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Surface Modes in Layered Inhomogeneous Metamaterial Structures

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***Abstract*— A simple robust method for calculating dispersion curves of surface modes in layered media containing an inhomogeneous metamaterial is presented. The metal properties employed in the metamaterial are defined by published experimentally determined optical constants. The numerical procedure is based on solving the Helmholtz's equation. The approach is demonstrated on examples that are of the current technological interest. These include metamaterial/air and metamaterial/semiconductor structures. The main outcomes of this paper, i. e. the surface plasmon characteristics, may provide wide avenues for creating plasmon devices for integrated optoelectronic applications.**

***Index Terms*— surface mode, inhomogeneous metamaterial, wave equation, permittivity profile.**

I. INTRODUCTION

Metamaterials are engineered composites tailored for specific electromagnetic properties that are not found in nature and not observed in the constituent materials. They are constructed by periodic arrays of small metal and/or dielectric particles. As such, they are considered artificial electromagnetic materials and are usually designed for sub wavelength levels. The electromagnetic resonators or "particles" such as split-ring resonators and nanowires are the structural units of the metamaterials [1]. It is interesting to note that the evidence for a composite medium – interlaced lattices of conducting rings and wires – presenting negative permittivity and permeability has been suggested by Smith and co-workers [2, 3].

Due to the fact that the progress in the study of metamaterials is impressive, it is worthwhile to investigate a material with permittivity and permeability varying in space. We refer to such materials as *inhomogeneous metamaterials*. There have been some proposals in designing inhomogeneous metamaterials for cloaking by means of spectral representation [4, 5] as well as studying the electromagnetic field distributions in the designed imperfect cloaks.

It is worthwhile mentioning that the negative refractive index metamaterials (NRM) with spatially varying effective permittivity and permeability within the NRM structure and with a gradual transition from the positive refractive index metamaterials (PRM) to NRM and vice versa are of special interest. A graded refractive index deserves attention in the field of transformation optics including hyper lenses [6] and invisibility cloaks [7]. The investigation of gradient refractive index (GRIN) metamaterials opens wide avenues for applications, including beam shaping and directing, enhancement of nonlinear effects [8], super lenses [9] etc.

In this paper, we consider properties of surface waves existing at a boundary between a semiconductor and a metamaterial. Graded index optical structures have already been studied in the framework of metamaterial gradient index lenses by a few authors [9, 10]. They have shown that such a structure reduces geometrical aberrations; a gradient index metamaterial lens was also validated experimentally in [11]. These works addressed the propagation problems through graded index structures approximately with geometrical optics while dealing with a one-layer structure. We suggest to investigate a more complex problem, i.e. introducing a second layer such as a semiconductor.

Here we present a numerical solution of the Helmholtz's equation for the propagation of electromagnetic waves through an inhomogeneous metamaterial as well as through a semiconductor. We choose a profile of the metamaterial for which the permittivity varies according to a sinusoidal function. Section 2 briefly reviews the field equations describing magnetic fields in inhomogeneous and homogeneous media. In section 3, we present magnetic field distributions of the surface wave propagating at a boundary separating the mentioned two layers.

II. MATHEMATICAL BACKGROUND

Using Maxwell's equations for nonmagnetic medium free of charges, one can derive a second order differential equation for a TM mode of the electromagnetic field in inhomogeneous and homogeneous media. Introducing a plane wave in the form:

$$H = H(x)e^{i(\omega t-\beta z)}, \quad (1)$$

Where $\omega = 2\pi f$ with f being the frequency, β – is the longitudinal propagation constant, z – is direction of the wave propagation, x – is direction transversal to the direction of the wave propagation.

