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The Mathematical Safe with Arbitrary Locks Problem Solving

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Abstract- Mathematical safes with two possible states of locks are considered. Necessary and sufficient conditions for existence of the problem solution are derived. General approach to solving the problem on safe with arbitrary number of lock types is proposed.

Index terms- **the mathematical safe. the locks of safe. a set of states locks, incidence matrix of locks, matrix of a system of linear equations, module of system equations, inverse matrix of system equations, matrix of solutions of system equations.**

I. INTRODUCTION

We continue here the research of topic stated in [1], where a problem of mathematical safe with a prime number of locks was posed and solved. All symbols and term names are reserved. Safes where each lock has its own set of states are under consideration. First, the problem for two types of locks is solved, then it is generalized to arbitrary number.

II. THE PROBLEM FORMULATING

In [2] and [3] the safes set on directed and undirected graphs were considered. It follows from the mathematical safe global definition, that any of them can be set using some graph adjacency matrix. Let us consider in general case some safe defined in [1], in which all locks are arranged in a rectangular table of size $m \times n$. Any initial state $\vec{\mathbf{b}}$ of the safe corresponds to the matrix

$B = (b_{ij})_{m,n}$ where $b_{ij} \in \{0, 1, \ldots, k_{ij} - 1\}$.

It is necessary to find such a sequence of locks and the corresponding number of turns in them to "open the safe", that is to transit into the safe state $B_{fin} = (b_{ij} = 0)_{m,n}$.

Let $X = (x_{ij})_{m,n}$ be a solution to the problem, where x_{ij} is equal to the number of key turns in the lock z_l.

Then the condition that an element b_{ij} is converted by the matrix X to zero is represented by the ratio:

$$\sum_{k=1}^{n} x_{ik} + \sum_{\substack{k=1 \\ k \neq i}}^{m} x_{kj} + b_{ij} \equiv 0 (\mathrm{mod}\, k_{ij})$$

where $i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$.

III. THE PROBLEM SOLUTION

Let us denote $\vec{x} = (x_{11}, x_{12}, \ldots\ldots, x_{1n}, x_{21}, x_{22}, \ldots\ldots, x_{2n}, \ldots\ldots, x_{m,n-1}, x_{mn})$ column vector obtained from the matrix X by sequentially recording her lines.

Similarly, we obtain the column vector $\vec{b}$ from the matrix B, and a vector $\vec{K} = (k_{11}, k_{12}, \ldots, k_{1n}, k_{22}, \ldots, k_{2n}, \ldots, k_{m,n-1}, k_{mn})$ from lock states. In addition, let $\Im_n$ be a matrix of size $n \times n$, consisting of ones, E_n be the identity matrix of the same size, and I_n be a row vector of n ones. The condition of transformation (1) for the whole matrix B can be written as a system of equations

$$A\vec{x} + \vec{b} \equiv 0 (\mathrm{mod}\, \vec{K}) \qquad (2)$$

Where matrix A of order $mn \times mn$ consists of m^2 sells:

$$A = \begin{pmatrix} \Im_n & E_n & E_n & \ldots & E_n \\ E_n & \Im_n & E_n & \ldots & E_n \\ E_n & E_n & \Im_n & \ldots & E_n \\ .. & .. & .. & \ldots & .. \\ E_n & E_n & E_n & \ldots & \Im_n \end{pmatrix} \qquad (3)$$

If the matrix A rank equals to mn, then system (2) solution is $\vec{x} = -A^{-1}\vec{b} (\mathrm{mod}\, \vec{K})$

Thus, the problem is reduced to finding the inverse matrix A^{-1}. In the general case for arbitrary m, n and K, it may not exist. Then the system (2) may have a solution if the initial state satisfies certain constraints. Therefore, in such cases, the problem arises of the initial state correction, after which the solution of the problem will exist. Depending on the specific values of m, n and K the problem solutions of different complexity arise. Safes with two state locks were studied in [4], safes with the same type locks with an arbitrary number of states were described in [5] and [6]. In this paper we study safes with arbitrary locks. To begin we solve the problem for safes, for which the first p rows of the matrix B are the locks of the first type with the number of states k_1, and the rest of the lines are the locks of the second type with the number of states k_2. The condition that the element b_{ij} is converted to zero by the matrix B is represented by the ratios (2), which are specifically written in the form

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$$\sum_{k=1}^{n} x_{ik} + \sum_{\substack{k=1 \\ k\neq i}}^{m} x_{kj} + b_{ij} \equiv 0 \pmod{k_1} \qquad (4)$$

$(i = 1, 2, \ldots, p)$;

$$\sum_{k=1}^{n} x_{ik} + \sum_{\substack{k=1 \\ k\neq i}}^{m} x_{kj} + b_{ij} \equiv 0 \pmod{k_2}$$

$, (i = p+1, p+2, \ldots, m)$.

