull D.G., 2013 optimal control theory for applications, Springer-Verlag, New York, USA.
- [6] B. Fahimnia, R. Molae, M. Ebrahimi (2008). Genetic Algorithm Optimization of Fuel Consumption in Compressor Stations. *WSEAS Transactions on CIRCUITS and SYSTEMS ISSUE 1*, Volume 3.
- [7] Ioslovich I. and Seginer I., 1998. Approximate seasonal optimization of the greenhouse environment for a multi-state-variable tomato model. *Transactions of the ASAE* 41.
- [8] Md. Shahidul I., Toshiyuki M., Yuichi Y., 1996. Effect of carbon dioxide enrichment on physico-chemical and enzymatic changes in tomato fruits at various stages of maturity. *Scientia horticulturae* 65, 137--149.
- [9] Mortensen M. L., 1987. CO<sub>2</sub> enrichment in greenhouse. *Sci. Horticulturae* 33, 1--25.
- [10] Seginer I. and Ioslovich I., 1998. Seasonal optimization of the greenhouse environment for a simple two-stage crop growth model. *Journal of agricultural engineering research* 70, 145--155.
- [11] Tchamitchian M. and Ioslovich I., 1998. Equivalence of the temperature integral and the carbon dynamics concepts in plants: Utility for control. *Acta horticulturae* 519, 171--180.
- [12] Van Straten G., Challa H. and Buwalda F., 2000. Towards user accepted optimal control of greenhouse climate. *Computers and electronics in agriculture* 26, No. 3, 221--238.
- [13] Van Straten G., van Willigenburg L. G., van Henten E. and van Ooteghem R., 2011. Optimal control of greenhouse cultivation. CRC. Press, United States of America.
- [14] Van Henten E. J., 1994. Greenhouse climate management: An optimal control approach. Doctorate dissertation. Agricultural University, Wageningen.
- [15] Robert L. Mott, *Mecánica de Fluidos*, 6ta edición.
- [16] Saptarshi das, Indranil Pan, Kaushik Halder, Shantanu Das, Amitava Gupta. LQR based improved discrete PID controller design via optimum selection of weighting matrices using fractional order integral performance index. *Journal: Applied mathematical Modelling* 37 (2013) 4253-4268.