ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 5, Issue 3, September 2015

Thermal Stresses of a Thick Circular Plate: Steady State Problem

Ritesh Ganar, Shalu Barai and N. W. Khobragade
Department of Mathematics, MJP Educational Campus,
RTM Nagpur University, Nagpur 440 033, India.

ABSTRACT- In this paper, an attempt has been made to study thermoelastic response of a thick circular plate occupying the space $D: 0 \le r \le a,\ -h \le z \le h,$ ***with radiation type boundary conditions. We apply integral transform technique to find the thermoelastic solution.***

Keywords: **Thermo elastic Response, thick circular plate, Thermal Stresses.**

I. INTRODUCTION

Khobragade et al. [3 - 12] have derived temperature distribution, displacement function, thermal stresses and thermal deflection of a thick and thin circular plate. Further Khobragade et al. [13] have established displacement function, temperature distribution and stresses and deflection of a triangular plate.

This paper is concerned with transient thermoelastic problem of a thick circular plate occupying the space $D: 0 \le r \le a,\ -h \le z \le h,$ due to heat generation with radiation type boundary conditions.

II. STATEMENT OF THE PROBLEM

Consider thick circular plate of thickness $2h$ occupying the space $D: 0 \le r \le a,\ -h \le z \le h,$ the material is homogenous and isotropic. The differential equation governing the displacement potential function $\phi(r,z,t)$ as Nowacki [2] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu}\right)\alpha_t T \tag{1}$$

Where ν and a_t are Poisson's ratio and linear coefficient of thermal expansion of the material of the plate and T is the temperature of the plate satisfying the differential equation as Noda [3] is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g(r,z)}{k} = 0 \tag{2}$$

The boundary conditions are

$$M_r(T,0,1,a) = g\ (z), \quad -h \le z \le h, \tag{3}$$

$$\left.\begin{aligned} M_z(T,1,k_1,h) &= f_1(r) \\ M_z(T,1,k_2,-h) &= f_2(r) \end{aligned}\right\},\ 0 \le r \le a, \tag{4}$$

Where k is thermal diffusivity of material of the plate. The displacement functions in the cylindrical coordinate system are represented by as Khobragade [4] are

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 L}{\partial r \partial z} \tag{5}$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2} \tag{6}$$

Where L is the Love's function [14] and must satisfy

$$\nabla^2 \nabla^2 L = 0 \tag{7}$$

Where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

The component of stresses are represented by the thermoelastic displacement potential ϕ and Love's function L as Noda [3] are

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu \nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right)\right\} \tag{8}$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial \phi}{\partial r} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu \nabla^2 L - \frac{1}{r}\frac{\partial^2 L}{\partial r^2}\right)\right\} \tag{9}$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial z^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left\{\left((2-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\}\right\} \tag{10}$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left\{\left((1-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\}\right\} \tag{11}$$

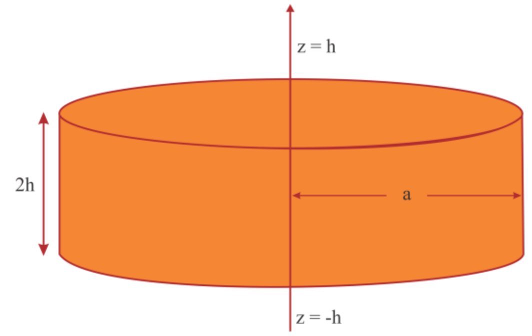

Fig. 1: Shows the geometry of the problem

For traction free surface stress function $\sigma_z = \sigma_{r\theta} = 0$ at $z = \pm h$ for thick plate

Equations (1) to (11) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

Applying Marchi-Fasulo transform to the equation (2), we get

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 5, Issue 3, September 2015

$$\frac{d^2\overline{T}}{dr^2}+\frac{1}{r}\frac{d\overline{T}}{dr}-\lambda_n^2\overline{T}=\psi \tag{12}$$

where $\psi=\dfrac{P_n(-h)}{k_2}f_2(r)-\dfrac{P_n(h)}{k_1}f_1(r)-\dfrac{\overline{g}}{k}$

