ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 9, March 2015

Solution of Partial Differential Equation by Undetermined Coefficients Method

PratimaPatil
Department Science and Humanities (Mathematics)
K. J. Institute of Technology & Engineering, Vadodara
AtulKolte
Department Science and Humanities (Mathematics)
Vadodara Institute of Engineering Kotambi, Vadodara

***Abstract:** Generally we compute the particular integration of two dimensional partial differential equation by general method. In this paper we apply the method of undetermined coefficient to the partial differential equation. The purpose of this method is to compute the particular integration of same two-dimensional partial differential equation with constant coefficient. One of the main advantages of this method is that to reduce the problem to an algebra problem.*

Keywords: **undetermined coefficient method, poission equation, partial differential equation, General method**.

I. INTRODUCTION

The undetermined coefficient generally used to compute the particular integration of non-homogeneos linear ordinary differential equation ($f(D)y = X$) with constant coefficients. In this method we assume a trial solution containing unknown constants, which are obtain by substitution in ($f(D)y = X$). The formula for trial solution is different for$X$. Same as, here we are going to apply undermined coefficient to compute particular integration of two dimensional non-homogeneos partial differential equation with constant coefficients ($f(D,D')u = g(x,y)$). Still here we are applying this method on two dimensional poission equation which is the known as the standard equation in partial differential equation and comparing it to the general method which is also known as short-cut method.

II. THEMETHOD OF UNDETERMINED COEFFICIENT FOR PARTIAL DIFFERENTIAL EQUATION

Let the given equation is $f(D,D')u = g(x,y)$, where$D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$. We denote particular integration by u_p.

Here we guess the formulas of $u_p$which depends on $g(x,y)$ as per following table:

No.	$g(x,y)$	P.I. (u_p)
1	$e^{(ax+by)}$	$Ae^{(ax+by)}$
2	$\cos(ax+by)$ or $\sin(ax+by)$	$A\cos(ax+by) + B\sin(ax+by)$
3	$x^m y^n$	$c_1x^{m+2}y^n + c_2x^{m+4}y^{n-2} + c_3x^{m+6}y^{n-4} + \cdots . c_nx^{m+n-2}y^0$ if $(n - even)$ $c_1x^{m+2}y^n + c_2x^{m+4}y^{n-2} + c_3x^{m+6}y^{n-4} + \cdots . c_nx^{m+n-1}y^1$ if $(n - odd)$

Notations: In the following examples we use these notations:

P.I. = u_p= particular integral, $= \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$,

u_{px}= first derivative of P.I. with respect to x

u_{py}= first derivative of P.I. with respect to y

u_{pxx}=second derivative of P.I. with respect to x

u_{pyy}= second derivative of P.I. with respect to y

Example -1) Find the particular integral of $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial^2 u}{\partial y^2} = e^{x+2y}$.

Given equation is $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial^2 u}{\partial y^2} = e^{x+2y}$ -------------- (1)

By general method:

P.I. = $u_p = \frac{1}{D^2 - 16D'^2} e^{x+2y}$

$= \frac{1}{(D-4D')(D+4D')} e^{x+2y}$

$= \frac{1}{(-7)(9)} e^{x+2y}$ $(\because D = 1, D' = 2)$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 9, March 2015

$= \frac{-1}{63} e^{x+2y}$

By U.C. method:

P.I. = $u_p = Ae^{x+2y}$

$\Rightarrow u_{px} = Ae^{x+2y} \Rightarrow u_{pxx} = Ae^{x+2y}$

and $u_{py} = 2Ae^{x+2y} \Rightarrow u_{pyy} = 4Ae^{x+2y}$

Equation (1) becomes, $Ae^{x+2y} - 16(4)Ae^{x+2y} = e^{x+2y}$

$\Rightarrow (A - 64A)e^{x+2y} = e^{x+2y}$

Comparing both the sides, we have $(A - 64A) = 1$

$\Rightarrow (-63)A = 1$

$\Rightarrow A = \frac{-1}{63}$

$\therefore u_p = \frac{-1}{63} e^{x+2y}$

Example -2) Find the particular integral of $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial^2 u}{\partial y^2} = sin(2x + y)$.

Given equation is $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial^2 u}{\partial y^2} = sin(2x + y)$.-------------- (1)

By general method:

P.I. = $u_p = \frac{1}{D^2 - 16D'^2} \sin(2x + y)$

$= \frac{1}{-4-16(-1)} \sin(2x + y)$ $(\because D^2 = -4, D'^2 = -1)$

$= \frac{1}{12} \sin(2x + y)$

By U.C. method:

P.I. = $u_p = Acos(2x + y) + Bsin(2x + y)$

$\Rightarrow u_{px} = -2Asin(2x + y) + 2Bcos(2x + y)$

$\Rightarrow u_{pxx} = -4Acos(2x + y) - 4Bsin(2x + y)$

and $u_{py} = -Asin(2x + y) + Bcos(2x + y)$

$\Rightarrow u_{py} = -Acos(2x + y) - Bsin(2x + y)$

Equation (1) becomes,

$-4Acos(2x + y) - 4Bsin(2x + y) + 16Acos(2x + y) + 16Bsin(2x + y) = \sin(2x + y)$

$\Rightarrow 12Acos(2x + y) + 12Bsin(2x + y) = \sin(2x + y)$

Comparing both the sides, we have $12A = 0$ and $12B = 1$

$\Rightarrow A = 0 \text{ and } B = \frac{1}{12}$

$\therefore u_p = \frac{1}{12} \sin(2x + y)$

Example -3) Find the particular integral of $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial^2 u}{\partial y^2} = x^3y^3$.

