ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 5, Issue 1, July 2015

Semi Compatibility and Weak Compatibility in Fuzzy Metric Space and Fixed Point Theorems

Chandrajeet Singh Yadav
Vadodara Institute of Engineering, Vadodara (Gujarat)

Abstract: ***The Main object of this paper is to establish some common fixed point theorems, for six maps with the concept of semi-compatibility and weak-compatibility, which generalizes the results of Vasuki[10] and Singh and Jain[9].***

Keywords: **Fuzzy metric space, common fixed point, semi-compatibility and weak-compatibility, self-maps.** ***AMS Subject Classification: Primary 47H10, Secondary 54H25.***

I. INTRODUCTION

The tenant of fuzzy sets was introduced by Zadeh[11] in 1965. Kramosil and Michalek [5] initiated the concept of fuzzy metric space and it is modified by George and Veeramani[2]. Banach Contraction Principle in the setting of fuzzy metric space established by Grebies[3], which is milestone to developing fixed point theorems. Vasuki[10] defined R-weakly commutativity of fuzzy metric spaces. In 1977, Singh and Chauhan [7] introduce the notion of compatible maps in fuzzy metric spaces and then proved some fixed point theorems. An important fact about R-weakly commutativity and compatibility is that each pair of R-weakly commuting self-maps is compatible. Weak Compatibility has been introduced by Jungck and Rhoads [4]. According to them two self-maps are weakly compatible if they commute their coincidence points. An idea of semi-compatibility in d-topological spaces has been investigated by Cho et.al.[1].They defined a pair of self-maps (S, T) to be semi-compatible if (i) $Sy = Ty \Rightarrow Sty = TSy$ and (ii) for sequence $\{x_n\} \in X$ and $x \in X$, $STx_n \to Tx$ whenever $\{Tx_n\}$ and $\{Sx_n\} \to x$ as $n \to \infty$. However, (ii) implies (i) bt taking $x_n = y$ and $x = Sy = Ty$. After a while B. Singh and S. Jain[10] defined semi-compatible pair of self-maps in fuzzy metric spaces by condition (ii) only. In this paper, we prove some common fixed point theorems for six self-maps through semi-compatibility and weak-compatibility.

II. PRELIMINARIES

Definition 2.1 : A binary operation $* : [0, 1]\times[0, 1]\to [0, 1]$ is called a *t-norm* if ([0, 1], $*$) is an abelian topological manoid with unit 1 such that $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for a, b, c and d are in [0, 1]. For example $a * b = ab$ and $ab = \min.\{a, b\}$ are t-norm.

Definition 2.2[10] : The three touple (X, M, $*$) is said to be fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^2\times [0, \infty)$ satisfying the following conditions :

For all $x, y, z \in X$ and $s, t > 0$,

(FM-1) $M(x, y, o) = 0$,

(FM-2) $M(x, y, t) = 1$ if and only if $x – y$,

(FM-3) $M(x, y, t) = M(y, x, t)$,

(FM-4) $M(x, y, t) * M(y, z, s) \le M(x, z, t+s)$,

(FM-5) $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left continuous,

(FM-6) $\lim_{t\to\infty} M(x, y, t) = 1$

Note that M(x, y, t) can be considered as the degree of nearness between x and y with respect to t. we identify x = y with M(x, y, t) = 1 for all t > 0. The following example shows that every metric space induces a fuzzy metric space.

Example 2.1[2] : Let (X, d) be a metric space. Define $a * b = \min \{a, b\}$ and

$$M(x, y, t) = \frac{t}{t + d(x,y)}$$

for all $x, y \in X$ and all $t > 0$. Then (X, M, $*$) is a fuzzy metric space induced by d.

