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Steady-State Problem Thermoelastic Problem of Semi-Infinite Rectangular Beam

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Abstract- ***This paper is concerned with steady state thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a semi-infinite rectangular beam when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.***

Key Words: **Semi-infinite rectangular beam, steady-state problem, Integral transform, heat source**

I. INTRODUCTION

Khobragade et al. [2-7, 9] have investigated temperature distribution, displacement function, and stresses of a thin rectangular plate and Khobragade et al. [8] have established displacement function, temperature distribution and stresses of a semi-infinite rectangular beam. In this paper, an attempt has been made to determine the temperature distribution, displacement function and thermal stresses of a semi-infinite square beam occupying the region **D: -*a* ≤ x ≤ *a* ; 0 ≤ y ≤ *b*, 0 ≤ z ≤ ∞.** with known boundary conditions. Here Marchi-Fasulo transforms and Fourier cosine transform techniques have been used to find the solution of the problem.

II. STATEMENT OF THE PROBLEM

Consider a thin rectangular plate occupying the space D: -a ≤ x ≤ a ; 0 ≤ y ≤ b, 0 ≤ z ≤ ∞. The displacement components ux , uy and uz in the x and y and z directions respectively as Noda et al. (2003) [1] are

$$u_x = \int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-v\frac{\partial^2 U}{\partial x^2}\right)+\lambda T\right]dx \tag{1}$$

$$u_y = \int_{0}^{b}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}-v\frac{\partial^2 U}{\partial y^2}\right)+\lambda T\right]dy \tag{2}$$

$$u_z = \int_{0}^{\infty}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}-v\frac{\partial^2 U}{\partial z^2}\right)+\lambda T\right]dz \tag{3}$$

where E, ν, and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and *U* (x,y,z) is the Airy's stress functions which satisfy the differential equation as **Noda et al. (2003)** [**1**] is

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z) = -\lambda E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)T(x,y,z) \tag{4}$$

where T(x,y,z) denotes the temperature of a rectangular beam satisfy the following differential equation as **Noda et al. (2003)** [**1**] is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g(x,y,z)}{k}=0 \tag{5}$$

where *k* is the thermal conductivity of the material, subject to the boundary conditions

$$\left[T(x,y,z)+k_1\frac{\partial T(x,y,z)}{\partial x}\right]_{x=a} = f_1(y,z) \tag{6}$$

$$\left[T(x,y,z)+k_2\frac{\partial T(x,y,z)}{\partial x}\right]_{x=-a} = f_2(y,z) \tag{7}$$

$$\left[\frac{\partial T(x,y,z)}{\partial y}\right]_{y=0} = f_3(x,z) \tag{8}$$

$$\left[\frac{\partial T(x,y,z)}{\partial y}\right]_{y=b} = f_4(x,z) \tag{9}$$

$$[T(x,y,z)]_{Z=0} = 0 \tag{10}$$

$$[T(x,y,z)]_{Z=\infty} = 0 \tag{11}$$

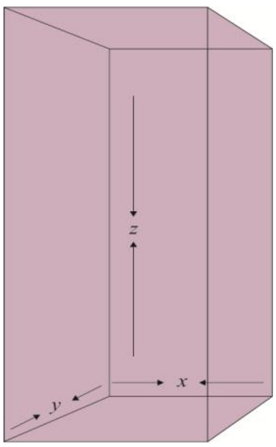

Fig 1: Geometry of the problem

The stress components in terms of U(x, y, z) **Noda et al. (2003)** [**1**] are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right] \tag{12}$$

$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}\right] \tag{13}$$

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right] \tag{14}$$

Equations (1) to (14) constitute the mathematical formulation of the problem under consideration.

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III. SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo transform, finite Fourier cosine transform and Fourier sine transform to the equations, we get

$$\frac{d\bar{T}^*}{dy} - q^2\bar{T}^* = \Psi \tag{15}$$

where, $q^2 = \lambda_n^{\ 2} + p^2$

$$\psi = \frac{P_n(-a)}{k_2} f_2^* - \frac{P_n(a)}{k_1} f_1^* - \frac{\bar{g}^*}{k}$$

This is a linear differential equation whose solution is given by

$$\bar{T}^* = Ae^{qy} + Be^{-qy} + F(y) \tag{16}$$

Where $F(y)$ is the P.I.