Equation (12) is a Bessel's equation whose solution is given by

$$\overline{T}=AI_0(\lambda_n r)+BK_0(\lambda_n r)+P.I. \tag{13}$$

Now, as $r\to 0, K_0\to\infty$, but $\overline{T}$ is finite

$\therefore$ *B =0.*

Using (3), we get

$$A=\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}$$

Substituting these values in equation (13) we get

$$\therefore\overline{T}=\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}I_0(\lambda_n r)+P.I. \tag{14}$$

Appling inverse Marchi-Fasulo transform we obtain

$$T=\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\left[\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}I_0(\lambda_n r)+P.I.\right] \tag{15}$$

Where,

$P_n(z)=Q_n\cos(a_n z)-W_n\sin(a_n z)$,

$Q_n=a_n(\alpha_1+\alpha_2)\cos(a_n h)+(\beta_1-\beta_2)\sin(a_n h)$

$W_n=(\beta_1-\beta_2)\cos(a_n h)+a_n(\alpha_1-\alpha_2)\sin(a_n h)$

Equation (15) is the desired solution of the given problem.

Let us assume Love's function L, which satisfy condition (10) as

$$L=\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\left[\frac{(\overline{f}-P.I.\big|_{r=\xi})}{I_0(\lambda_n\xi)}I_0(\lambda_n r)\right] \tag{16}$$

Using (1) and (16), we get displacement potential ϕ *as*

$$\therefore\phi=\frac{r^2\alpha_t}{4}\left(\frac{1+\nu}{1-\nu}\right)\sum\frac{P_n(z)}{\lambda_n}$$
$$\times\left[\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}I_0(\lambda_n r)+P.I.\right] \tag{17}$$

IV. DETERMINATION OF DISPLACEMENT FUNCTION

Substituting equations (16) and (17) in equation (6), we get

$$u_r=\frac{\alpha_t}{4}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\left\{\frac{P_n(z)}{\lambda_n}\times\frac{(\overline{f}-P.I.\big|_{r=\xi})}{I_0(\lambda_n\xi)}\right.$$
$$\left.\left(r^2\lambda_n I_0'(\lambda_n r)+2rI_0(\lambda_n r)\right)+r^2\frac{\partial}{\partial r}(P.I.)+2r(P.I.)\right\}$$
$$-\sum_{n=1}^{\infty}P_n{}'(z)\times\frac{(\overline{f}-P.I.\big|_{r=\xi})}{I_0(\lambda_n\xi)}I_0{}'(\lambda_n r) \tag{18}$$

$$u_z=\frac{r^2\alpha_t}{4}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{P_n{}'(z)}{\lambda_n}\left[\frac{(\overline{f}-P.I.\big|_{r=\xi})}{I_0(\lambda_n\xi)}I_0(\lambda_n r)+P.I.\right]$$
$$+2(1-\nu)\sum_{n=1}^{\infty}\frac{\overline{f}-P.I.\big|_{r=\xi}}{\lambda_n I_0(\lambda_n\xi)}$$
$$\left[P_n(z)\lambda_n{}^2I_0''(\lambda_n r)+\frac{P_n(z)}{r}\lambda_n I_0'(\lambda_n r)+P_n{}''(z)I_0(\lambda_n r)\right]$$
$$-\sum_{n=1}^{\infty}\frac{P_n{}'(z)}{\lambda_n}\times\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}\times I_0(\lambda_n r)$$

(19)

Substituting equations (16) and (17) in equations (9) to (12), we obtain

$$\sigma_{rr}=\frac{-G\alpha_t}{2}\left(\frac{1+\nu}{1-\nu}\right)$$
$$\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left\{\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}\left[P_n(z)\left(r\lambda_n I_0'(\lambda_n r)+2I_0(\lambda_n r)\right)+rP''(z)r^2(P.I.)\right\}\right.$$
$$+2G\sum_{n=1}^{\infty}\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}$$
$$\left\{P_n{}'(z)\left[(\nu-1)\lambda_n{}^2I_0''(\lambda_n r)+\frac{\lambda_n I_0'(\lambda_n r)}{r}+P_n'''(z)I_0(\lambda_n r)\right\}\right. \tag{20}$$

