ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 9, March 2015

Flow of an Incompressible Second-Order Fluid past a Body of Revolution

M.S.Saroa
Department of Mathematics, M.M.E.C., Maharishi Markandeshwar University, Mullana (Ambala), Haryana, India

ABSTRACT- The equations of fluid motion are obtained in compact form using four dimensional coordinate system. Boundary layer equations for the flow of an incompressible second-order fluid past a body of revolution have been derived .Boundary layer equations for the axially symmetrical flow past a sphere have been transformed to those for a two-dimensional flow past a cylinder. The effect of second-order parameters which also depend on shape of the body has been determined on the location of ring of separation. The ring of separation shifts towards the stagnation point for the second-order fluid as compared to Newtonian fluid.

***KEYWORDS*-Axially symmetrical flow, body of revolution, incompressible second-order fluid, ring of separation, sphere, and two-dimensional flow.**

I. INTRODUCTION

The general three-dimensional case of a boundary layer in which the three velocity components depend on all three co-ordinates encounters enormous mathematical difficulties. On the other hand, the mathematical difficulties encountered in the study of axially symmetrical boundary layer and two-dimensional boundary layers are considerably smaller.

The two-dimensional boundary layer which exists on a cylindrical body when it is placed in a stream whose direction is perpendicular to its axis depends only on the potential flow around the cylinder. The shape of the cross-section of the cylinder does not involve explicitly in the equations. On the other hand, in axial flow past a body of revolution, the boundary layer depends directly on the shape of the body in addition to its dependence on the potential flow. Mangler's transformations [1] reduce the calculations of laminar boundary layer flow of a viscous fluid for an axially symmetrical body to that on a cylindrical body, In this paper, boundary layer equations have been derived for axially symmetrical flow of an incompressible second-order fluid past a body of revolution from general equations of motion valid for any three-dimensional flow of the fluid. We have applied Mangler's transformations [1] to examine whether the boundary layer equations for the axially symmetrical flow of an incompressible second-order fluid past a sphere reduce to those for a two-dimensional flow of the fluid past a circular cylinder.

It is found that the Mangler's transformations do not hold completely for an incompressible second-order fluid. The shape of the body enters the equations through second-order parameters. We have solved the problem on the lines of the solution obtained in [2] for the two-dimensional flow of an incompressible second-order past a circular cylinder. The locations of the rings of separations of the boundary layer on the surface of the sphere have been obtained for different values of the second-order parameters.

II. GENERAL EQUATIONS OF MOTION

Consider four dimensional space where(x^1, x^2, x^3, x^4) are the curvilinear coordinates with $x^4 = c\,t$ and (v^1, v^2, v^3, v^4) are the corresponding velocity components with $v^4 = c$ where c , usually taken as velocity of light, is constant .Then, the equation of continuity and the equations of motion for a general unsteady flow of a compressible fluid are :

$$\nabla.(\rho v) = 0 \text{ and } \nabla.S = 0 \qquad (2.1)$$

Or, in tensor notation, are:

$$(\rho v^i)_{,i} = 0 \text{ and } S^{ij}_{,j} = 0,$$

$$i, j = 1,2,3,4 \qquad (2.2)$$

Respectively. The tensor S^{ij} is defined as

$$S^{ij} = T^{ij} - \rho v^i v^j \;,\; T^{i4} = 0 \qquad (2.3)$$

The stress tensor T^{ij} for an incompressible second-order fluid [3] is given as

$$T^i_j = -p\ \delta^i_j + \mu_1 A^i_j + \mu_2 A^i_k A^k_j + \mu_3 B^i_j. \quad (2.4)$$

The coefficients μ_i are material constants, p is indeterminate pressure and the tensors A_{ij} and B_{ij} are given as

$$A_{ij} = v_{i,j} + v_{j,i} \qquad \text{and}$$

$$B_{ij} = a_{i,j} + a_{j,i} + 2\, v^k_{,i}\, v_{k,j} \qquad (2.5)$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 9, March 2015