$$A = \frac{e^{-q\xi}\left(F'(0) - \bar{f}_3^*\right) + F'(\xi)}{2q\sinh(q\xi)} \tag{17}$$

$$B = \frac{e^{q\xi}\left(F'(0) - \bar{f}_3^*\right) + \bar{f}_4^* - F'(\xi)}{2q\sinh(q\xi)} \tag{18}$$

Substituting the values of A and B in equation (16) one obtains

$$\bar{T}^* = \frac{\left(F'(0) - \bar{f}_3^*\right)\cosh(q(y-\xi)) + \left(\bar{f}_4^* - F'(\xi)\right)\cosh(qy)}{q\sinh(q\xi)} + F(y) \tag{19}$$

Applying inverse Fourier sine transform and inverse Marchi-Fasulo transform to the equation (19) we get,

$$T = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n} \int_0^\infty \left\{ \frac{\left(F'(0) - \bar{f}_3^*\right)\cosh\sqrt{p^2+\lambda_n^2}(y-b) + \left(\bar{f}_4^* - F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)} + F(y) \right\}\sin(pz)dp \tag{20}$$

Equation (20) is the required solution.

IV. AIRY'S STRESS FUNCTIONS

Substituting the value of temperature distribution T(x,y,z) from (19) in equation (18) one obtains

$$U = \frac{-2\lambda E}{\pi}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n} \int_0^\infty \left\{ \frac{\left(F'(0) - \bar{f}_3^*\right)\cosh\sqrt{p^2+\lambda_n^2}(y-b) + \left(\bar{f}_4^* - F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)} + F(y) \right\}\times\sin(pz)dp \tag{21}$$

V. DISPLACEMENT COMPONENTS

Substituting the values of Airy's stress function U from equation (21) in the equation (1) to (3), one obtains

$$u_x = \frac{-2\lambda}{\pi}\int_{-a}^{a}\int_0^\infty\sum_{n=1}^{\infty}\frac{1}{\lambda_n} \left\{ \frac{\left(F'(0) - \bar{f}_3^*\right)\cosh\sqrt{p^2+\lambda_n^2}(y-b) + \left(\bar{f}_4^* - F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)} \left[\left(\lambda_n^2\right)P_n(x) - \nu\, P_n''(x)\right] + \left[F''(y) - \left(p^2+1\right)F(y)\right]P_n(n) - \nu\, F(y)P_n''(n) \right\} \sin(pz)dp\,dx \tag{22}$$

$$u_y = \frac{2\lambda}{\pi}\int_0^b\int_0^\infty\sum_{n=1}^{\infty}\frac{1}{\lambda_n} \left\{ \frac{\left(F'(0) - \bar{f}_3^*\right)\cosh\left(\sqrt{p^2+\lambda_n^2}(y-b)\right) + \left(\bar{f}_4^* - F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)} \left[\left((\nu+1)p^2 + \nu\,\lambda_n^2 + 1\right)P_n(x) - \nu\, P_n''(x)\right] + \left[\left(p^2+1\right)F(y) + \nu\, F''(y)\right]P_n(n) - F(y)P_n''(n) \right\} \sin(pz)dp\,dy \tag{23}$$

$$u_z = \frac{-2\lambda}{\pi}\int_0^\infty\int_0^\infty\sum_{n=1}^{\infty}\frac{1}{\lambda_n} \left\{ \frac{\left(F'(0) - \bar{f}_3^*\right)\cosh\sqrt{p^2+\lambda_n^2}(y-b) + \left(\bar{f}_4^* - F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)} \left[\left((\nu-1)p^2 + \lambda_n^2 - 1\right)P_n(x) + P_n''(x)\right] + \left[\left(\nu\, p^2 - 1\right)F(y) + \nu\, F''(y)\right]P_n(n) + F(y)P_n''(n) \right\} \sin(pz)dp\,dz \tag{24}$$

VI. DETERMINATION OF STRESS FUNCTION

Substituting the value of Airy's stress function U(x,y,z) from equation (22) in the equation (23) to (24) one obtain the stress functions as,

$$\sigma_{xn} = \frac{-2\lambda E}{\pi}\sum_{n=1}^{\infty}\int_0^\infty\frac{P_n(x)}{\lambda_n}$$

