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Experimental and FE Analysis of Funicular Shells

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Abstract: ***Shells belong to the class of stressed skin structures which, because of their geometry and small flexural rigidity, tend to carry loads primarily by direct stresses acting in their plane. Concrete funicular shells of square ground plan, double curvature with 80 mm rise are analysed for uniformly distributed load (One-way action). Specimens of size 1080 mm x 1080 mm in plan with rectangular edge beam of 50 mm x 40 mm are prepared using cement concrete of grade M20 for which the mix design is carried by Indian Standard method. The precasting of the Concrete shallow funicular shell specimens is carried by masonry mould method. The specimens are prepared with thickness of 25 mm and 20 mm. The specimens are moist cured for 28 days before testing. The uniformly distributed load over the shell specimen is applied and the corresponding deflections, strains are measured. The coordinates of funicular shells are determined by developing a computer program. To relate experimental results to theory, the finite element technique (SAP 2000 Program) is utilized to analyse a similar model in the elastic range. Finite element models of funicular shells are developed by discretizing the shell specimens into 20 elements along x direction and 20 elements along y direction. Behaviour of funicular shells under uniformly distributed load is carried out. Conclusions are made by comparing the experimental and analytical results.***

Keywords: Funicular shell, Edge beam, One-way action, Finite element models, Mix-design, Discretization, SAP.

I. INTRODUCTION

Environmental degradation witnessed today is a result of an irresponsible use of materials. Materials are being made to perform contrary to their natural qualities. Most materials behave best in compression but the over reliance on RCC has resulted in tensile structures which are made to perform contrary to the natural qualities of the materials. For example, a conventional beam upon loading tends to bend at the centre. The upper region of the beam is in compression while the lower part is in tension. To counter the tensile stresses steel reinforcements are required in the lower portion against its natural capabilities. Upon inverting the structure, it is converted into a compression structure, with a considerable reduction in the amount of steel and cement. In this case, a nominal ring beam is capable of taking the lateral thrust developed in the structure. Traditionally compression structures in the form arches, vaults, domes, catenaries and doubly curved structures also called funicular shells have been used extensively in the temples / forts. The problem of prefabricated shells of double curvature, subjected to concentrated load and supported at two edges, has previously been investigated. In practice there are cases in which shells are supported on four edges, however, a survey of the related literature reveals not much research has done in this area.

II. LITERATURE REVIEW

The selection of available documents (both published and un published) on the topic, which contain information, ideas, data and evidence written from a particular standpoint to ful fill certain aims or express certain views on the nature of the topic and how it is to be investigated, and the effective evaluation of these documents in relation to the research being proposed. A literature review is a survey of already existing writings (usually published) on a given topic or area with a view to assessing their relevance to a proposed project. The behavior of prefabricated shell units has been investigated, both theoretically and experimentally, by many researchers. Apart from construction schemes, the use of pre fabricated shell units creates certain problems which have to be investigated from the research point of view. One of the most important is the structural behavior of the units when subjected to the concentrated loads at various points along their span. Another consideration of interest is the ultimate strength and pattern of failure for these types of shells. The shells of double curvature usually offer a higher ultimate strength than the shells of single curvature, such as cylindrical shells. The funicular shell roof is one such compression structure, which ensures conservation of natural resources by utilizing waste materials effectively and optimizing the use of expensive steel and cement. Further, the

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arch distributes the point load in all direction equally thus, is able to withstand impact loading at any point. Diagonal grid of funicular shell gives the illusion of a larger space.

III. REVIEW OF SELECTED TECHNICAL PAPERS

Albolhassan Vafai and Mehdi Farshad (1979) have done a research work on theoretical and experimental study of prefabricated funicular shell units. This paper presents the results of a theoretical and experimental study of funicular shell structures. A particular design is given for the geometry, the form, and the type of reinforcement of the units. Ten models are constructed based on this design. First, six samples supported at two edges are loaded to a specified load within the elastic region. Electrical resistance gauges are mounted both inside and outside at several locations on the surface of the shell on two different specimens. Also, dial gauges are installed at several locations on the surface of four shells. Following these non-destructive tests, all ten samples are loaded to failure, subjected to a concentrated load at the center. To relate experimental results to theory, the finite technique is utilized to analyze a similar model. The experimental values of membrane stresses along the central section in the direction of the supports are calculated and compared with the theory. The results are in close agreement at some distance away from the supports, but the difference becomes noticeable closer to the support. The same phenomenon appears to be true when the experimental values of vertical deflections along the longitudinal and transverse sections of the shells are compared with the theory. Also, the experimental failure loads are found to be directly related to the amount of reinforcement, and the age of the concrete shells.