Definition 2.3[6] : A sequence $\{x_n\}$ is said to *converge* to a point $x \in X$ if and only if for each $\varepsilon > 0$, $t > 0$, there exists $n_0 \in N$ such that

$$M(x_n, x, t) > 1 - \varepsilon \quad \text{for all } n \ge n_0.$$

The sequence $\{x_n\}$ in a fuzzy metric space (X, M, $*$) is said to be a *Cauchy sequence* if and only if for each $\varepsilon > 0$, $t > 0$, there existsn$_0 \in N$ such that $(x_n, x_m, t) > 1 - \varepsilon$ for all $n, m \ge n_0$.

A fuzzy metric space (X, M, $*$) is said to be *complete* if every Cauchy sequence in it converges to a point in it.

Definition 2.4[10]: Two self-maps A and B in a fuzzy metric space (X, M, $*$) are said to be *weakly commuting* if for each $x \in X$,

$$M(ABx, BAx, t) \ge M(Ax, Bx, t) \text{for all } t > 0.$$

Definition 2.5[10] : Two self-maps A and B in a fuzzy metric space (X, M, $*$) are said to be *R-weakly commuting* if there exists a positive real number R such that for each $x \in X$,

$$M(ABx, BAx, Rt) \ge M(Ax, Bx, t) \quad \text{for all } t > 0.$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 5, Issue 1, July 2015

Definition 2.6[6] **:** Let A and B be two self-maps of a fuzzy metric space (X, M, ∗). Then A and B are said to be *compatible* if and only if

$$M(ABx_n, BAx_n, t) \to 1 \quad \text{for all } t > 0,$$

whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to p$ for some p in X as $n \to \infty$.

Definition 2.7[9] **:** Let A and B be two maps from a fuzzy metric space (X, M, ∗) into itself. Then A and B are said to be *weak-compatible* if they commute their coincidence points, that is,

$$Ax = Bx \Rightarrow ABx = BAx.$$

Proposition 2.1[1] **:** If two self-maps A and B of a fuzzy metric space (X, M, ∗) are compatible then they are weak-compatible.

The following example shows that a pair of self-maps (A, B)is not compatible though it is weak compatible.

Example 2.2 : Let (X, M, ∗) be a fuzzy metric space where X = [0, 16]. t-norm is defined by

$$a * b = \min\{a, b\} \quad \text{for all } a, b \in X \text{ and}$$

$$M(x, y, t) = e^{-\frac{|x-y|}{t}} \quad \text{for all } x, y \in X.$$

Define self-maps A and B on X as follows:

$$Ax = \begin{cases} 16-x & \text{if } 0 \le x < 8 \\ 16 & \text{if } 8 \le x \le 16 \end{cases} \quad \text{and} \quad Bx = \begin{cases} x & \text{if } 0 \le x < 8 \\ 16 & \text{if } 8 \le x \le 16 \end{cases}.$$

Taking $x_n = 8 - \frac{1}{n}$; n = 1, 2, 3 …

Then $x_n \to 8$, $x_n < 8$ and $16 - x_n > 8$ for all n.

Also $Ax_n, Bx_n \to 8$ as $n \to \infty$. Now

$$M(ABx_n, BAx_n, t) = e^{-\frac{|ABx_n - BAx_n|}{t}} \; e^{\frac{-8}{t}} \neq 1 \text{ as } n \to \infty.$$

Hence the pair (A, B) is not compatible.

Definition 2.8[9] **:** A pair (A, B) of self-maps A and B of a fuzzy metric space (X, M, ∗) is called *semi-compatible* if $\lim_{n\to\infty} ABx_n = Bx$, whenever $\{x_n\}$ ia sequence such that

$$\lim_{n\to\infty} Ax_n = \lim_{t\to\infty} Bx_n = x \in X.$$

Proposition 2.2[9] **:** Let A and B be two self-maps on a fuzzy metric space (M, X, ∗). Assume that B is continuous. Then (A, B) is semi-compatible if and only if (A, B) is compatible.

According to proposition-2.2 compatibility implies semi-compatibility. We have a counter example of it. Also, this example shows that the semi-compatibility of the pair (A, B) need not imply the semi-compatibility of the pair (B, A).