Combining Maxwell's equations, a differential equation for the inhomogeneous part of the structure is obtained as:

$$\frac{d^2H}{dx^2} - \frac{1}{\varepsilon}\frac{d\varepsilon}{dx}\frac{dH}{dx} + \left[\omega^2\mu\varepsilon - \beta^2\right]H(x) = 0 \quad (2)$$

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Where $\varepsilon = \varepsilon(f, x)$ is spatially-varying frequency-dependent dielectric permittivity. In the homogeneous part of the structure Eq. (2) reduces to the standard wave equation:

$$\frac{d^2H}{dx^2} + \left[\omega^2 \mu\varepsilon - \beta^2\right] H(x) = 0 \tag{3}$$

III. NUMERICAL SOLUTIONS

We consider an inhomogeneous medium shown in figure 1 for which the effective permittivity varies as:

$$\varepsilon = \varepsilon_0 \varepsilon_{eff}(f) \cos(x) \tag{4}$$

where $\varepsilon_{eff}(f) = \varepsilon_\infty - f_p^2 / [f(f + if_c)]$, ε_∞ is the material's background permittivity, fp and fc are plasma and collision frequencies, respectively. The physical structure of the described metamaterial may consist of a two-dimensional array of single split rings deposited onto a dielectric substrate, as implemented in [12]. It is interesting to note that a widely-used Drude model [10] in the field of metamaterials for gold has $\varepsilon_\infty = 1$, $f_p = 2175$ THz, and f_c=10.725 THz, with f_c being three times larger than the normal value for the bulk metal shown in [13]. Alternatively, an improved Drude model for gold [14] fits Johnson-Christy data better. Thus we have found, that $\varepsilon_\infty = 9.6$, $f_p = 2184$ THz, and f_c=17 THz. The improved model agrees with Johnson-Christy data perfectly up to the certain frequency [14].

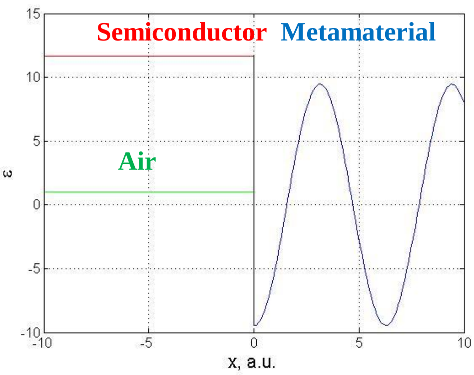

Fig. 1. Schematic of metamaterial/semiconductor interface in terms of permittivity function

IV. NUMERICAL RESULTS AND DISCUSSION

In this section we present numerical results first for a real ε, and then for a complex ε. To demonstrate the performance of our calculation method, we present two examples of structures:

1) Inhomogeneous metamaterial/air
2) Inhomogeneous metamaterial/semiconductor.

The dispersion curves (frequency, f, versus propagation constant, β) of the surface plasmon calculated by solving the wave equation numerically are shown in figures 2 and 4 for real and complex permittivity's, respectively.

The plasma frequency of the semiconductor is lower than that of the metamaterial, i. e. $f_{ps} < f_{pm}$. Consider an n-doped silicon sample as the second layer of the semiconductor/metamaterial compound with a carrier concentration of $N_b = 9 \cdot 10^{19}$ cm^{-3}. With an average effective mass m^* for electrons being $0.26 m_0$, and m_0 being the free-electron mass, and $\varepsilon_\infty = 11.68$, this leads to $f_{ps} = 4.8 \cdot 10^{13}$ Hz.

In figure 2 the calculated dispersion is shown for a real ε for two different cases, the metamaterial/semiconductor and the metamaterial/air structures. It may be noted that in the metamaterial/air case one can observe an increase of the frequency band within which the surface wave exists.

One can observe two different branches that correspond to the investigated structures. It is interesting to note, that the presented curves correspond to the usual surface-plasmon branches or to the branch I in the nomenclature of [15]. For frequencies below the plasma frequency corresponding to each case, the typical bound surface-plasmon mode is observed, approaching the light line for short wave-vectors.