Where, $a_i = \frac{\partial v_i}{\partial t} + v^j v_{i,j}$ are the acceleration components? In the equations (2.4) *and* (2.5) the indices take the values 1, 2, 3 only. For all practical problems, the coordinate systems used are orthogonal. Thus, for an orthogonal curvilinear coordinate system, let the metric tensor be diag.$[(h_1)^2, (h_2)^2, (h_3)^2]$. Then, the equation of continuity is :

$$\frac{\partial \rho}{\partial t} + \frac{1}{h_1 h_2 h_3} \sum_{i=1}^{3} \frac{\partial}{\partial x^i}\left[\frac{h_1 h_2 h_3}{h_i} \rho v_{(i)}\right] = 0 \qquad (2.6)$$

and the equations of motions are :

$$\rho \frac{\partial v_{(i)}}{\partial t} + \sum_{i=1}^{3}\left[\frac{h_i}{h_1 h_2 h_3}\frac{\partial}{\partial x^i}\left\{\frac{h_1 h_2 h_3}{h_i h_j} T_{(ij)}\right\} + 2\frac{T_{(ij)}}{h_i h_j}\frac{\partial h_i}{\partial x^j} - \frac{T_{(jj)}}{h_i h_j}\frac{\partial h_j}{\partial x^i} - \rho v_{(j)} v_{(i,j)}\right] = 0 \qquad (2.7)$$

Where $v_{(i)}, v_{(i,j)}$ and $T_{(ij)}$ are the physical components of their corresponding tensor components? The physical components $v_{(i,j)}$ are given by

$$v_{(i,j)} = \frac{1}{h_j}\frac{\partial v_{(i)}}{\partial x^j} - \frac{v_{(j)}}{h_i h_j}\frac{\partial h_j}{\partial x^i} + \delta_j^i \sum_{k=1}^{3} \frac{v_{(k)}}{h_i h_k}\frac{\partial h_i}{\partial x^k}$$

III. BOUNDARY LAYER NEAR A BODY OF REVOLUTION

Consider the flow of an incompressible second-order fluid past a body of revolution, when the stream is parallel to its axis. Assuming an orthogonal system of curvilinear coordinates, let $x^1 = x$ *be* the coordinate of a point P on a meridian along the increasing tangent to the meridian ; let $x^2 = y$ be the second coordinate of P along the principal normal to the meridian at P in the negative direction. The third coordinate $x^3 = \varphi$ is along the binormal to the meridian at P. Since the flow is axially symmetric, all the quantities are independent of φ i.e. $\frac{\partial}{\partial \varphi} \equiv 0$.

Let $(u, v, 0)$ be the velocity field corresponding to the coordinate system (x, y, φ) and let $U(x)$ be the velocity of the potential flow. For the assumed coordinate system, we have $h_1 = h_2 = 1$ *and* $h_3 = r(x)$, which is the radius of a section cut at right angle to the axis of revolution and is independent of y. Assuming $o(\mu_2)$ and $o(\mu_3)$ equal to $o(\mu_1)$, and applying boundary layer approximations to the equation of continuity (2.6) and to the equations of motion (2.7) through (2.8) , we obtain the boundary layer equations as :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{r}\frac{dr}{dx} u = 0 \qquad (3.1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial}{\partial x}\left\{p - (\mu_2 + 2\mu_3)\left(\frac{\partial u}{\partial y}\right)^2\right\} - \frac{1}{\rho r}\frac{dr}{dx}\left\{\mu_2\left(\frac{\partial u}{\partial y}\right)^2 + 2\mu_2 u \frac{\partial^2 u}{\partial y^2} + 2\mu_3 u \frac{\partial^2 u}{\partial y^2}\right\} + \frac{\mu_3}{\rho}\left\{u\frac{\partial^3 u}{\partial y^2 \partial x} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right\} = 0 \qquad (3.2)$$

$$\frac{\partial}{\partial y}\left\{-\frac{p}{\rho} + \frac{1}{\rho}(\mu_2 + 2\mu_3)\left(\frac{\partial u}{\partial y}\right)^2\right\} \sim o(1) \qquad (3.3)$$