Example 2.3 : Let X =[0, 4] and (X, M, t) be a induced fuzzy metric space with (x, y, t) = $\frac{t}{t+|x-y|}$. Define self-map A on X as follows :

$$Ax = \begin{cases} x & if \quad 0 \le x < 2 \\ 4 & if \quad x \ge 2 \end{cases}$$

Let I be the identity map on X and $x_n = 2 - (1/n)$. Then $\{Ix_n\} = \{x_n\} \to 2$ and $\{Ax_n\} \to 2 \neq A(2)$ (because A(2) = 4). Thus (I, A) is not semi-compatible.

For a sequence $\{x_n\}$ in X, such that $\{x_n\} \to x$ and $\{Ax_n\} \to x$, we have $\{AIx_n\} = \{Ax_n\} \to x = Ix$. Thus (A, I) is semi-compatible.

In 1999, Vasuki[10] proved the following theorem for R-weakly commuting pair of self-maps :

Theorem 2.1[10] **:** Let f, g be R-weakly commuting self-maps on a complete matric space (X, M, ∗) such that

$$M(fx, fy, t) \ge r\,(M(gx, gy, t))$$

Where r : [0, 1] → [0, 1] is a continuous function such that r(t) > t for each 0 < t < 1. If f(X)⊂ g(X) and either f or g is continuous then f and g have a unique common fixed point.

In 2005, Singh and Jain[9] generalized the result of Vasuki[10] and proved the following theorem for four self-maps using the concept of semi and weak-compatibilities :

Theorem 2.2[9]**:** Let A, B, S, T be self maps on a complete fuzzy matric space (X, M, ∗) with continuous t-norm ∗ defined by a ∗ b = min{a, b} satisfying the following conditions :

(i) A(X) ⊂ T(X), B(X) ⊂ S(X),
(ii) One of A and S is continuous,
(iii) [A, S] is compatible and [B, T] is weak-compatible and
(iv) For all x, y in X and t > 0

$$M(Ax, By, t) \ge r(M(Sx, Ty, t))$$

Where r : [0, 1] → [0, 1] is a continuous function such that r(t) > t for each 0 < t < 1. Then A, B, S and T have a unique common fixed point.

III. MAIN RESULTS

Theorem3.1: Let A, B, S, T, P and Q be self-maps on a complete fuzzy metric space (X, M, ∗) satisfying the following conditions:

(1) P(X) ⊂ST(X), Q(X)⊂AB(X);
(2) AB = BA, PB = BP, ST = TS, QT = TQ;
(3) Either P or AB is continuous;
(4) (P, AB) is semi-compatible, (Q, ST) is weak-compatible and
(5) for all x, y ∈X and t > 0

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 5, Issue 1, July 2015

$$M(Px, Qy, t) \geq r(M(ABx, STy, t));$$

Where $r : [0, 1] \rightarrow [0, 1]$ is a continuous function such that $r(t) > t$ for each $0 < t < 1$. Then A, B, S, T, P and Q have a unique common fixed point.

Proof : Let $(X, M, *)$ be a complete fuzzy metric space and A, B, S, T, P and Q be six self-maps define on it. Now we prove the theorem n three steps.

***Step – I* :** In this step we construct a Cauchy sequence in X and prove that it is convergent in X. For this let x_0 is an arbitrary point in X. From (1) there exists $x_1, x_2 \in X$ such that

$$Px_0 = STx_1 \text{ and } Qx_1 = ABx_2 = y_1.$$

Inductively, we construct sequence $\{y_n\}$ and $\{x_n\}$ in X such that for n = 0, 1, 2, …

` $$Px_{2n} = STx_{2n+1} = y_{2n} \text{ and } Qx_{2n+1} = ABx_{2n+2} = y_{2n+1}.$$