Abolhassan Vafai, Massoud Mofid and Homayoon E.Estekanchi (1997) have done a research work on experimental study of prefabricated funicular shell units. The experimental values of membrane stresses in the elastic range along diagonal sections of funicular shell structures are calculated and compared with the theory. The results are generally found to be in close agreement with the theory. Experimental values of vertical deflections along the longitudinal and transverse sections of the shells also compare favorably with the theory. The experimental failure and crack loads are found and empirical equations, expressing the relation between rise and failure crack loads, are given. This paper contains the results of numerical and experimental studies carried out in the area of funicular shell units with square bases supported at four edges and subjected to concentrated loading at the apex. A particular design is given for the geometry, the form, the rise and the type of reinforcement of the units. Forty-five models are constructed based on this design. First, eight samples supported at four edges are loaded to a specified load within the elastic region. Electrical resistance gauges are mounted both inside and outside at several locations along the diagonal on the surface of the shell on two different specimens with a rise of 6 and 9cm, respectively. Also, dial gauges are installed at several locations on the surface of the other six specimens with varying rise and reinforcement types. Following these non-destructive tests, all 45 samples are loaded to failure, subjected to concentrated load at the centre. To relate experimental results to theory, the finite element technique (SAP90 Program) is utilized to analyze a similar model in the elastic range.

John W. Weber, Kwong-chi Wu and Adholhassan Vafai (1984) have done an evaluation of ultimate loads for shallow funicular concrete shells. Shells belong to the class of stressed skin structures which, because of their geometry and small flexural rigidity of the skin, tend to carry loads primarily by direct stresses acting in their plane. Ten shallow funicular concrete shells were loaded to failure with a concentrated central force. Five shells were randomly reinforced with steel wires and the reminder with a wire mesh through the middle surface. All of the shells were 90 cm x 90 cm in plan form. Strain gauges showed a linear relationship between load and strain in the elastic range of the concrete, whereas measured deflections were larger than those determined analytically by small deflection theory, would be more appropriate for theoretical investigations of shallow funicular shells subjected to large concentrated loads. Ultimate loads were not clearly related to type of reinforcement, but were a function of the rise and thickness of the shell in general the larger the rise parameter (square of the ratio of rise to thickness), the larger the ultimate load. Failure patterns for shells with both kinds of reinforcement were the same. They observed that the mathematical investigations of shallow funicular shells with large concentrated loads should be based on large deflection theory and the deflection characteristics of a shell vary closely with its rise parameter.

S.Elangovan and A.R.Santhakumar (1988) have done a research work on parametric study of funicular shells. The funicular shell was analyzed by the finite element method using isoperimetric elements with five degrees of freedom at each node. A computer programme developed by the authors was used in the analysis of funicular shells with clamped boundaries. The behaviour of the shell under uniformly distributed load, for various ratios of spans and rise/span was studied analytically. Approximate expressions for the calculation of bending moment at the edge of the shell, in-plane force at crown, and deflection at crown had been proposed. The funicular shell is a shallow shell of double

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curvature. The shape of such shells to suit any boundary geometry can be found by analytical or experimental methods. The behaviour of the shell under uniformly distributed load, for various ratios of spans and rise/span was studied analytically. The load applied was within the elastic range and hence, the deflections, forces, and moments are proportional to the load. So the intensity of the uniformly distributed load was taken equal in all cases.

IV. METHODOLOGY

Funicular shells in square ground plan are considered for the investigation. Numerical approach using advanced finite element analysis based software called as SAP 2000 is adopted. Based on reported literature of similar nature a FE model is developed and an experimental study is conducted to investigate the behaviour of funicular shells under uniformly distributed. Concrete shallow funicular shells of square plan, double curvature with various rises are analyzed for concentrated central load.