The boundary conditions of the problem are:

$$u(x, 0) = 0,\ v(x, 0) = 0,$$

$$u(x, \infty) = U(x) \qquad (3.4)$$

The term on the left of (3.3) is of order $o(\delta^{-1})$, where δ is the boundary layer thickness, so that we can assume

$$p - (\mu_2 + 2\mu_3)\left(\frac{\partial u}{\partial y}\right)^2 = \Pi(x, t) \qquad (3.5)$$

where $\pi(x, t)$ is the pressure at the outer edge of the boundary layer . For the outer flow, we get

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} = -\frac{1}{\rho}\frac{\partial \Pi}{\partial x} \qquad (3.6)$$

For a steady flow, the pressure distribution within the boundary layer region is given by

$$p(x, y) = C - \frac{1}{2}\rho U^2 + (\mu_2 + 2\mu_3)\left(\frac{\partial u}{\partial y}\right)^2 \qquad (3.7)$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 9, March 2015

We observe that the pressure in the boundary layer varies for an incompressible second-order fluid whereas it is constant for a Newtonian fluid. In the region of accelerated flow, p decreases from its maximum value C at the forward stagnation point to a minimum at the end of this region, and then increases in the region of decelerated flow .In the region of accelerated flow, the pressure difference, due to the term $(\mu_2 + 2\mu_3)\left(\frac{\partial u}{\partial y}\right)^2$,is less for a second-order fluid as compared to the Newtonian fluid , and therefore, the kinetic energy produced for a second-order fluid is less for a second-order fluid. In both fluids, the friction in the boundary layer is the same, hence the consumption of kinetic energy is the same for both the fluids. Thus, a particle of a second-order fluid, moving with less kinetic energy in the immediate vicinity of the surface is forced to move back by the increasing pressure in the decelerated region earlier than a similar particle of a Newtonian fluid. Therefore, the separation for a second-order fluid occurs earlier than that for a Newtonian fluid. The solution of the problem corroborates this.

Using (3.6), the boundary layer equation (3.2) becomes

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \frac{1}{\rho}\left(\mu_1 + \mu_3\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho r}\frac{dr}{dx}\left\{\mu_2\left(\frac{\partial u}{\partial y}\right)^2\right\}$$

$$-\frac{2}{\rho r}\frac{dr}{dx}\left\{(\mu_2+\mu_3)u\frac{\partial^2 u}{\partial y^2}\right\} + \frac{\mu_3}{\rho}\left\{u\frac{\partial^3 u}{\partial y^2 \partial x} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right\} = 0 \quad (3.8)$$

We observe that the boundary layer equations of continuity (3.1) and motion (3.8) for the axially symmetric flow of an incompressible second-order fluid past a body of revolution depend on the shape of the body through $r(x)$ and on both of the material coefficients μ_2 and μ_3 . But, for the corresponding two - dimensional flow past a cylinder, the equations are independent of the shape of the cylinder and as well of μ_2. We examine whether there is a relationship between the two flows.

IV. RELATIONSHIP BETWEEN AXIALLY SYMMETRIC AND TWO- DIMENSIONAL BOUNDARY LAYERS

A transformation due to [1] referred by [4] permits the use of solutions of the two-dimensional flow of a viscous (Newtonian) fluid past a cylinder to derive solutions of axially symmetric flow past a body of revolution.

The two-dimensional steady flow of an incompressible second-order fluid past a circular cylinder has been discussed [2] where the effect of second-order parameters on the point of separation of the boundary layer has been examined. The method used gives a good approximation to the exact solution in the case of a viscous fluid.