With $x = x_{2n}$ and $y = x_{2n+1}$ in (**5**), we get

$$M(Px_{2n}, Qx_{2n+1}, t) \geq r(M(ABx_{2n}, STx_{2n+1}, t))$$

$$M(y_{2n}, y_{2n+1}, t) \geq r(M(y_{2n-1}, y_{2n}, t)) > M(y_{2n-1}, y_{2n}, t)$$

i.e., $M(y_{2n}, y_{2n+1}, t) > M(y_{2n-1}, y_{2n}, t)$

Similarly,

$$M(y_{2n+1}, y_{2n+2}, t) > M(y_{2n}, y_{2n+1}, t)$$

In general,

$$M(y_{n+1}, y_n, t) > M(y_n, y_{n-1}, t).$$

Thus $\{M(y_{n+1}, y_n, t)\}$ is an increasing sequence of positive real numbers in [0, 1] and tends to a limit$l \leq 1$. If $l = 1$then

$$\lim_{n\to\infty} M(y_{n+1}, y_n, t) = l > r(l) > 1.$$

Which is a contradiction and so $l = 1$.

Now let p be any positive integer such that

$$M(y_n, y_{n+p}, t) \geq M(y_n, y_{n+1}, t/p) * M(y_{n+1}, y_{n+2}, t/p) * \ldots * M(y_{n+p-1}, y_{n+p}, t/p)$$

taking limit as $n \rightarrow \infty$, we get

$$\lim_{n\to\infty} M(y_n, y_{n+p}, t) \geq 1 * 1 * \ldots * 1$$

$$\Rightarrow \lim_{n\to\infty} M(y_n, y_{n+p}, t) = 1$$

Thus $\{y_n\}$ is a Cauchy sequence in X and also $\{y_n\}$ is converges to z in X because of completeness of x. Hence

$$Px_{2n} \rightarrow z, ABx_{2n} \rightarrow z, Qx_{2n+1} \rightarrow z, Stx_{2n+1} \rightarrow z \quad (1.1)$$

***Step – II* :** Now we prove that z in X is a common fixed point of six self-maps A, B, S, T, P and Q.

<u>When P is Continuous</u>

Since P is Continuous and and (P, AB) is semi-compatible, we get

$$PABx_{2n} \rightarrow Pz, ABPx_{2n} \rightarrow ABz$$

Since the limit in fuzzy metric space is unique, we get

$$Pz = ABz \quad (1.2)$$

[I]. First we prove that Pz = z. Put x = z and $y = x_{2n+1}$ in (5) and suppose on the contrary that $Pz \neq z$.

$$M(Pz, Qx_{2n+1}, t) \geq r(M(ABz, STx_{2n+1}, t)),$$

taking limit as $n \rightarrow \infty$, using (1.1) & (1.2), we get

$$M(Pz, z, t) \geq r(M(Pz, z, t)) > M(Pz, z, t)$$

Which is a contradiction, therefore Pz = z.

[II]. Since $P(X) \subset ST(X)$, there exists $u \in X$ such that Z = Pz = STu. Put $x = x_{2n}$ and y = u in (5), we get

$$M(Px_{2n}, Qu, t) \geq r(M(ABx_{2n}, STu, t)),$$

taking limit as $n \rightarrow \infty$, we get and using (1.1), we get

$$M(z, Qu, t) \geq r(M(z, z, t)) = r(1) = 1.$$

Thus z = Qu, which gives z = Qu = STu and the weak-compatibility of (Q, ST) implies that

$$(ST)Qu = Q(ST)u$$

i.e. $STz = Qz.$

[III]. We now show that Pz = Qz. Suppose on the contrary that Pz ≠Qz and put x = y = z in (5) such that

$$M(Pz, Qz, t) \geq r\,(M(ABz, STz, t))$$

Then by an obvious relation we have

$$M(Pz, Qz, t) \geq r\,(M(Pz, Qz, t)) > M(Pz, Qz, t)$$

Which is a contradiction and so Pz = Qz.