V. GENERATING COORDINATES FOR FUNICULAR SHELLS

The coordinates of funicular shells are determined by developing a computer program. Here we are using Turbo C as the compiler to compile the program. Steps followed in the program are,

1. Declare integer values of x, y, s. c;
2. Initialize the integer values of f, a, b;
3. Declare Z as a float value
4. Take choice from user whether to continue or to stop by pressing 1 or 0;
5. Take values of x and y by user;
6. Then calculate the value of Z, by using formula of

$$Z = \frac{f}{a^2 b^2}(a^2 - x^2)(b^2 - y^2)$$

Where, Z = vertical ordinate at point x, y
f = maximum central rise
a = half length of the shell
b = half width of the shell
x, y = the co-ordinates of the grid point from the origin, which is taken as the centre of the shell unit

7. The print the value of Z;
8. Take choice of user whether to continue or not;

VI. EXECUTION OF PROGRAM

To correlate experimental results with theory, a theoretical study is carried out. The model consisted of a funicular shell uniformly loaded at its top. The finite element technique and a related computer program (SAP 2000) were utilized to analyze structure. The finite element model used to analyse the shell is based on the assumption that the material is linearly elastic. Hence, the theoretical prediction is expected to be valid only in the elastic region of the shell behavior. The plan of the shell, its dimensions and other dimensions are represented in the table.1.

Table.1. Shell Dimensions

Shell	Plan in m	Rise in mm	Thick in mm	Edge Beam Reinforcement
FS I	1.08 X 1.08	80	25	6mm Dia rod all round
FS II	1.08 X 1.08	80	20	6mm Dia rod all round

The finite element model of funicular shell with square ground plan is developed by using SAP 2000 finite element package is shown in figure 1.

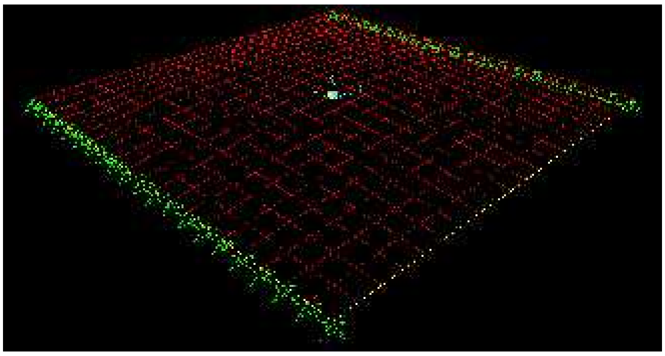

Fig.1 Funicular shell Model in SAP2000(3D View)

The plan of the shell, its dimensions, and mesh consisting of 400 elements, 54 X 54 mm each is shown in figure.2.

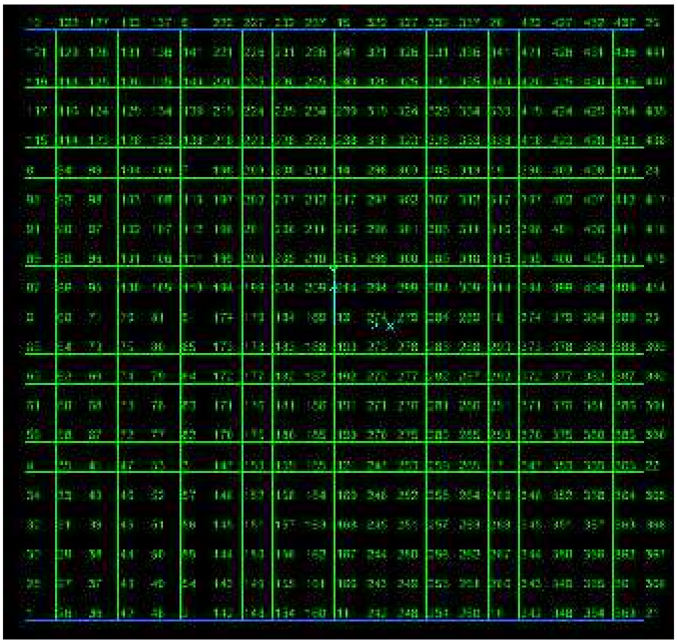

Fig.2. Finite Element Modelling and corresponding Node Numbers.