If you multiply the first part of (4) by k_2, and the second part by k_1, the ratios will not change, but now they can be written as

$$\mathbf{A}'\vec{x} + \mathbf{b}' \equiv 0 \pmod{k_1 k_2}, \qquad (5)$$

Where $\mathbf{A}'$ is a square matrix of order $mn \times mn$, consisting of m^2 sub matrices

$$= \begin{pmatrix} k_2\mathfrak{I}_n & k_2E_n & k_2E_n & \ldots & k_2E_n \\ k_2E_n & k_2\mathfrak{I}_n & k_2E_n & \ldots & k_2E_n \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ k_1E_n & k_1E_n & k_1E_n & \ldots & k_1E_n \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ k_1E_n & k_1E_n & k_1E_n & \ldots & k_1\mathfrak{I}_n \end{pmatrix}$$

and $\mathbf{b}'$ $= (k_2b_1, k_2b_2, \ldots k_2b_{pn}, k_1b_{pn+1}, k_1b_{pn+2} \ldots, k_1b_{mn})$

Theorem 1. System (5) solution satisfies the comparison system

$$A x + \mathrm{b} \equiv 0 \pmod{k_1 k_2}$$

where A is the matrix (3).

Proof. To solve the system (5) we have to find its inverse matrix . It is easy to make sure that **det** $\mathbf{A}'$ = $k_2^{pn}\, k_1^{n(m-p)} = k_2^{pn}\, k_1^{n(m-p)}$ **det** A.

In calculating the minors for inverse matrices it can be noted that we have $a_{ij}^{-1} = (a'_{ij})^{-1} / k_2$ for $j \le p$, and $a_{ij}^{-1} = (a'_{ij})^{-/} / k_1$ for $j \ge p$.

This leads to the equation $\mathbf{A}'^{-1}\,\mathbf{b}' = A^{-1}\,\mathbf{b}$. This implies the theorem , since we get from the two systems $\vec{x} = -A^{-1}\mathbf{b} = -\mathbf{A}'^{-1}\,\mathbf{b}'$.

Thus, the problem is reduced to finding the inverse matrix A^{-1}. In the general case for arbitrary m, n, k_1 and k_2, and it may not exist. Then the system (1) may have a solution if the initial state satisfies certain constraints.

Consider the symmetric square matrix of order *n*, depending on two parameters $H(\alpha,\beta) = (\alpha - \beta)E_n + \beta\mathfrak{I}_n$.

$$T_{m,n}(\alpha,\beta,\gamma,\delta) = \begin{pmatrix} H_n(\alpha,\beta) & H_n(\gamma,\delta) & \ldots & H_n(\gamma,\delta) \\ H_n(\gamma,\delta) & H_n(\alpha,\beta) & \ldots & H_n(\gamma,\delta) \\ \ldots & \ldots & \ldots & \ldots \\ H_n(\gamma,\delta) & H_n(\gamma,\delta) & \ldots & H_n(\alpha,\beta) \end{pmatrix} \qquad (6)$$

It follows from [6] that $\mathbf{A}^{-1} = T_{m,n}(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$, where

$$\left.\begin{aligned} \alpha_1 &\equiv \frac{1}{m-1} + \frac{1}{n-1} - 1 + \alpha_4, \\ \alpha_2 &\equiv \frac{1}{n-1} + \alpha_4, \\ \alpha_3 &\equiv \frac{1}{m-1} + \alpha_4, \\ \alpha_4 &\equiv -\left(\frac{1}{n-1} + \frac{1}{m-1}\right)\frac{1}{m+n-1} \end{aligned}\right\} \pmod{k_1 k_2}. \qquad (7)$$

Hence the <u>**condition of solvability of the system (1)**</u> is:

$$m \neq 1 \pmod{k_i};\; n \neq 1 \pmod{k_i};\; m + n \neq 1 \pmod{k_i}$$

, where $i = 1, 2$.

IV. INVESTIGATES

Example 1. Let $m = 5$, $n = 3$, $k_1 = 3$, $k_2 = 5$, $p = 2$, and the matrix B has the form

$$B = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 4 & 1 & 0 \\ 4 & 2 & 1 \\ 0 & 2 & 0 \end{pmatrix};$$

We calculate the fractional values:

$$\begin{cases} \dfrac{1}{m-1} = \dfrac{1}{4} \equiv 4 \pmod{3 \cdot 5}, \\ \dfrac{1}{n-1} = \dfrac{1}{2} \equiv 8 \pmod{3 \cdot 5}, \\ \dfrac{1}{m+n-1} = \dfrac{1}{7} \equiv 13 \pmod{3 \cdot 5} \end{cases}$$

Substituting these values into (7), we obtain: $\alpha_4 = 6$, $\alpha_1 = 2$, $\alpha_2 = 14$, $\alpha_3 = 10$.

Hence we obtain the inverse matrix (6) $\mathbf{A}^{-1} = T_{5,3}(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$.