[IV]. Now we show that Bz = z. For this put x = Bz and y = x_{2n+1} in (5), and let Bz ≠z, then

$$M(PBz, Qx_{2n+1}, t) \geq r\,(M(ABBz, STx_{2n+1}, t))$$

Or $$M(BPz, Qx_{2n+1}, t) \geq r\,(M(BABz, STx_{2n+1}, t))$$

Since PB = BP and AB = BA. Now taking limit as $n \to \infty$, using (1.1) and above relations, we get

$$M(Bz, z, t) \geq r\,(M(Bz, z, t)) > M(Bz, z, t)$$

Which is a contradiction and so Bz = z.

Also ABz = z and Bz = z implies Az = z. Thus we have

$$Az = Bz = Pz = Qz = z. \qquad (1.3)$$

[V]. Now let Tz ≠z and put x = x_{2n} & y = Tz in (5), we get

$$M(Px_{2n}, QTz, t) \geq r\,(M(ABx_{2n}, STTz, t))$$

Or $$M(Px_{2n}, TQz, t) \geq r\,(M(ABx_{2n}, TSTz, t))$$

Since QT = TQ and ST = TS. By taking limit as as $n \to \infty$, using above results and from (1.1), we obtain

$$M(z, Tz, t) \geq r\,(M(z, Tz, t)) > M(z, Tz, t)$$

Which is not possible and so Tz = z. Also STz = z and Tz = z implies Sz = z. Thus we have

$$Tz = Sz = z \qquad (1.4)$$

From (1.3) and (1.4), we have

$$z = Az = Bz = Sz = Tz = Pz = Qz.$$

Hence, z is a common fixed point of six self-maps A, B, S, T, P and Q.

<u>When AB is Continuous</u>

Suppose AB is continuous and (P, AB) is semi-compatible. Thus we have

$$ABPx_{2n} \to ABz,\ (AB)^2x_{2n} \to ABz,\ PABx_{2n} \to ABz \quad (1.5)$$

[VI]. We first prove that ABz = z. Suppose on the contrary that ABz ≠z and put x = ABx_{2n} and y = x_{2n+1}in (5), we get

$$M(PABx_{2n}, Qx_{2n+1}, t) \geq r\,(M(ABABx_{2n}, STx_{2n+1}, t))$$

By taking limit as as $n \to \infty$ and using equations (1.1) & (1.5), we get

$$M(ABz, z, t) \geq r\,(M(ABz, z, t)) > M(ABz, z, t)$$

Which is a contradiction and therefore ABz = z.

[VII]. Now since Q(X) ⊂AB(X), there exists w in X such that z = Qz = ABw. Now put x = w and y = x_{2n+1} in (5), we get

$$M(Pw, Qx_{2n+1}, t) \geq r\,(M(ABw, STx_{2n+1}, t))$$

taking limit as as $n \to \infty$ and using equation (1.1), we get

$$M(Pw, z, t) \geq r\,(M(z, z, t)) = r(1) = 1.$$

Thus Pw = z. Since ABw = z and hence Pw = ABw. As(P, AB) is semi-compatible and hence (P, AB) is compatible according to preposition. Then we have

$$Pz = ABz = z.$$

Also all the remaining relation are from [II], [III], [IV] and [V]. Hence, we conclude that

$$z = Az = Bz = Sz = Tz = Pz = Qz.$$

Finally, z∈X is a common fixed point of six self-maps A, B, S, T, P& Q in both the conditions.

Step – III **: Uniqueness :** we finally show that fixed point z of six self-maps A, B, S, T, P & Q is unique.