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VII. EXPERIMENTAL INVESTIGATIONS

Following materials are used in construction of shell specimens.

1) Cement: OPC 53 Grade.
2) Fly ash: 30% of cement is replaced by fly ash.
3) Aggregates: Fine and Coarse.
4) Super plasticizer: CONPLAST 430
   CERAPLAST 400

Physical properties of cement are given in table.2.

Table.2. Physical properties of procured OPC

Particulars	Test Results
Fineness of cement	2.96 %
Standard Consistency	29 %
Setting Time Initial Final	 200 minutes 4 hr 10 minutes
Compressive Strength 3 Days 7 Days	 16.22 N/mm^2 19.86 N/mm^2

VIII. CONSTRUCTION OF FUNICULAR SHELL SPECIMENS

Precasting of the doubly curved shell units may be carried out by any one of the methods given below.

1. Sagging Fabric Method.
2. Masonry Mould Method.
3. Mechanized Process.

In the present work masonary mould method has been adopted.

To study the stress distribution, deflections of funicular shells under uniformly distributed load specimens are prepared and designated as

FS1: Funicular shell with rise 8 cm and thickness 25 mm.
FS2 : Funicular shell with rise 8 cm and thickness 20 mm.

The Mix Design for Funicular shell 1 has been arrived at 1:1.4:2.34. Cement is partially replaced with 30% of fly ash and for better workability 0.2% Superplasticizer (Conplast 430) was added. Five concrete cubes and cylinders are cast for compression and split tensile tests at 28 days.

The Mix Design for Funicular shell 2 has been arrived at 1:1.5:1.6. Cement is partially replaced with 30% of fly ash and for better workability 0.2% Superplasticizer (Ceraplast 400) was added. Seven concrete cubes and cylinders are cast for compression and split tensile tests at 28 days.

In masonary mould method, for any desired raise of the shell of rectangular or square plan the ordinates on various points can also be calculated using approximate formula. (Fig 3).

Fig.3. Fixing shell coordinates

The ordinates obtained shall be set off from a level platform and the surface concreted , finished smooth. (Fig 4).

Fig.4. Finishing the surface with cement mortar

The finished surface shall be coated with oil, grease or any other releasing agents. (Fig 5).

Fig.5. Oil coating

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Outer mould for the edge beams shall be set up. (Fig 6).

Fig.6. Fixing outer mould for the shell

The designed reinforcement for the edge beam shall be placed between the outer edge beam mould and the masonry platform built up. The minimum reinforcement in the edge beams of the shell shall be one 6 mm diameter mild steel bar. (Fig 7).

Fig.7. Placing the reinforcement in edge beam mould

Concrete of the specified mix shall be laid in the edge beam and over the shell mould. (Fig 8).

Fig.8. Casting of edge beam

The thickness over the shell mould equal to the designed thickness of the shell shall be controlled by thickness gauges. (Fig 9).

Fig.9. Finished shell Surface

The edge beam mould should be released three hours after casting. (Fig 10).

Fig.10. Removing of edge beam mould

The shell should be lifted off the mould using levers at the four corners 24 to 48 hours after casting. Longer time up to 72 hours may be necessary in cold climates (below 25^{0}c) as also when pozzolana cement is used. In using levers ensure that the levers are operated only on one side at a time and never at the end at diagonals. (Fig 11).

Fig.11. Finished funicular shell

The shell should be kept stacked and cured in the normal way. The stacking of the shells may be done one above the other supported at four corners only. Shells units up to 1.5 m in size may be handled manually. Shells heavier than this will need the help of hoisting equipment and appliances. For small size shells where handling is done manually, provision of lifting hooks are not necessary. The procedure is repeated for casting the specimens FS1 and FS2 with rise 8 cm and varying thickness. The shell specimens are moist cured for 28 days before testing.