We use the above mentioned transformations to examine if the solutions obtained for the two-dimensional case in [2] may be used to derive solutions for the present axially symmetric flow. The transformations are as below:

$$\bar{x} = \frac{1}{L^2}\int_0^x r^2\,dx \quad, \bar{y} = \frac{r}{L}\,y \quad,$$

$$\bar{t} = \frac{r^2}{L^2}\,t\,, \bar{U} = U, \bar{u} = u \quad,$$

$$\bar{v} = \frac{L}{r}\left(v + \frac{1}{r}\frac{dr}{dx}\,y\,u\right).\ \bar{r} = r \quad (4.1)$$

Then, we have

$$\frac{\partial f}{\partial x} = \frac{r^2}{L^2}\frac{\partial f}{\partial \bar{x}} + \frac{1}{r}\frac{dr}{dx}\,\bar{y}\,\frac{\partial f}{\partial \bar{y}} \quad,$$

$$\frac{\partial f}{\partial y} = \frac{r}{L}\frac{\partial f}{\partial \bar{y}}.$$

With these transformations, the equation of continuity changes to

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (4.2)$$

And the boundary layer equation (3.7) transforms to

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}$$
$$=$$

IJEIT
ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 9, March 2015

$$\frac{\partial \bar{U}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{x}} + \frac{r^2}{L^2}\frac{\mu_3}{\rho}\left(\bar{u}\,\frac{\partial^3 \bar{u}}{\partial \bar{x}\partial \bar{y}^2} + \bar{v}\,\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} + \frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) - \frac{r}{\rho L^2}(\mu_2 + 2\mu_3)\frac{d\bar{r}}{d\bar{x}}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 - 2\,\mu_2\frac{r}{\rho L^2}\frac{d\bar{r}}{d\bar{x}}\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}$$

(4.3)

The boundary conditions (3.4) become

$$\bar{u} = 0 \;, \bar{v} = 0 \quad at \quad \bar{y} = 0\;,$$
$$\bar{u} = \bar{U}(\bar{x}) \quad at \quad \bar{y} = \delta \qquad (4.4)$$

We observe that that the transformed equation of continuity (4.2) is the same as that for a two-dimensional flow. But , the equation (4.3), although holds for a two-dimensional flow when μ_2 and μ_3 are zero , yet in the case of an incompressible second-order fluid, the equation (4.3) is different as it contains $r(x)$ explicitly in addition to terms with μ_2 and μ_3 . On the other hand, $r(x)$ enters the equation (4.3) only as a multiple of the second-order parameters.

V. FLOW PAST A SPHERE

To examine the effect of second-parameters on the ring of separation of the boundary layer, we take the body of revolution as a sphere. We consider the steady flow of an incompressible second-order past a sphere of radius R when the sphere is at rest and the free stream has velocity U_∞. The ideal potential velocity distribution $U(x)$ is given by

$$U(x) = \frac{3}{2}\,U_\infty \sin\theta \qquad (5.1)$$

where, $= \frac{x}{R}$. In equation (5.1) and in subsequent equations we have taken off the bars over the symbols. The radius $r(x)$ is given by

$$r(x) = R\sin\theta \qquad (5.2)$$

Since the method of solution used in [2] gives sufficiently good approximation to the exact solutions, we use the same method to derive the solution of the present problem. We supplement the boundary conditions (4.4) by

$$\frac{\partial u}{\partial y} = 0\;, \quad \frac{\partial^2 u}{\partial y^2} = 0 \quad and \quad \frac{\partial^3 u}{\partial y^3} = 0 \quad at \quad y = \delta \qquad (5.3)$$

On the assumption that the solution within the boundary layer passes smoothly to that outside it. Satisfying the boundary conditions (4.4) and (5.3) , we assume the velocity profile within the boundary layer in the form

$$\frac{u(x,y)}{U(x)} = \left\{1 - \left(1 - K\frac{y}{\delta}\right)\left(1 - \frac{y}{\delta}\right)^4\right\} \qquad (5.4)$$