Suppose v∈X be another common fixed point of A, B, S, T, P & Q, then we have

$$v = Av = Bv = Sv = Tv = Pv = Qv.$$

Put x = z and y = v in (5), we get

$$M(Pz, Qv, t) \geq r\,(M(ABz, STv, t))$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 5, Issue 1, July 2015

$\Rightarrow$ $M(z, v, t) \geq r(M(z, v, t)) > M(z, v, t)$

Which is a contradiction and thus z = v. Hence common fixed point z is unique for A, B, S, T, P and Q in both the condition. This completes the proof. ■

Theorem3.2 : Let A, B, S, T, P and Q be self-maps on a complete fuzzy metric space (X, M, ∗) satisfying the following conditions :

(6) $P(X) \subset ST(X)$, $Q(X) \subset AB(X)$;
(7) AB = BA, PB = BP, ST = TS, QT = TQ;
(8) Either P or AB is continuous;
(9) (P, AB) is compatible, (Q, ST) is weak-compatible and
(10) for all $x, y \in X$ and $t > 0$
$$M(Px, Qy, t) \geq r(M(ABx, STy, t));$$

Where $r : [0, 1] \to [0, 1]$ is a continuous function such that $r(t) > t$ for each $0 < t < 1$. Then A, B, S, T, P and Q have a unique common fixed point.

Proof : In the view of proposition-2.2 and theorem-3.1, it suffice to prove the theorem when P is continuous. As the proof of theorem-3.1, construct a sequence $\{y_n\}$ which is a Cauchy sequence in X and hence converges to some $z \in X$ and (6) is true. Since P is continuous and (P, AB) is compatible, we get

$$PABx_{2n} \to Pz,\ (Px_{2n})^2 \to ABz, ABPx_{2n} \to Pz \quad (1.6)$$

[I]. We first we prove that Pz = z. Let $Pz \neq z$. Now put $x = Px_{2n}$ and $y = x_{2n+1}$ in (10), we get

$$M(PPx_{2n}, Qx_{2n+1}, t) \geq r(M(ABPx_{2n}, STx_{2n+1}, t))$$

taking limit as $n \to \infty$ and using (1.6) & (1.1), we obtain

$$M(Pz, z, t) \geq r(M(Pz, z, t)) > M(Pz, z, t)$$

Which is a contradiction, therefore Pz = z.

[II]. Since $P(X) \subset ST(X)$, there exists $u \in X$ such that z = Pz = STu. Put $x = x_{2n}$ and y = u in (10), we get

$$M(Px_{2n}, Qu, t) \geq r(M(ABx_{2n}, STu, t)),$$

taking limit as $n \to \infty$, we get and using (1.1), we get

$$M(z, Qu, t) \geq r(M(z, z, t)) = r(1) = 1.$$

Thus z = Qu, which gives z = Qu = STu and the weak compatibility of (Q, ST) implies that

$$(ST)Qu = Q(ST)u \text{ i.e. } \quad STz = Qz.$$

[III]. Now since z = Qu and $Q(X) \subset AB(X)$, there exists v in X such that z = Qz = ABv. Now put x = vand y = u in (10), we get

$$M(Pv, Qu, t) \geq r(M(ABv, STu, t))$$
$$= r(M(z, z, t)) = r(1) = 1.$$

Thus Pv = Qu. Since z = ABv = Pv. Again since (P, AB) is semi-compatible (according to proposition-1), we obtained PABv = ABPv and thus Pz = ABz = z.

[IV]. We now show that Pz = Qz. Suppose on the contrary that $Pz \neq Qz$ and put x = y = z in (10) such that

$$M(Pz, Qz, t) \geq r(M(ABz, STz, t))$$

$$\Rightarrow \quad M(Pz, Qz, t) \geq r(M(Pz, Qz, t)) > M(Pz, Qz, t)$$

Which is a contradiction and so Pz = Qz.

According to theorem-3.1 (when P is continuous) we get

Bz = z and Tz = z, and since ABz = z $\Rightarrow$Az = z.