IX. INSTRUMENTATION AND TESTING

To investigate the behaviour of the funicular shell subjected to a uniformly distributed load at its top surface, the structure was instrumented and loaded as follows.

Shells FS1 and FS2 are simply supported on a loading frame and subjected to uniformly distributed load. To simulate uniformly distributed load a number of 30 kg sand bags have been prepared and placed on top of the shell in a uniform manner. To check the elastic behavior of the shells, the structures were loaded and unloaded several times. The two shell specimens FS1 and FS2 were placed in a loading apparatus designed for this purpose, as shown in figure 12.

Fig.12. Sand bags loading

Deflection measurements require more thought than might be supposed. Since the shell is flexible, it is not very easy to measure tangential displacements with a reasonable degree of accuracy, except at its edges and it is best to confine oneself, on the main part of the shell proper, to the measurement of radial deflections. Two specimens with rise of 8cm, thickness 25 mm and 20 mm were used for dial gauge readings. Figure 7.2 shows the location of installed dial gauges. Deflection measurements were also carried out for shells FS1 and FS2 with rise of 8cm, thickness 25 mm and 20 mm respectively. (Fig 13).

Fig.13. Location of dial gauges

Two specimens with rise of 8cm, thickness 25 mm and 20 mm respectively, were chosen for strain measurement. Electrical resistance strain gauge rosettes were mounted on the top and bottom surface of the two shells (FS1 and FS2) at different locations. Electrical resistance strain gauges have a suitable sensitivity, they occupy little space, do not require access once fixed and being read from a control panel so electrical strain gauges were chosen for measuring strains. Figure 14 shows the instrumented specimens and the installed strain gauges respectively. Of these, the two instrumented shells, FS1 and FS2 with 25 mm and 20 mm thicknesses respectively, were loaded uniformly at the top surface in the elastic range. Strain gauge readings were recorded at different load increments. The results are presented.

Fig.14. Instrumented shell and location of strain gauges

X. RESULTS AND DISCUSSIONS

Two shells of double curvature were constructed and tested. From testing the specimens FS1 and FS2, the values of strains and deflections for different loadings were recorded. From the experimental strain values stresses are found out and are compared with analytical stress values. Funicular shell I and Funicular shell II are loaded up to 1145 Kg and 1140 Kg respectively. It is observed that the deflection of the shell due to applied loads is small. From results obtained it is found out that when the shell thickness is reduced the

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deflection of the shell also decreases for the same applied load. Deflection is more in edge beam compared to shell center.

Analysis of funicular shell 1(FS I)

The analysis of Funicular shell over square ground plan FS I is made and from the results of the analysis, a graph is plotted between the uniformly distributed load and membrane stress distribution. From the visualization, the maximum deflection under the load, the maximum stresses for the each analysis are observed. Maximum tension stresses are developed at shell corners. The comparisons of maximum stress with the different applied load are shown in figure.15.

It should be noted that at most locations the theory yields lower values than experiment. This would be because of non-uniform applying of loads experimentally due to practical difficulty of applying loads uniformly. The discrepancies between the theory and experiment both in the case of stress and deflection calculations could be attributed to the possible errors involved in the computations in the process of matrix inversion by single precision and also non-consideration of material nonlinearity in stiffness matrix. Also, the element sizes in the theoretical analysis are not fine enough to simulate the physical structure. A stress contour for maximum stress of Funicular shell I over square ground plan is shown in figure 16. The maximum value of tensile stress is observed on the shell edges.