The coefficient K and the boundary layer thickness δ are functions of x and are to be determined by the two equations which we obtain below: Integrating the equation (4.3) over the boundary layer, we obtain

$$\frac{\partial}{\partial x}\int_0^\delta u^2\,dy - U\frac{\partial}{\partial x}\int_0^\delta u\,dy = \delta\,U\frac{\partial U}{\partial x} - \frac{\mu_1}{\rho}\left(\frac{\partial u}{\partial y}\right)_{y=0} + \mu_2\frac{r}{\rho L^2}\frac{dr}{dx}\int_0^\delta \left(\frac{\partial u}{\partial y}\right)^2 dy - 2\mu_3\frac{r}{\rho L^2}\frac{dr}{dx}\int_0^\delta \left(\frac{\partial u}{\partial y}\right)^2 dy$$

$$-2\mu_3\frac{r^2}{\rho L^2}\frac{\partial}{\partial x}\int_0^\delta \left(\frac{\partial u}{\partial y}\right)^2 dy \qquad (5.5)$$

The equation (4.3) at the meridian $y = 0$ is

$$\left[U\frac{\partial U}{\partial x} + \frac{\mu_1}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{\mu_3}{\rho}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right]_{y=0} = 0 \quad (5.6)$$

Substituting (5 .4) in the equations (5.5) and (5.6) , we have

$$2M(4F^2 + 23F - 530)\cos\theta + \frac{dM}{d\theta}(2F^2 - 5F - 100)\sin\theta + 2M(4F - 5)\frac{dF}{d\theta}\sin\theta + 1980F + \frac{220}{7}sin^2\theta\{2\beta(8F - 5)\frac{dF}{d\theta}\sin\theta\} - \frac{220}{7}sin^2\theta\left(\frac{\beta}{M}\frac{dM}{d\theta}\sin\theta\right) - \frac{220}{7}sin^2\theta(\alpha - 6\beta) \times \cos\theta(4F^2 - 5F + 100) = 0 \qquad (5.7)$$

2M

$$2M\cos\theta + 8(5 - 2F) - 2\beta F\left(F\cos\theta + \frac{dF}{d\theta}\sin\theta\right) + \beta F^2\frac{1}{M}\frac{dM}{d\theta}\sin\theta = 0 \qquad (5.8)$$

where

$$F=4+K\;, \quad M = \frac{3\rho U_\infty}{2R\,\mu_1}\delta^2\;,$$

$$\alpha = \frac{3U_\infty\mu_2}{2R\,\mu_1}\;, \quad \beta = \frac{3U_\infty\mu_3}{2R\,\mu_1} < 0 \qquad (5.9)$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 9, March 2015

The differential equations (5.7) and (5.8) are solved by series method. Since the flow is axially symmetric, F and M have only even powers of θ . Thus, let

$$F = \sum_{n=0}^{\infty} F_{2n} \theta^{2n} , M = \sum_{n=0}^{\infty} M_{2n} \theta^{2n}$$

Substituting the series for F and M in the equations (5.7) and (5.8) , and equating the coefficients of different powers of θ to zero , we obtain the pairs of simultaneous linear equations in F_{2n} and M_{2n} except for F_0 and M_0.

The values F_{2n} **and** M_{2n} **are** given in Table I for n = 0,1,2,3,4,5 against **(α, β) =(0,0) , (0.05 ,−0.025) , (0.1,−0.05) ,(0.15 ,−0.075) and (0.2 ,−0.1)**

VI. DISCUSSION

The shearing stress on the surface of the sphere is given by

$$\tau_0 = \mu_1 \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{9\rho U_\infty^2}{4\sqrt{R_e M}} F \sin\theta \quad (6.1)$$

where $R_e = \frac{3U_\infty R\rho}{\mu_1}$

The variation of τ_0 along the meridian is not qualitatively different from that along the circumference of a cylinder [2] and hence , is not shown here .The location of the ring of separation on the surface of the sphere can be obtained from the condition that the shearing stress τ_0 must vanish there. For a Newtonian fluid ($(\alpha, \beta) = (0,0)$) , the location of the ring of separation is given by

$$3.650064 - 1.409859\theta^2 + 0.152959\theta^4 - 0.00919\theta^6 + , (0.15, -0.075) \; 0.000105\theta^8 - 0.00009\theta^{10} = 0 \quad (6.2)$$