$$\sigma_{\theta\theta}=\frac{-G\alpha_t}{2}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{\lambda_n}$$
$$\left\{\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}\left[P_n(z)\left(r\lambda_n{}^2I_0''(\lambda_n r)+4r\lambda_n I_0'(\lambda_n r)\right)+P_n''(z)r^2I_0(\lambda_n r)\right]\right.$$
$$+2G\sum_{n=1}^{\infty}\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}$$
$$\left.+P_n(z)\left[r^2\frac{\partial^2}{\partial r^2}(P.I.)+4r\frac{\partial}{\partial r}(P.I.)+(P.I.)\right]+P_n''(z)r^2(P.I.)\right\}$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 5, Issue 3, September 2015

$$\left\{P_n'(z)\left[\left(\nu-\frac{1}{r}\right)\lambda_n^2 I_0''(\lambda_n r)+\frac{\nu}{r}I_0(\lambda_n r)+P_n''(z)\nu\, I_0(\lambda_n r)\right\}$$

(21)

$$\sigma_{zz}=\frac{-G\alpha_t}{2}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}$$

$$\left\{\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}\left[\left(r^2\lambda_n^{\ 2}I_0''(\lambda_n r)+5r\lambda_n I_0'(\lambda_n r)\right)+4I_0(\lambda_n r)\right]\right.$$

$$\left. r^2\frac{\partial^2}{\partial r^2}(P.I.)+5r\frac{\partial}{\partial r}(P.I.)+(P.I.)+4(P.I.)\right\}$$

$$+2G\sum_{n=1}^{\infty}\frac{\overline{f}-P.I.\big|_{r=\xi}}{\lambda_n I_0(\lambda_n\xi)}$$

$$\left\{P_n'(z)\left[(2-\nu)\lambda_n^{\ 2}I_0''(\lambda_n r)+\left(\frac{2-\nu}{r}\right)\lambda_n I_0'(\lambda_n r)\right]\right.$$

$$\left.+P_n'''(z)(1-\nu)\, I_0(\lambda_n r)\right\} \qquad (22)$$

$$\sigma_{rz}=\frac{G\alpha_t}{2}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{P_n'(z)}{\lambda_n}\left\{\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}\right.$$

$$\left.\left[r^2\lambda_n I_0'(\lambda_n r)+2rI_0(\lambda_n r)\right]+r^2\frac{\partial}{\partial r}(P.I.)+2r(P.I.)\right\}$$

$$+2G\sum_{h=1}^{\infty}\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}$$

$$\left\{P_n(z)\lambda_n\left[(1-\nu)\lambda_n I_0'''(\lambda_n r)+\frac{1-\nu}{r}I_0''(\lambda_n r)-\left(\frac{1-\nu}{r^2}\right)I_0''(\lambda_n r)\right]\right.$$

$$\left.+P_n'''(z)\nu\, I_0'(\lambda_n r)\right\} \qquad (23)$$

V. SPECIAL CASE

Set $F(r,z)=z^2(1-r^2)$ (24)

Applying Marchi-Fasulo transform, we obtain

$$\overline{F}(r,n)=(1-r^2)\Phi_n\left[\frac{2h^2\sin(a_n h)}{a_n}+\frac{4h\cos(a_n h)}{a_n^{\ 2}}-\frac{4\sin(a_n h)}{a_n^{\ 3}}\right]$$

(25)

where

$\Phi_n=a_n(\alpha_1+\alpha_2)\cos(a_n h)+(\beta_1-\beta_2)\sin(a_n h).$

Using equation (25) in equation (15), one obtains

$$T=\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\left[\frac{\overline{f}-P.I.\big|_{r=\zeta}}{I_0(\lambda_n\xi)}I_0(\lambda_n r)+P.I.\right] \qquad (26)$$

VI. NUMERICAL RESULTS

Set $a=2,\xi=1.5.\ k=15.9\times10^6, t=1$ second in equation (26), we get

$$T=\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\left[\frac{\overline{f}-P.I.\big|_{r=\xi}}{I_0(\lambda_n\xi)}I_0(\lambda_n r)+P.I.\right] \qquad (27)$$

VII. CONCLUSION

In this paper, the temperature distribution, displacement and thermal stresses of a thick circular plate are investigated with known boundary conditions. Finite integral transform technique have been used to obtain numerical results. The results are obtained in terms of Bessel's function in the form of infinite series.

Any particular cases of special interest can be assigned to the parameters and functions in expressions. The results that are obtained can be useful to the design of structure or machines in engineering applications.

REFERENCES

[1] **Nowacki, W**: the state of stress in thick circular plate due to temperature field. Ball. Sci. Acad. Polon Sci. Tech.5 (1957).