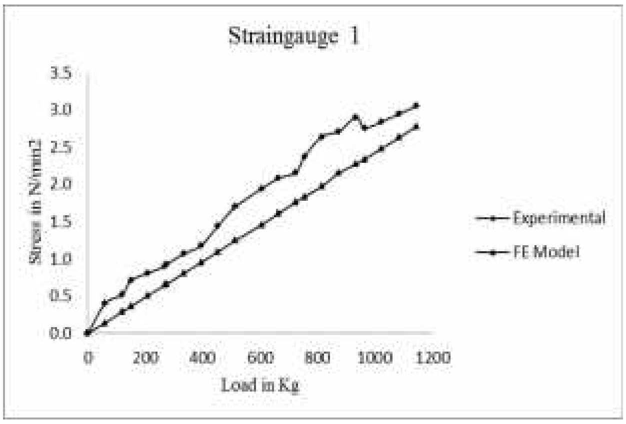

Fig.15. Stress at strain gauge 1 for FS I

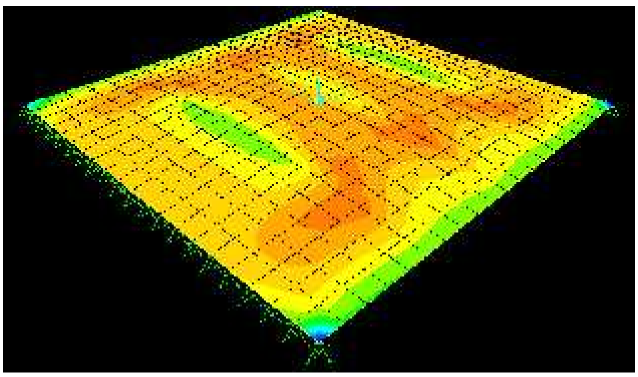

Fig.16. Stress contour for FS I

From the analysis results, the deflections of the concrete funicular shell over square ground plan along vertical direction are calculated and a plot is made between the various points considered in the shell and the corresponding deflections as shown in figure 17.

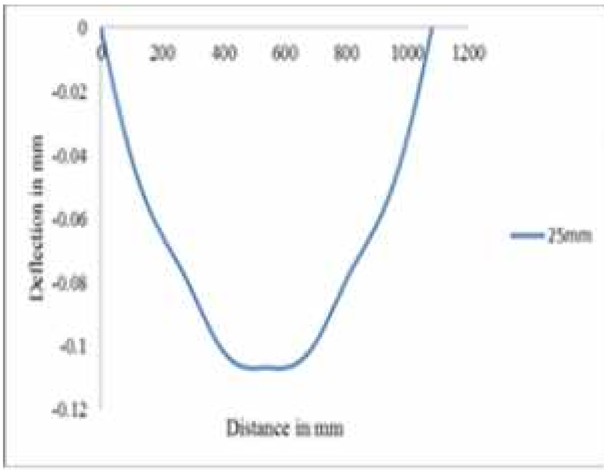

Fig.17. Deflection Curve for concrete funicular shell I

Analysis of funicular shell 1I (FS II)

The analysis of Funicular shell over square ground plan FS II is made and from the results of the analysis, a graph is plotted between the uniformly distributed load and membrane stress distribution. From the visualization, the maximum deflection under the load, the maximum stresses for the each analysis are observed. Maximum tension stresses are developed at shell corners. The comparisons of maximum stress with the different applied load are shown in figure.18.

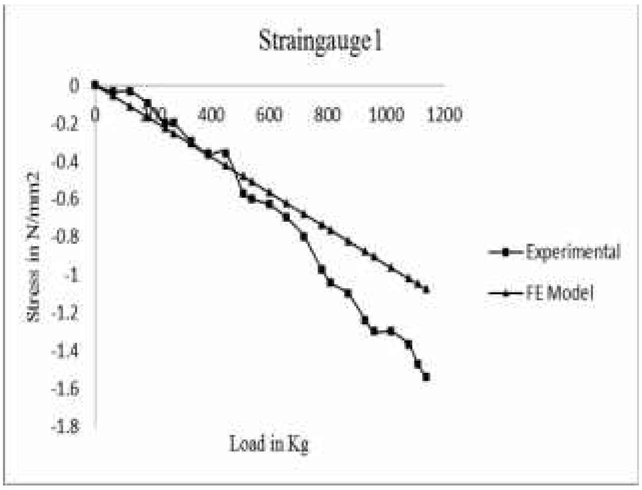

Fig.18. Stress at strain gauge 1 for FS II

It should be noted that at most locations the theory yields lower values than experiment. This would be because of non-uniform applying of loads experimentally due to practical difficulty of applying loads uniformly. The discrepancies between the theory and experiment both in the case of stress and deflection calculations could be attributed to the possible errors involved in the computations in the