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$$\left\{ \lambda_n^2 \frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}(y-b) + \left(\bar{f}_4^{\,*}-F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)} + F''(y)-p^2F(y)\right\}\sin(pz)dp \quad (25)$$

$$\sigma_{yy} = \frac{2\lambda E}{\pi}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{P_n(x)-P_n''(x)}{\lambda_n}\left[\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}(y-b) + \left(\bar{f}_4^{\,*}-F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)} + F(y)\right]\sin(pz)dp \quad (26)$$

$$\sigma_{zz} = \frac{-2\lambda E}{\pi}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}(y-b) + \left(\bar{f}_4^{\,*}-F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)}\left[\left(p^2+\lambda_n^2\right)P_n(x)+P_n''(x)\right] + F''(y)P_n(x)+F(y)P_n''(x)\right\}\sin(pz)dp \quad (27)$$

VII. SPECIAL CASE

Set $f(x,y,z,t)=(x-a)^2(x+a)^2(z+e^{-z})(e^{y-t})$ (28)

Applying Marchi-Fasulo transform to the equation (4.7.1) we get

$$\overline{f}(n,y,z,t)=(z+e^{-z})(e^{y-t}) \times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{a_n{}^2}\right] \quad (29)$$

Substituting equation (4.7.2) in equations (4.3.6), (4.4.1), (4.5.1)- (4.5.3), (4.6.1)-(4.6.3) we obtain

$$T = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\int_0^{\infty}\left\{\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}(y-b) + \left(\bar{f}_4^{\,*}-F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)} + F(y)\right\}\sin(pz)dp \quad (30)$$

$$U = \frac{-2\lambda E}{\pi}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\int_0^{\infty}\left\{\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}(y-\xi) + \left(\bar{f}_4^{\,*}-F'(\xi)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(\xi\sqrt{p^2+\lambda_n^2}\right)} + F(y)\right\}\sin(pz)dp \quad (31)$$

$$u_x = \frac{-2\lambda}{\pi}\int_{-a}^{a}\int_0^{\infty}\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}(y-b)+\left(\bar{f}_4^{\,*}-F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)}\left[\left(\lambda_n^2\right)P_n(x)-\nu\,P_n''(x)\right] + \left[F''(y)-\left(p^2+1\right)F(y)\right]P_n(n)-\nu\,F(y)P_n''(n)\right\}\sin(pz)dp\,dx \quad (32)$$

$$u_y = \frac{2\lambda}{\pi}\int_0^{b}\int_0^{\infty}\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\left(\sqrt{p^2+\lambda_n^2}(y-b)\right) + \left(\bar{f}_4^{\,*}-F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)}\left[\left((\nu+1)p^2+\nu\,\lambda_n^2+1\right)P_n(x)-\nu\,P_n''(x)\right] + \left[\left(p^2+1\right)F(y)+\nu\,F''(y)\right]P_n(n)-F(y)P_n''(n)\right\}\sin(pz)dp\,dy \quad (33)$$

$$u_z = \frac{-2\lambda}{\pi}\int_0^{\infty}\int_0^{\infty}\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}(y-b) + \left(\bar{f}_4^{\,*}-F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)}\left[\left((\nu-1)p^2+\lambda_n^2-1\right)P_n(x)+P_n''(x)\right]\right.$$

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$$+\left[\left(\nu\, p^2-1\right)F(y)+\nu\, F''(y)\right]P_n(n)+F(y)P_n''(n)\Big\}$$

$$\sin(pz)dp\, dz \qquad (34)$$

$$\sigma_{xx}=\frac{-2\lambda E}{\pi}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{P_n(x)}{\lambda_n}\left\{\lambda_n^2\frac{\left(F'(0)-\bar f_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-b)+\left(\bar f_4^{\,*}-F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)}\right.$$

$$\left.+F''(y)-p^2F(y)\right\}\sin(pz)dp \qquad (35)$$

$$\sigma_{yy}=\frac{2\lambda E}{\pi}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{P_n(x)-P_n''(x)}{\lambda_n}$$

$$\left[\frac{\left(F'(0)-\bar f_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-b)+\left(\bar f_4^{\,*}-F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)}+F(y)\right]$$

$$\sin(pz)dp \qquad (36)$$

$$\sigma_{zz}=\frac{-2\lambda E}{\pi}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar f_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-b)+\left(\bar f_4^{\,*}-F'(b)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(b\sqrt{p^2+\lambda_n^2}\right)}\right.$$

