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Thermal Stress Analysis of a Thick Rectangular Plate: Steady-State Problem

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Abstract-* *This paper is concerned with inverse steady statet thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a thick rectangular plate when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.

***Key Words:* Thick rectangular plate, inverse steady state problem, Integral transform.**

I. INTRODUCTION

Khobragade et al. [1, 2] have derived thermal deflection of a thick clamped rectangular plate , Khobragade et al. [5, 6, 8-10] have investigated displacement function, temperature distribution and stresses of a thin rectangular plate and Khobragade et al. [11] have established displacement function , temperature distribution and stresses of a thick rectangular plate.

In the present paper, an attempt is made to determine the temperature distribution, unknown temperature gradient, thermal stresses and deflection of the plate occupying the space *D:* $\{(x,y,z)\in R^3 : -a \le x \le a,\ 0 \le y \le b, 0 \le z \le h\}$ with the known boundary conditions.

Finite Fourier cosine transform and Marchi-Fasulo transform techniques are used to find the solution of the problem. Numerical estimate for the temperature distribution is obtained.

II. STATEMENT OF THE PROBLEM

Consider a thick isotropic rectangular plate occupying the space D. The temperature of the plate satisfying the differential equation as Nowacki [15] is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g(x,y,z)}{k}=0 \tag{1}$$

Where k is the thermal diffusivity of the material of the plate,

subject to the boundary conditions**:**

$$\left[T(x,y,z)+k_1\frac{\partial T(x,y,z)}{\partial x}\right]_{x=-a}=f_1(y,z) \tag{2}$$

$$\left[T(x,y,z)+k_2\frac{\partial T(x,y,z)}{\partial x}\right]_{x=a}=f_2(y,z) \tag{3}$$

$$\left[\frac{\partial T(x,y,z)}{\partial y}\right]_{y=0}=g_1(x,z) \tag{4}$$

$$\left[\frac{\partial T(x,y,z)}{\partial y}\right]_{y=b}=g_2(x,z) \tag{5}$$

$$[T(x,y,z)]_{z=0}=0 \tag{6}$$

$$[T(x,y,z)]_{z=\xi}=g(x,y) \tag{7}$$

$$[T(x,y,z)]_{z=h}=f(x,y)\quad \text{(Unknown)} \tag{8}$$

The displacement components u_x and u_y u_z in the x and y and z directions respectively as **Tanigawa et al.** [1] are

$$u_x=\int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-\nu\frac{\partial^2 U}{\partial x^2}\right)+\lambda T\right]dx \tag{9}$$

$$u_y=\int_{0}^{b}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}-\nu\frac{\partial^2 U}{\partial y^2}\right)+\lambda T\right]dy \tag{10}$$

$$u_z=\int_{0}^{h}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}-\nu\frac{\partial^2 U}{\partial z^2}\right)+\lambda T\right]dz \tag{11}$$

where E, ν, and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and U (x,y,z) is the Airy's stress functions which satisfy the differential equation as **Tanigawa et al**. [1] is

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z)=-\lambda E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) T(x,y,z) \tag{12}$$

The stress components in terms of U(x, y, z) **Tanigawa et al**. [1] are given by

$$\sigma_{xx}=\left[\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right] \tag{13}$$

$$\sigma_{yy}=\left[\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}\right] \tag{14}$$

$$\sigma_{zz}=\left[\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right] \tag{15}$$

Equations (2.2.1) to (2.2.15) constitute the mathematical formulation of the problem under consideration.

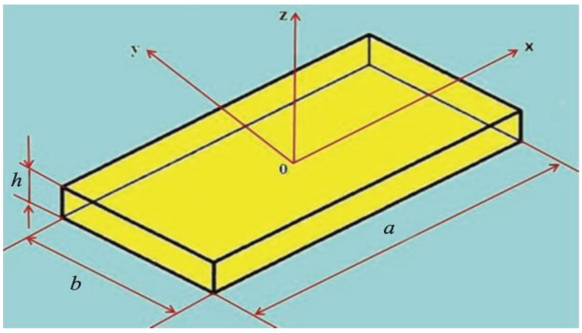

Fig 1: Geometry of the problem

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III. SOLUTION OF THE PROBLEM

Applying Marchi-Fasulo, transform and finite Fourier cosine Transform to the equations one obtains.

$$\frac{\partial^2 \bar{T}^*}{\partial z^2} - p^2 \bar{T}^* = \psi \tag{8}$$

where,

$$p^2 = \lambda_n^2 + \frac{m^2\pi^2}{b^2} \tag{9}$$

and

$$\psi = \frac{P_n(-a)}{k_1} f_2^* - \frac{P_n(a)}{k_2} f_2^* - (-1)^m \bar{g}_2 + \bar{g}_1 - \frac{\bar{g}^*}{k} \tag{10}$$

solution of equation (8) is

$$\bar{T}^* = Ae^{pz} + Be^{-pz} + F(z) \tag{11}$$

where $F(z)$ is the P.I.

where

$$A = \frac{\frac{Q_0}{\lambda}\bar{f}^* e^{-ph} + \bar{g}^* + F(0)e^{-ph} - F(h)}{2\sinh(ph)} \tag{12}$$

$$B = \frac{-\frac{Q_0}{\lambda}\bar{f}^* e^{ph} - \bar{g}^* - F(0)e^{ph} + F(h)}{2\sinh(ph)} \tag{13}$$

$$\bar{T}^* = \frac{\left(\frac{Q_0}{\lambda}\bar{f}^* + F(0)\right)\sinh(p(z-h)) + \left(\bar{g}^* - F(h)\right)\sinh(pz)}{2\sinh(ph)} + F(z) \tag{14}$$