Given equation is $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial^2 u}{\partial y^2} = x^3y^3$ -------------- (1)

By general method:

P.I. = $u_p = \frac{1}{D^2}\left[1 + \frac{16D'^2}{D^2}\right] x^3y^3$

$= \frac{1}{D^2}\left[x^3y^3 + \frac{16x^3y}{D^2}\right]$

$= = \frac{1}{D^2}\left[x^3y^3 + \frac{96}{20}x^5y\right]$

$= \frac{1}{20}x^5y^3 + \frac{4}{35}x^7y$

By U.C. method:

P.I. = $u_p = c_1x^5y^3 + c_2x^7y$

$\Rightarrow u_{px} = 5c_1x^4y^3 + 7c_2x^6y$

$\Rightarrow u_{pxx} = 20c_1x^3y^3 + 42c_2x^5y$

and $u_{py} = 3c_1x^5y^2 + c_2x^7$

$\Rightarrow u_{pyy} = 6c_1x^5y$

Equation (1) becomes, $20c_1x^3y^3 + 42c_2x^5y - 96c_1x^5y = x^3y^3$

Comparing both the sides, we have $20c_1 = 1$ and $42c_2 - 96c_1 = 0$

$\Rightarrow c_1 = \frac{1}{20}$ and $c_2 = \frac{4}{35}$

$\therefore u_p = = \frac{1}{20}x^5y^3 + \frac{4}{35}x^7y$

Example -4) Find the particular integral of $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial^2 u}{\partial y^2} = x^2y^2$.

Given equation is $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial^2 u}{\partial y^2} = x^2y^2$-------------- (1)

By general method:

P.I. = $u_p = \frac{1}{D^2}\left[1 + \frac{16D'^2}{D^2}\right] x^2y^2$

$= \frac{1}{D^2}\left[x^2y^2 + \frac{16x^2(2)}{D^2}\right]$

$= = \frac{1}{D^2}\left[x^2y^2 + \frac{8}{3}x^4\right]$

$= \frac{1}{12}x^4y^2 + \frac{8}{90}x^6$

By U.C. method:

P.I. = $u_p = c_1x^4y^2 + c_2x^6$

$\Rightarrow u_{px} = 4c_1x^3y^2 + 6c_2x^5$

$\Rightarrow u_{pxx} = 12c_1x^2y^2 + 30c_2x^4$

and $u_{py} = 2c_1x^4y$

$\Rightarrow u_{pyy} = 2c_1x^4$

Equation (1) becomes,

$12c_1x^2y^2 + 30c_2x^4 - 32c_1x^4 = x^2y^2$

$\Rightarrow 12c_1x^2y^2 + (30c_2 - 32c_1)x^4 = x^2y^2$

Comparing both the sides, we have $12c_1 = 1$ and $30c_2 - 32c_1 = 0$

$\Rightarrow c_1 = \frac{1}{12}$ and $c_2 = \frac{8}{90}$

$\therefore u_p = = \frac{1}{12}x^4y^2 + \frac{8}{90}x^6$

III. CONCLUSION

The method of undetermined coefficients use only simple algebra and polynomial calculus whenever general requires linear algebra. This method consist of decomposing $f(D, D')u = g(x, y)$ into a number easy to solve equation, each of which is solved by determining a polynomial and the values of undetermined coefficients are find by back substitution. The disadvantage of this method is that it will only work for a small class of $g(x, y)$. The class of $g(x, y)$ for which the method works does include some of the more common functions however there are many functions for which this method will not work. It is generally only useful for the equation having constant coefficients.

REFERENCES

[1] Advanced Engineering Mathematics (8th Edition), by E. Kreyszig, Wiley-India (2007).

[2] Ordinary and Partial Differential Equations (11th Edition) by M.D.Raisinghania, S.Chand, 2005.

[3] W. E. Boyce and R. DiPrima, Elementary Differential Equations (8th Edition), John Wiley (2005).

[4] R. V. Churchill and J. W. Brown, Fourier series and boundary value problems (7th Edition), McGraw-Hill (2006).

[5] T.M.Apostol, Calculus, Volume-2 (2nd Edition), Wiley Eastern, 1980.

[6] Paul Blanchard, Robert L. Devaney, Glenn R. Hall, Differential Equations (Preliminary Edition), PWS Publishing, Boston, 1996.

[7] William H. Boyce and Richard C. Diprima, Elementary Differential Equations and Boundary Value Problems (6th Edition), Wiley, New York, 1996.

[8] C.H. Edwards, Jr., David E. Penney, Elementary Differential Equations with Applications (Third Edition), Prentice-Hall, Englewood Cliffs, NJ, 1996.