$$\left[\left(p^2+\lambda_n^2\right)P_n(x)+P_n''(x)\right]$$

$$\left.+F''(y)P_n(x)+F(y)P_n''(x)\right\}\sin(pz)dp \qquad (37)$$

VIII. NUMERICAL RESULTS

Set $a=2$, $k=0.86$, $b=3$, $t=1$ sec in the equations (30)-(37) to obtain

$$T=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\int_0^{\infty}\left\{\frac{\left(F'(0)-\bar f_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-3)+\left(\bar f_4^{\,*}-F'(3)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(3\sqrt{p^2+\lambda_n^2}\right)}+F(y)\right\}$$

$$\sin(pz)dp \qquad (38)$$

$$U=\frac{-2\lambda E}{\pi}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\int_0^{\infty}\left\{\frac{\left(F'(0)-\bar f_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-3)+\left(\bar f_4^{\,*}-F'(3)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(3\sqrt{p^2+\lambda_n^2}\right)}+F(y)\right\}\sin(pz)dp \qquad (39)$$

$$u_x=\frac{-2\lambda}{\pi}\int_{-2}^{2}\int_0^{\infty}\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar f_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-3)+\left(\bar f_4^{\,*}-F'(3)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(3\sqrt{p^2+\lambda_n^2}\right)}\right.$$

$$\left[\left(\lambda_n^2\right)P_n(x)-\nu\, P_n''(x)\right]$$

$$\left.+\left[F''(y)-\left(p^2+1\right)F(y)\right]P_n(n)-\nu\, F(y)P_n''(n)\right\}$$

$$\sin(pz)dp\, dx \qquad (40)$$

$$u_y=\frac{2\lambda}{\pi}\int_0^{3}\int_0^{\infty}\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar f_3^{\,*}\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,(y-3)\right)+\left(\bar f_4^{\,*}-F'(3)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(3\sqrt{p^2+\lambda_n^2}\right)}\right.$$

$$\left[\left((\nu+1)p^2+\nu\,\lambda_n^2+1\right)P_n(x)-\nu\, P_n''(x)\right]$$

$$\left.+\left[\left(p^2+1\right)F(y)+\nu\, F''(y)\right]P_n(n)-F(y)P_n''(n)\right\}$$

$$\sin(pz)dp\, dy \qquad (41)$$

$$u_z=\frac{-2\lambda}{\pi}\int_0^{\infty}\int_0^{\infty}\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar f_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-3)+\left(\bar f_4^{\,*}-F'(3)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(3\sqrt{p^2+\lambda_n^2}\right)}\right.$$

$$\left[\left((\nu-1)p^2+\lambda_n^2-1\right)P_n(x)+P_n''(x)\right]$$

$$\left.+\left[\left(\nu\, p^2-1\right)F(y)+\nu\, F''(y)\right]P_n(n)+F(y)P_n''(n)\right\}$$

$$\sin(pz)dp\, dz \qquad (42)$$

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$$\sigma_{xx}=\frac{-2\lambda E}{\pi}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{P_n(x)}{\lambda_n}\left\{\lambda_n^2\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-3)+\left(\bar{f}_4^{\,*}-F'(3)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(3\sqrt{p^2+\lambda_n^2}\right)}+F''(y)-p^2F(y)\right\}\sin(pz)\,dp \quad (43)$$

$$\sigma_{yy}=\frac{2\lambda E}{\pi}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{P_n(x)-P_n''(x)}{\lambda_n}\left[\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-3)+\left(\bar{f}_4^{\,*}-F'(3)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(3\sqrt{p^2+\lambda_n^2}\right)}+F(y)\right]\sin(pz)\,dp \quad (44)$$

$$\sigma_{zz}=\frac{-2\lambda E}{\pi}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-3)+\left(\bar{f}_4^{\,*}-F'(3)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(3\sqrt{p^2+\lambda_n^2}\right)}\left[\left(p^2+\lambda_n^2\right)P_n(x)+P_n''(x)\right]+F''(y)P_n(x)+F(y)P_n''(x)\right\}\sin(pz)\,dp \quad (45)$$

IX. MATERIAL PROPERTIES

The numerical calculations has been carried out for an Aluminum (pure) rectangular beam with the material properties as,