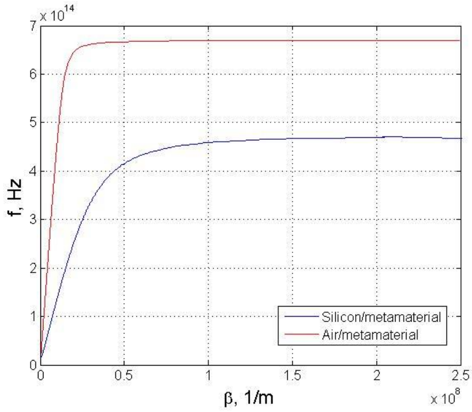

Fig. 2. Dispersion for real ε

To demonstrate the behavior of the fields, figure 3 shows magnetic field amplitudes versus the distance into the structure, x, for both discussed cases, i.e. the air/metamaterial and the semiconductor/metamaterial structures. In all cases for large enough x one expects a decay following an exponential function. While in the homogeneous media the decay follows an exponential function, in the inhomogeneous part one obtains a numerical solution of the wave equation very close to the exponential function. The most of the field is contained on the metamaterial side.

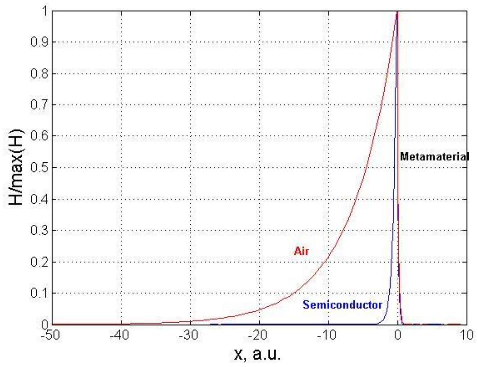

Fig. 3. Magnetic field amplitudes normalized to the surface values versus depth below the surface for real ε

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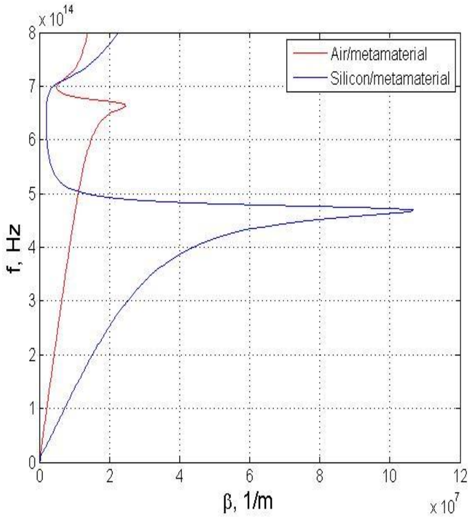

Fig. 4. Dispersion for complex ε

Figure 4 shows the dispersion of figure 2 after the introduction of what seems physically reasonable damping. Below $f = 4.7 \cdot 10^{14}$ Hz the surface-plasmon mode is observed for the silicon/metamaterial case and below $f = 6.6 \cdot 10^{14}$ Hz for the air/metamaterial compound. It approaches the light line at short wave-vectors, but terminating at a finite wave-vector on resonance. Above $f = 7 \cdot 10^{14}$ Hz the radiative mode is observed for both investigated cases. The radiative surface plasmons are coupled with propagating electromagnetic waves; however, for perfectly flat surfaces, surface plasma is always nonradioactive.

Aside from the differences, all of the curves show the same qualitative behavior. At the frequency range between surface-plasmon and radiative surface-plasmon modes the quasibound modes appear to exist. It is interesting to notice that in the case of a silicon/metamaterial structure, this frequency range is wider. The presented modes can be described by the real mathematical components and therefore are not *a priori* forbidden. It is of particular interest to study negative phase velocities in naturally existing materials with the help of quasibound modes. On the other hand, the transition regimes between surface-plasmon/quasibound modes and quasibound/radiative surface-plasmon modes are marked by the infinite group velocity, that is, at first glance, a confusing feature. In normal dispersive media, the group velocity is defined by a linear relation. However, in regions of anomalous dispersion this linearization is not valid, and one should modify the propagation velocity of the wave packet to account for amplitude damping and the wave profile deformation [16].