Again since STz = z $\Rightarrow$Sz = z. Therefore we have

$$z = Az = Bz = Sz = Tz = Pz = Qz.$$

That $z \in X$ is a common fixed point of six self-maps A, B, S, T, P & Q and the uniqueness of z follows as in thetheorem-3.1. This completes the proof. ■

By assuming P = Q = f, A = S = g and B = T = 1 in theorem-3.2, we obtain the following result :

Theorem3.3 : Let f and g be compatible self-maps on a complete fuzzy metric space (X, M, ∗) satisfying

$$M(fx, fy, t) \geq r(M(gx, gy, t))$$

Where $r : [0, 1] \to [0, 1]$ is a continuous function such that $r(t) > t$ for each $0 < t < 1$. If $f(X) \subset g(X)$ and either f or g is continuous then f and g have a unique common fixed point.

IV. CONCLUSION

The existence of a fixed point is nothing but the existence of a solution of a particular equation. It is often an important piece of information. It will helpful to a qualitative understanding of the mathematical models. In addition to that, here, we developed some fixed point theorems.Theorem-3.3 generalizes the result of Vasuki[10] by assuming only compatibility of the pair (f, g) in the place of its R-weakly commutativity. Thus thorem-3.3 for six self-maps is a quite better generalization of a result of Vasuki[10]. Also theorem-3.1 and 3.2 for six self-maps also generalizes the results of Singh and Jain[9], whenever B = T = I.

V. ACKNOWLEDGMENT

I am grateful to **Late Dr. Bijendra Singh**, former Professor and Head of the Department of School of Studies in Mathematics, Vikram University, Ujjain (M.P.) INDIA, for his contribution of knowledge towards the research. His continuous encouragement, valuable suggestions and support for the upliftment of this work. His role for the society of mathematics is stimulating

REFERENCES

[1] Y. J. Cho, B. K. Sharma and D. R. Sahu, Semi-compatibility and fixed points, Math Japonica 42 (1995), 91-98.

[2] George, A. and Veeramani, P., on some results in fuzzy metric spaces, Fuzzy Sets and System 64 (1994), 395-399.

[3] Grebiec, M., Fixed point in fuzzy metric space, Fuzzy Sets and Systems 27(1998), 385-389.

[4] G. Jungck and B. E. Rhoades, Fixed point for set valued functions without continuity, Indian J., Pure App. Math. 29 (1998), 227-238.

[5] I. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetica 11 (1975), 326-324.

[6] S. N. Mishra, N. Mishra and S. L. Singh, Common fixed point of maps in fuzzy metric space, Int. J. Math. Math. Sci. 17(1994), 253-258.

[7] B. Singh and M. S. Chauhan, Common fixed point of compatible maps in fuzzy metric spaces, Fuzzy Sets and System 97 (1998), 395-397.

[8] B. Singh and A.Jain, Fixed point theorem using weak-compability in fuzzy metric space, V. J. M. S. 5 (1), (2005), 297-306.

[9] B. Singh and S. Jain, Semi-compatibility, compatibility and fixed point theorems in fuzzy metric spaces, Jour. of the Chungcheong Math. Soc. 18 (1), (2005), 1-22.

[10] R. Vasuki, Common fixed points for R-weakly commuting maps infuzzy metric spaces, Indian J. Pure Appl. Math. 31(4), (1999), 419-423.

[11] L.A. Zadeh, Fuzzy sets, Inform and control, 89 (1965), 338-353.

AUTHOR'S PROFILE

Chandrajeet Singh Yadav is born in 1982 at Ujjain, a city of Temples of Madhya-Pradesh (INDIA). He has received Bachelor degree in 2003, master degree in Mathematics in 2005 and Master of Philosophy in Mathematics in 2006 from Vikram University, Ujjain. He is working as Assistant Professor in the department of Mathematics of Vadodara Institute of Engineering, Vadodara (GUJARAT). He has around eight years of teaching experience. He has life membership of **Indian Mathematical Society** (IMS) and **Indian Society for Technical Education** (ISTE).