[2] **Noda, N; Hetnarski, R.B; Tanigawa, Y**: Thermal Stresses, second edition Taylor & Francis, New York (2003), 260.

[3] **Ghume, R.S, Ashwini Mahakalkar and Khobragade, N.W**: Thermoelastic solution of a thin circular plate due to partially distributed heat supply, Int. J. of Engg. And Information Technology, vol. 3, Issue 6, pp.314-317, (2013).

[4] **Khobragade, N.W:** Thermoelastic analysis of a thick circular plate, Int. J. of Engg. And Information Technology, vol. 3, Issue 5, pp.94-100, (2013).

[5] **Khobragade, N.W** (2013): Thermal stresses of a thin circular plate with internal heat source, Int. J. of Engg. And Information Technology, vol. 3, Issue 5, pp.66-70,

[6] **Khobragade N. W., Khalsa L. H. , Gahane T. T. and A. C. Pathak**: Transient Thermo elastic Problem of a Circular Plate With Heat Generation, Int. J. of Engg. And Information Technology, vol. 3, Issue 1, pp. 361-367, (2013).

[7] **Hamna Parveen, N. K. Lamba and Khobragade, N.W**: "Thermal Stresses Of A Circular Disk With Internal Heat Sources", Journal of Statistics and Mathematics, Vol. 3, Issue 3, pp. 125-129, (2012).

[8] **Gahane, T. T, Khalsa, L H and Khobragade, N.W**: "Thermal Stresses in A Thick Circular Plate With Internal Heat Sources", Journal of Statistics and Mathematics, Vol. 3, Issue 2, pp. 94-98, (2012).

[9] **Hamna Parveen; Navneet Kumar and Khobragade, N. W**: "Thermal deflection of a thin circular plate with radiation", African journal of mathematics and computer science research, vol.5 (4), 66-70, (2012).

[10] **Hamna Parveen and Khobragade, N. W**: "Thermal Stresses Of A Thick Circular Plate Due To Heat Generation", Canadian Journal on Science and Engg. Mathematics Research, Vol. 3 No. 2, pp. 65-69, (2012).

[11] **Lamba, N.K; and Khobragade, N.W**: "Analytical Thermal Stress Analysis in a thin circular plate due to diametrical compression", Int. Journal of Latest Trends in Maths, Vol. 1, No. 1, pp.13-17, (2011).

[12] **Varghese, V., and Khobragade, N. W.**: "Alternative Solution of a Transient Heat Conduction in a Circular Plate with Radiation", Int. Journal of Appl. Math., vol.20 No.8,pp 1133-1141, (2007).

[13] **Dange, W. K; Khobragade, N.W, and Durge, M. H**: "Deflection of Isosceles Triangular Plate under Unsteady Temperature Distribution", Int. J. of Appl. Maths, Vol.23, No.3, 395-412, (2010).

[14] **Love, A.E.H**. Treatise on the Mathematical Theory of Elasticity, Oxford, (1927).

[15] **Khobragade, N. W:** Some Aspects Of Inverse Thermoelastic Problems, LAP LAMBERT Publishing Germany (2012).

[16] **Khobragade, N. W**: Selected Papers on Thermoelasticity Vol. I, LAP LAMBERT Publishing Germany (2013).

[17] **Khobragade, N. W**: Selected Papers on Thermoelasticity Vol. II, LAP LAMBERT Publishing Germany (2014).

[18] **Khobragade, N. W**: Selected Papers on Thermoelasticity Vol. III, LAP LAMBERT Publishing Germany (2015).

AUTHOR BIOGRAPHY

Dr. N.W. Khobragade for being M.Sc in statistics and Maths, he attained Ph.D in both subjects. He has been teaching since 1986 for 28 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities.

At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems (Thermoelasticity in particular) and Operations Research. Published more than 180 research papers in reputed journals. Fourteen students awarded Ph.D Degree and six students submitted their thesis in University for award of Ph.D Degree under their guidance.

Miss Shalu Barai M.Sc in Mathematics, she is a research scholar of Gondwana University Gadchiroli.

Mr. Ritesh Ganar M. Sc in maths, he is a research scholar of Maths, RTM Nagpur University, Nagpur and sucessfully handled different capacities.