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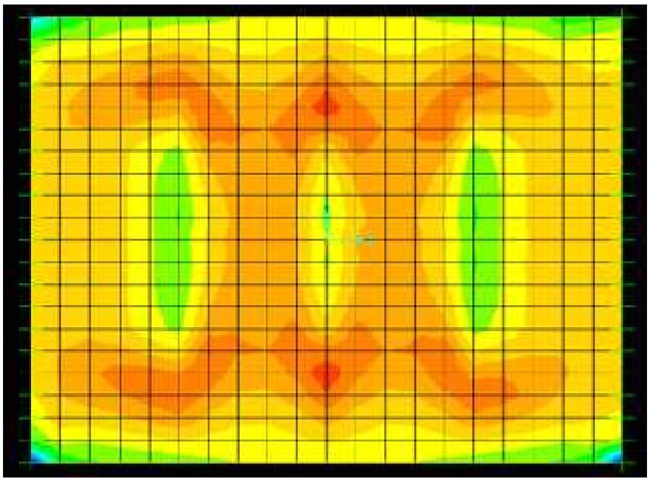

Fig.19. Stress contour for FS II

From the analysis results, the deflections of the concrete funicular shell over square ground plan along vertical direction are calculated and a plot is made between the various points considered in the shell and the corresponding deflections as shown in figure 20.

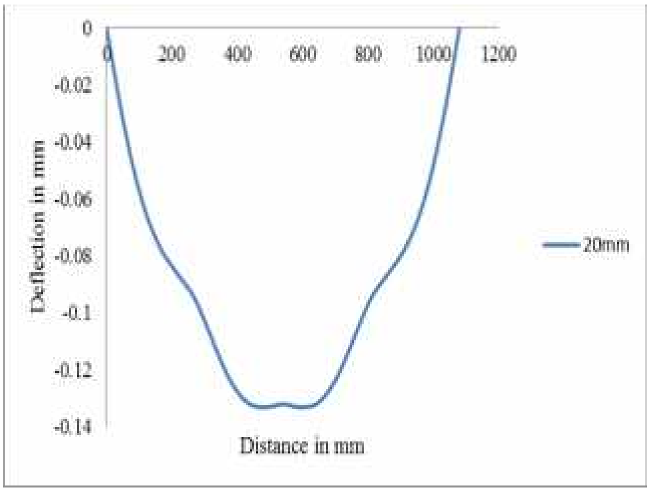

Fig.20. Deflection Curve for concrete funicular shell II

XI. SUMMARY AND CONCLUSIONS

An experimental and analytical investigation has been carried out to study the performance of funicular shells with varying thickness using SAP 2000 software. Various parameters like variation of deflection, stress distribution have been considered in the study. The performance of funicular shell under uniformly distributed load is presented in this study.

The following conclusions are drawn from the test results:

1. The deflection of concrete funicular shell decreases with increase in thickness.
2. Membrane stresses decreases with the increase in thickness of concrete funicular shell.
3. Maximum tension stresses are developed at shell corners.
4. Performance of funicular shell I is better when compared to funicular shell II under uniformly distributed load.
5. The deflection of shallow funicular concrete shell decreases with increase in rise.
6. Membrane stresses decreases with the increase in rise of shallow funicular concrete shell.

XII. SCOPE FOR FUTURE WORK

Precast doubly-curved shells have been in use for the past few years as roofing and flooring elements. These shell units are effectively used for replacing the solid RCC slabs. They may be used in conjunction with precast joints or battens or planks or as waffle shells by providing in situ ribs in two directions. This type of construction has many advantages over the conventional RCC slabs as it is lighter in weight and saves reinforcing steel and concrete. It is also suited for workshop floors and loading platforms carrying heavy uniform loads. It eliminates the use of shuttering fully and scaffolding partially. The sizes of the shells are chosen depending upon handling facility available. The experimental results presented in this work pertain to the size of specimens chosen in the study, which has a potential for application to industrial use. However, the exact nature of behavioural response of large dimension shells is a topic for further investigation. Analytical results can be improved by considering non-linearity and also by using finer meshing.

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