Applying inverse Fourier cosine transform and inverse Marchi-Fasulo transform on equation (14) we get,

$$T = \frac{2}{\pi}\sum_{m,h=1}^{\infty}\frac{P_n(x)}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}^* + F(0)\right)\sinh(p(z-h)) + \left(\bar{g}^* - F(h)\right)\sinh(pz)}{\sinh(ph)} + F(z)\right\} \tag{15}$$

$$U = \frac{-2\lambda E}{b}\sum_{m,n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}^* + F(0)\right)\sinh(p(z-h)) + \left(\bar{g}^* - F(h)\right)\sinh(pz)}{\sinh(ph)} + F(z)\right\}$$

(16)

$$u_x = \frac{-2\lambda}{\pi}\int_{-a}^{a}\int_{0}^{\infty}\sum_{m,n=1}^{\infty}\frac{1}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}^* + F(0)\right)\sinh(p(z-h)) + \left(\bar{g}^* - F(h)\right)\sinh(pz)}{\sinh(ph)}\left[\left(p^2 + \frac{m^2\pi^2}{b^2}\right)P_n(x) + \nu P_n''(x)\right] + \left[\left(\frac{m^2\pi^2}{b^2} + 1\right)P_n(x) + \nu P_n''(x)\right]F(z) - P_n(x)F''(z)\right\}dx \tag{17}$$

$$u_y = \frac{-2\lambda}{b}\int_{0}^{b}\sum_{m,n=1}^{\infty}\frac{1}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}^* + F(0)\right)\sinh(p(z-h)) + \left(\bar{g}^* - F(h)\right)\sinh(pz)}{\sinh(ph)}\left[\left(\frac{m^2\pi^2}{b^2}p^2 - 1\right)P_n(x) + P_n''(x)\right] + \left[\left(\frac{m^2\pi^2}{b^2} - 1\right)P_n(x) + P_n''(x)\right]F(z) + P_n(x)F''(z)\right\}dy \tag{18}$$

$$u_z = \frac{-2\lambda}{b}\int_{0}^{h}\sum_{m,n=1}^{\infty}\frac{1}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}^* + F(0)\right)\sinh(p(z-h)) + \left(\bar{g}^* - F(h)\right)\sinh(pz)}{\sinh(ph)}\left[\left(\nu p^2 + 1 - \frac{m^2\pi^2}{b^2}\right)P_n(x) + P_n''(x)\right] + \left[\left(1 - \frac{m^2\pi^2}{b^2}\right)P_n(x) + P_n''(x)\right]F(z) - \nu P_n(x)F''(z)\right\}dz \tag{19}$$

$$\sigma_{xn} = \frac{2\lambda E}{\pi}\sum_{m,n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}^* + F(0)\right)\sinh(p(z-h)) + \left(\bar{g}^* - F(h)\right)\sinh(pz)}{\sinh(ph)}\left[\frac{m^2\pi^2}{b^2} + p^2\right] - \frac{m^2\pi^2}{b^2}F(z) - F''(z)\right. \tag{20}$$

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$$\sigma_{yy} = \frac{-2\lambda E}{\pi}\sum \frac{1}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)$$

$$\left\{ \frac{\left(\frac{Q_0}{\lambda}\bar{f}^* + F(0)\right)\sinh(p(z-h)) + \left(\bar{g}^* - F(h)\right)\sinh(pz)}{\sinh(ph)} \right.$$

$$\left(-p^2 P_n(x) + P_n''(x)\right) + P_n(x)F''(z) + P_n''(x)F(z)\} \quad (21)$$

$$\sigma_{zz} = \frac{-2\lambda E}{\pi}\sum_{m,n=1}^{\infty} \frac{1}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)$$

$$\left\{ \frac{\left(\frac{Q_0}{\lambda}\bar{f}^* + F(0)\right)\sinh(p(z-h)) + \left(\bar{g}^* - F(h)\right)\sinh(pz)}{\sinh(ph)} + F(z) \right.$$

IV. SPECIAL CASE

Set $g(x,y) = (x^2 - ax)(y^2 - by)$, (23)

Substituting this value in equation (15) we get

$$T = \frac{2}{\pi}\sum_{m,n=1}^{\infty} \frac{P_n(x)}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)$$

$$\left\{ \frac{\left(\frac{Q_0}{\lambda}\bar{f}^* + F(0)\right)\sinh(p(z-h)) + \left(\bar{g}^* - F(h)\right)\sinh(pz)}{\sinh(ph)} + F(z) \right\} \quad (24)$$

V. NUMERICAL RESULTS

Set, $a = 1$, $b = 2$, $h = 2$, $t = 1\sec$ $\xi = 1.5$ and $k = 0.86$

in equations (24) we get

$$T = \frac{2}{\pi}\sum_{m,n=1}^{\infty} \frac{P_n(x)}{\lambda_n}\cos\left(\frac{m\pi y}{2}\right)$$

$$\left\{ \frac{\left(\frac{Q_0}{\lambda}\bar{f}^* + F(0)\right)\sinh(p(z-2)) + \left(\bar{g}^* - F(2)\right)\sinh(pz)}{\sinh(2p)} + F(z) \right\} \quad (25)$$

VI. CONCLUSION

In this paper, the temperature distribution, unknown temperature gradient, displacement function and thermal stresses of a thick rectangular plate have been investigated, with the aid of finite Fourier cosine transform and Marchi-Fasulo transform techniques when the stated boundary conditions are known. The results are obtained in the form of infinite series.

The results that are obtained can be applied to the design of useful structures or machines in engineering applications.

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$$P_n^*(x) = -\frac{m^2\pi^2}{b^2}P_n(x) \quad (22)$$

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