Density ρ =169 lb/ft ^{3}
Specific heat = 0.208 Btu/lbOF
Thermal conductivity K = 117 Btu/(hr. ftOF)
Thermal diffusivity α = 3.33 ft^2/hr.
Poisson ratio ν = 0.35
Coefficient of linear thermal expansion α_t = 12.84 x 10^{-6}1/F
Lame constant μ = 26.67
Young's modulus of elasticity E = 70G Pa

X. DIMENSIONS

The constants associated with the numerical calculation are taken as

Length of rectangular beam x = 4ft
Breath of rectangular beam y = 3 ft
Height of rectangular beam z = 10^{3}ft

XI. CONCLUSION

In this paper, the temperature distribution, displacement function and thermal stresses at any point of a semi-infinite rectangular beam have been obtained, when the boundary conditions are known with the aid of finite Marchi-Fasulo transform and finite Fourier cosine transform and Fourier sine transform techniques. The results are obtain in the form of infinite series in terms of Bessel's function and depicted graphically.

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handled different capacities. At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems (Thermoelasticity in particular) and Operations Research. Published more than 225 research papers in reputed journals and published more than twenty books. Seventeen students awarded Ph.D Degree and ten students submitted their thesis in University for award of Ph.D Degree under their guidance.

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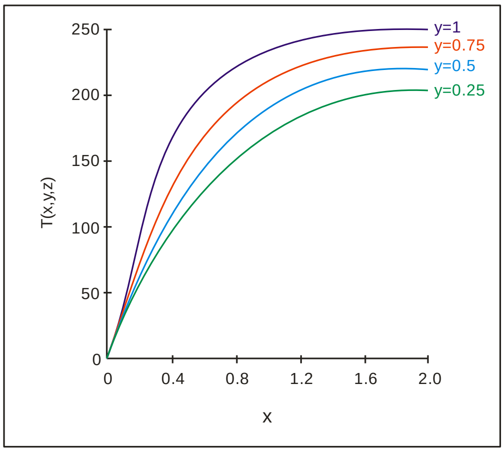
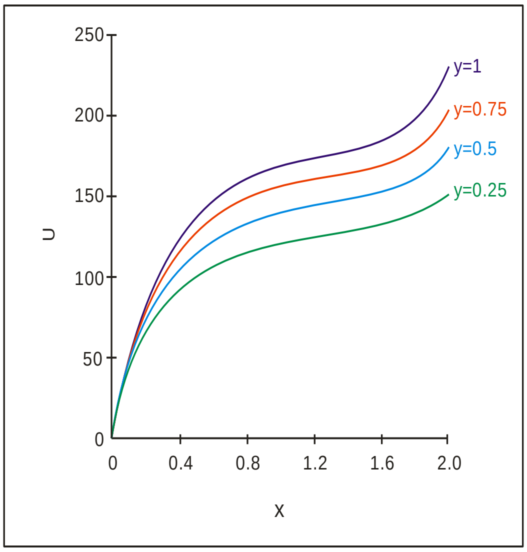
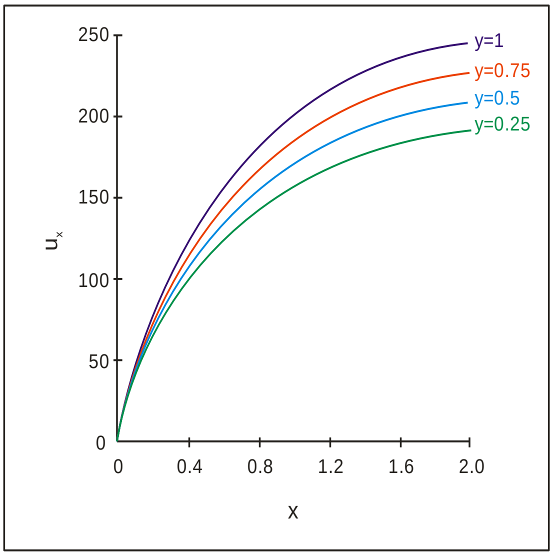