It should also be noted that the plasma frequency is affected by replacing air with silicon. The usage of semiconductors for low-frequency surface-plasmon propagation can be explained by the possibility to tune the carrier density and thus the plasma frequency by thermal excitation, photo carrier generation, or direct carrier injection. Comparing figures 3 and 5 clearly indicates damping effects on the magnetic field distribution.

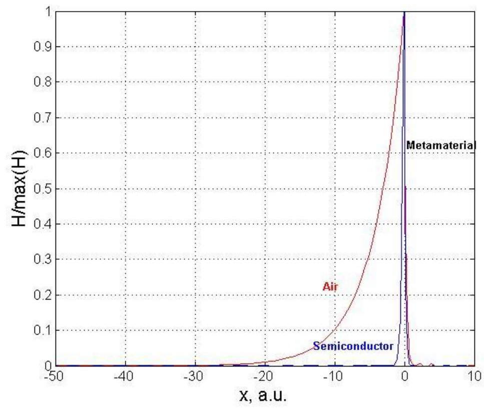

Fig. 5. Magnetic field amplitudes normalized to the surface values versus depth below the surface for complex ε

V. CONCLUSIONS

In summary, we have presented a simple and robust method for the calculation of the dispersion curves of surface modes in two-layered structures containing a metamaterial and possessing one inhomogeneous layer. Due to our algorithm enabling the exact numerical solution of the Helmholtz's equations, no *a priori* knowledge about the dispersion is necessary. Furthermore, the method is assumed to be valid for a broad range of possible structures. This has been demonstrated on two examples which are of current interest in the device technology. The dispersion curves were found to exhibit three distinct branches corresponding to surface plasmons, radiative surface plasmons, and a feature which is termed as quasibound modes. The application examples of the obtained results are considered as the future enhancement of the present study.

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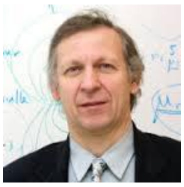
Dr. Gric’s research efforts have focused on the new functional material for electronic devices. She has been working on an extramural grants from the research grant “New creative teams in priorities of scientific research” that fully supports her salary in the past years. Dr. Gric has been conducting the independent research projects for the past eight years. Dr. Gric has published really well in her field of investigation with 26 peer-reviewed papers in top journals in physics, electrodynamics, and optics. It is worth noting that her recent publication rate is getting even higher her being the first author.



Dr. Cada studied with A.M. Prokchorov, a Nobel Prize Laureate for Physics. A former Canada Research Chair in Integrated Active Photonics, he has attracted more than C$10 million in research funds; published more than 250 refereed articles, conference papers and industrial reports; supervised more than 200 highly qualified personnel – and set up a high-tech company. Dr. Cada is a principal investigator in a European Union nanotechnology training project that has recently attracted more than C$2 million in funding.

Dr. Pistora is a researcher in optics and magnetism, focused on thin film optics, periodic structures, integrated optics, optical diffraction and scatterometry, and magnetic field sensors (activities: critical dimensions of spectral ellipsometry, spectral ellipsometry of periodic structures, MO-SPR sensors, magnetic field generators for magneto-optics, nonlinear optical effects, magneto-ellipsometry).

Dr. Pistora is an educator in Physics, Electromagnetic Theory, Magnetic Sensors and Defect scopes. Supervisor of MSc and PhD theses. Published more than 300 papers in international journals and conference proceedings including 12 invited talks. Member of IEEE, OSA, SPIE-Czech Chapter, Czech and Slovak Society for Photonics, Union of Czech Math. and Physicists, EOS-Czech Chapter. Visiting Professor at Kyushu University – 1999, 2001, Research Institute of Electronics, Shizuoka University – 2002, 2005-2006, Dalhousie University – 2007. Chairman of International Symposium on Microwave and Optical Technology (ISMOT), Ostrava 2003 and vice chairman of ISMOT 2005, Fukuoka, Japan.