Solving the equation (6.2),we find that the separation occurs at $\theta = 110.5^0$. This value differs from the exact value of 109.6^0 within an error of 1% only. Thus, it is expected that the locations of the rings of separation for the case of second-order fluids also will be within an error of same order. The equations for determining the locations of the rings of separations for $(\alpha, \beta) = (0.5, -0.025), (0.1, -0.05), (0.15, -0.075)$ and $(0.2, -0.1)$ are respectively:

$$.650064 - 1.27448\theta^2 + 0.003952\theta^4 + 0.054462\theta^6 - 0.00453\theta^8 - 0.006245666\theta^{10} = 0$$

$$3.650064 - 1.13911\theta^2 - 0.113448\theta^4 +$$

$$3.650064 - 1.00374\theta^2 - 0.199214\theta^4 + 0.06342\theta^6 + 0.052834\theta^8 - 0.0221544\theta^{10} = 0$$

$$3.650064 - 0.868368\theta^2 - 0.253378\theta^4 + 0.0349466\theta^6 + 0.0840453\theta^8 - 0.0231807\theta^{10} = 0$$

Solving these equations, we find that the separation occurs at $\theta = 95.5^0, 92.7^0, 96.5^0$ and $\theta = 102.5^0$ corresponding to (α, β)=(0.5, −0.025),(0.1,−0.05),(0.15,−0.075) , (0.2,−0.1

It is observed that for increase in the absolute values of second-order parameters, the location of separation first shifts towards the stagnation point and then away from it , but remains earlier than that for a Newtonian fluid. The second-order effects are exhibited through non-dimensional parameters α, β which depend upon not only on second-order coefficients μ_2 *and* μ_3 but also on velocity U_∞ at infinity and radius R of the sphere. This is a peculiarity of a second-order fluid. This method may be applied to other bodies of revolution when the solution for the corresponding two-dimensional problem is known or is obtained.

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 9, March 2015

$(\alpha, \beta))$	(0,0)	(0.05, −0.0250)	(0.1, −0.05)	(0.15, −0..075)	(0.2, −0.01)
M_0	9.200517	9.200517	9.200517	9.200517	9.200517
F_0	3.650064	3.650064	3.650064	3.650064	3.650064
M_2	3.087113	1.809362	0.53161	−0.74614	−2.02389
F_2	−0.18914	−0.30723	−0.42531	−0.5434	−0.66148
M_4	0.706818	1.343592	2.056321	2.845006	3.709646
F_4	−0.05667	0.045883	0.158722	0.281844	0.415249
M_6	0.138955	0.094679	−0.00215	−0.18295	−0.47916
F_6	−0.01233	−0.02681	−0.03725	−0.04417	−0.04809
M_8	0.026268	0.062717	0.233129	0.621621	1.318043
F_8	−0.00223	.002672	0.029799	0.093112	0.209222
M_{10}	0.005945	−0.01887	−0.14805	−0.47668	−1.12761
F_{10}	−0.00029	−0.00768	−0.03922	−0.109	−0.23342

REFERENCES

[1] W. Mangler, “Zusammenhang zwischen ebenen und rotations symmetrischen Grenzschichten in kompressiblen Flüssigkeiten”. ZAMM, vol.28, pp 97, 1948.

[2] A.C.Srivastava and M.S.Saroa,” Phenomenon of separation in second-order fluids”. Intern. Jour. Non-Linear Mechanics, vol 6, pp 607-614, 1970.

[3] B.D.Coleman and W.Noll,”An approximation theorem for functional, with application in continuum mechanics” Archive for Rational Mechanics and Analysis, vol 6, No. 5, pp 355-370 , 1960.

[4] H. Schlichting,” Boundary Layer Theory”, 4th Edn. McGraw-Hill, New York, 1960