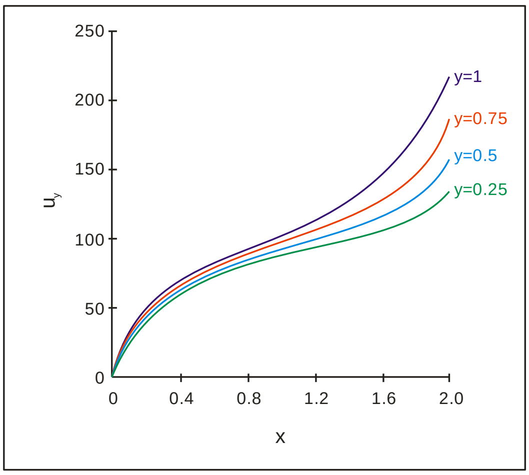
Graph 1: Temperature distribution versus x



Graph 2: Airy's stress function versus x



Graph 3: Displacement component versus x



Graph 4: Displacement component versus x

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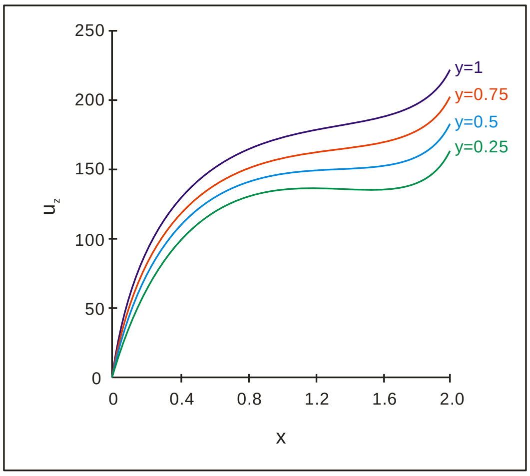
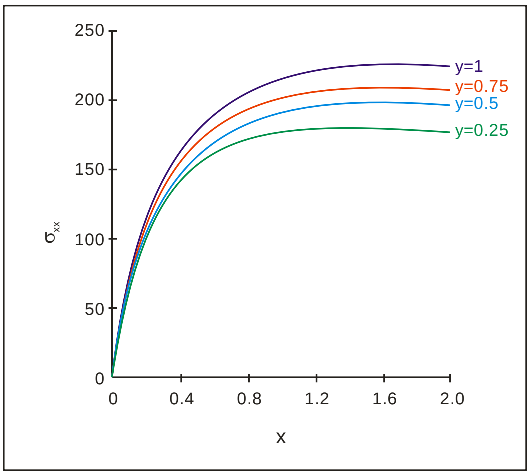
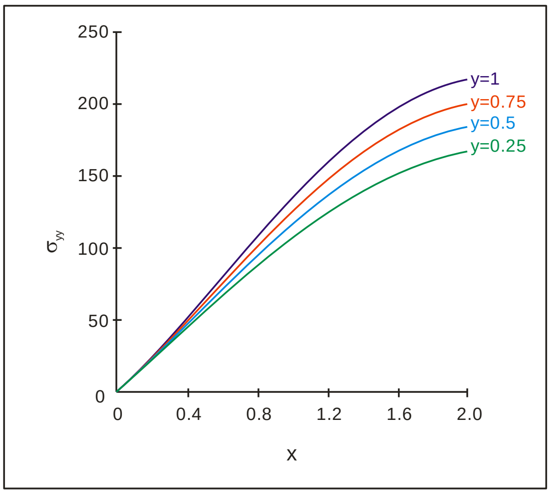


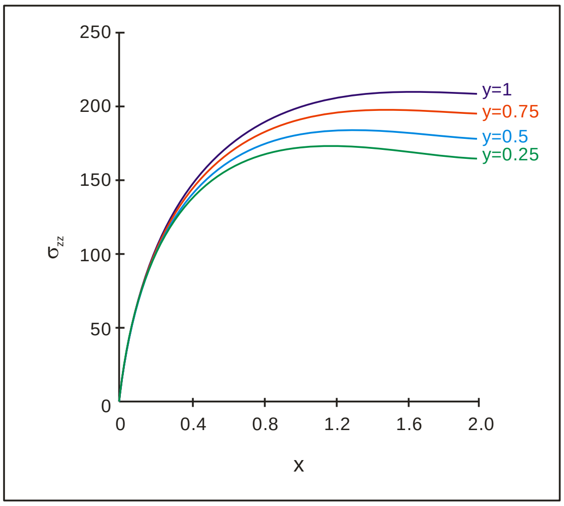
Graph 5: Displacement component versus x



Graph 6: Stress function versus x



Graph 7: Stress function versus x



Graph 8: Stress function versus x