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Let us calculate $\vec{x} = -A^{-1}\mathbf{b}$ = (12, 6, 13, 12, 5, 14, 14, 4, 12, 13, 4, 12, 4, 14, 6)

$$X=\begin{pmatrix}12 & 6 & 13\\ 12 & 5 & 14\\ 14 & 4 & 12\\ 13 & 4 & 12\\ 4 & 14 & 6\end{pmatrix} \pmod{15}$$

$$A^{-1}=\begin{pmatrix}
5&2&2&13&9&9&13&9&9&13&9&9&13&9&9\\
2&5&2&9&13&9&9&13&9&9&13&9&9&13&9\\
2&2&5&9&9&13&9&9&13&9&9&13&9&9&13\\
13&9&9&5&2&2&13&9&9&13&9&9&13&9&9\\
9&13&9&2&5&2&9&13&9&9&13&9&9&13&9\\
9&9&13&2&2&5&9&9&13&9&9&13&9&9&13\\
13&9&9&13&9&9&5&2&2&13&9&9&13&9&9\\
9&13&9&9&13&9&2&5&2&9&13&9&9&13&9\\
9&9&13&9&9&13&2&2&5&9&9&13&9&9&13\\
13&9&9&13&9&9&13&9&9&5&2&2&13&9&9\\
9&13&9&9&13&9&9&13&9&2&5&2&9&13&9\\
9&9&13&9&9&13&9&9&13&2&2&5&9&9&13\\
13&9&9&13&9&9&13&9&9&13&9&9&5&2&2\\
9&13&9&9&13&9&9&13&9&9&13&9&2&5&2\\
9&9&13&9&9&13&9&9&13&9&9&13&2&2&5
\end{pmatrix}$$

You can directly verify that this is the solution of (1).

+12 +6 +13

$$B=\begin{pmatrix}1&2&0\\1&1&1\\4&1&0\\4&2&1\\0&2&0\end{pmatrix}\Rightarrow\begin{pmatrix}13&14&12\\13&1&1\\1&1&0\\1&2&1\\12&2&0\end{pmatrix}\Rightarrow\begin{pmatrix}4&5&3\\13&7&1\\1&7&0\\1&8&1\\12&8&0\end{pmatrix}\Rightarrow{+12}\begin{pmatrix}2&3&1\\13&7&14\\1&7&13\\1&8&14\\12&8&13\end{pmatrix}$$

+5 +4

$$\Rightarrow\begin{pmatrix}14&3&1\\10&4&12\\13&7&13\\13&8&14\\9&8&13\end{pmatrix}\Rightarrow\begin{pmatrix}14&8&1\\0&9&1\\13&12&13\\13&13&14\\9&13&13\end{pmatrix}{+14}\Rightarrow{+14}\begin{pmatrix}14&8&0\\14&8&0\\13&12&12\\13&13&13\\9&13&12\end{pmatrix}\Rightarrow\begin{pmatrix}13&8&0\\13&8&0\\12&11&11\\12&13&13\\8&13&12\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}13&12&0\\13&12&0\\1&0&0\\12&2&13\\8&2&12\end{pmatrix}{+12}\Rightarrow{+13}\begin{pmatrix}13&12&12\\13&12&12\\13&12&12\\12&2&10\\8&2&9\end{pmatrix}\Rightarrow\begin{pmatrix}11&12&12\\11&12&12\\11&12&12\\10&0&8\\6&2&9\end{pmatrix}\Rightarrow\begin{pmatrix}11&1&12\\11&1&12\\11&1&12\\14&4&12\\6&6&9\end{pmatrix}{+12}$$

+4

$$\Rightarrow{+4}\begin{pmatrix}11&1&9\\11&1&9\\11&1&9\\11&1&9\\6&6&6\end{pmatrix}\Rightarrow\begin{pmatrix}0&1&9\\0&1&9\\0&1&9\\0&1&9\\10&10&10\end{pmatrix}\Rightarrow\begin{pmatrix}0&0&9\\0&0&9\\0&0&9\\0&0&9\\9&9&9\end{pmatrix}\Rightarrow\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\\0&0&0\\0&0&0\end{pmatrix}\pmod{15}$$

+14 +6

Theorem 2. The solution of system (1) for an arbitrary number q of lock types with relatively prime $k_1, k_2, \cdots, k_q$ satisfies the comparison system

$$A\mathbf{x}+\mathbf{b}\equiv 0 \pmod{k_1k_2\cdots k_q},$$

where A is the matrix (6).

The proof is carried in the same way as for Theorem 1.

V. CONCLUSIONS

In this paper, the theory of solving the matrix problem of mathematical safe with different types of locks is first proposed. It is considered that the numbers of lock states are relatively prime numbers. The cases where the solvability of the system (1) does not hold and when the numbers of lock states are composite ones were not solved. The solution of all these problems does not require fundamentally new approaches. The technical work related to the listing of the many options is needed only.

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