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Optimum Solution of Game Theory Problem by an Alternative Simplex Method

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Abstract -In this paper, an alternative simplex method for the solution of game theory problem is introduced. This method is easy to solve game theory problem which does not have a saddle point. It is a powerful method to reduce number of iterations and save valuable time.

Keywords: **Game theory problem, alternative simplex method, optimal solution, no saddle point.**

I. INTRODUCTION

Khobragade et al. [1-3, 6-14] suggested an alternative approach to solve linear programming problem.

In this paper, an attempt has been made to solve integer programming problem (IPP) by new method which is an alternative simplex method. This method is different from Khobragade et al. [1-3,6-14] Method.

II. SOLUTION OF $m \times n$ RECTANGULAR GAME

Consider an $m \times n$ rectangular payoff matrix (a_{ij}) for player A.

Let $S_m = \begin{bmatrix} A_1 & \dots & A_m \\ p_1 & \dots & p_m \end{bmatrix}$ and $S_n = \begin{bmatrix} B_1 & \dots & B_n \\ q_1 & \dots & q_n \end{bmatrix}$

where $\sum_{i=1}^{m} p_i = \sum_{j=1}^{n} q_j = 1$, be the mixed strategies for the two players respectively.

Player A select p_i that will maximize his minimum expected payoff in a column, while player B selects the q_j that will minimize his maximum expected loss in a row of the payoff matrix (a_{ij}).

Now, the expected gains $g_j (j = 1 2 \dots n)$ of player A against B's moves are given by

$$g_1 = a_{11} p_1 + a_{21} p_2 + \dots + a_{m1} p_m$$

$$g_2 = a_{12} p_1 + a_{22} p_2 + \dots + a_{m2} p_m$$

$$\vdots$$

$$g_n = a_{1n} p_1 + a_{2n} p_2 + \dots + a_{mn} p_m$$

and the expected losses $l_i (i = 1 2 \dots m)$ of player B against A's moves are given by

$$l_1 = a_{11} q_1 + a_{12} q_2 + \dots + a_{1n} q_n$$

$$l_2 = a_{21} q_1 + a_{22} q_2 + \dots + a_{2n} q_n$$

$$\vdots$$

$$l_m + a_{m1} q_1 + a_{m2} q_2 + \dots + a_{mn} q_n .$$

Thus, mathematically, minimax maximin principle suggests that player A should select $p_i (p_i \geq 0$, $\sum_{i=1}^{m} p_i = 1)$ that will yield $\max_i [\min_j (g_j)]$ for $j = 1 2 \dots n$ and the player B should select $q_j (q_j \geq 0$, $\sum_{j=1}^{n} q_j = 1)$ that will yield $\min_j [\max_i (l_i)]$ for $i = 1 2 \dots m$.

Let $u = \min_j (g_j)$ and $v = \max_i (l_i)$, then the problem for player A is to Maximize u subject to the constraints :

$$g_1 = \sum_{i=1}^{m} a_{i1} p_i \geq u \qquad \sum_{i=1}^{m} p_i = 1,$$

$$g_2 = \sum_{i=1}^{m} a_{i2} p_i \geq u , \quad p_i \geq 0 \text{ for all } i .$$

$$\vdots$$

$$g_n = \sum_{i=1}^{m} a_{in} p_i \geq u$$

and the problem for player B is to Minimize v subject to the constraints :

$$l_1 = \sum_{j=1}^{n} a_{1j} q_j \leq v$$

$$l_2 = \sum_{j=1}^{n} a_{2j} q_j \leq v , \quad \sum_{j=1}^{n} q_j = 1$$

$$\vdots$$

$$l_m = \sum_{j=1}^{n} a_{mj} q_j \leq v , \qquad q_j \geq 0 \text{ for all } j .$$

The above LPP formulation can be simplified by assuming that u and v both are positive. For, every element of (a_{ij}) can be made strictly greater than zero by adding some constant to all the entries of (a_{ij}).

After the optimum solution is obtained, the true value of the game is obtained by subtracting that constant. Thus assuming that $u > 0$, $v > 0$, we introduce the new variables

$$p'_i = \frac{p_i}{u} \quad i = 1 2 \dots m \text{ and } \quad q'_j = \frac{q_j}{v}, \quad j = 1 2 \dots n$$

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so that the two problem become :

Problem of Player A

Maximize u = Minimize $\frac{1}{u} = \sum_{i=1}^{m} \frac{p_i}{u} = \sum_{i=1}^{m} p_i'$

i.e. Minimize $p_0 = p_1' + p_2' + \ldots + p_m'$

subject to the constraints :

$$a_{11}p_1' + a_{21}p_2' + \ldots + a_{m1}p_m' \geq 1$$
$$a_{12}p_1' + a_{22}p_2' + \ldots + a_{m2}p_m' \geq 1$$
$$\vdots$$
$$a_{1n}p_1' + a_{2n}p_2' + \ldots + a_{mn}p_m' \geq 1$$
$$p_i' \geq 0,\ i = 1 2 \ldots m$$

Problem of Player B

Minimize $v =$ maximize $\frac{1}{v} = \sum_{j=1}^{n} \frac{q_j}{v} = \sum_{j=1}^{n} q_j'$

i.e. Maximize $q_0 = q_1' + q_2' + \ldots + q_n'$ Subject to the constraints:

$$a_{11}q_1' + a_{12}q_2' + \ldots + a_{1n}q_n' \leq 1$$
$$a_{21}q_1' + a_{22}q_2' + \ldots + a_{2n}q_n' \leq 1$$
$$\vdots$$
$$a_{m1}q_1' + a_{m2}q_2' + \ldots + a_{mn}q_n' \leq 1$$
$$q_j' \geq 0,\ \ j = 1 2 \ldots n.$$

After the optimum solution is obtained using alternative simplex method, the original optimum values can be determined.

III. AN ALTERNATIVE ALGORITHM FOR SIMPLEX METHOD

To find optimal solution of any LPP by an alternative method for simplex method, algorithm is given as follows:

Step (1). Check objective function of LPP is of maximization or minimization type. If it is to be minimization type then convert it into a maximization type by using the result:

$$\text{Min. } Z = -\text{ Max.}(-Z).$$

Step (2). Check whether all b_i (RHS) are non-negative. If any b_i is negative then multiply the corresponding equation of the constraints by(-1).

Step (3). Express the given LPP in standard form then obtain initial basic feasible solution.

Step (4). Select max $\sum x_{ij}$, $x_{ij} \geq 0$, for entering vector.

Step (5). Choose greatest coefficient of decision variables.

(i) If greatest coefficient is unique, then element corresponding to this row and column becomes pivotal (leading) element.

(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step (6). Use usual simplex method for this table and go to next step.

Step (7). Ignore corresponding row and column. Proceed to step 5 for remaining elements and repeat the same procedure until an optimal solution is obtained or there is an indication for unbounded solution.

Step (8). If all rows and columns are ignored, then current solution is an optimal solution.

IV. SOLVED EXAMPLES

PROBLEM- 1

Solve the following 3×3 game by linear programming:

	Player B		
Player A	3	–1	–3
	–3	3	-1
	–4	-3	3

SOLUTION:

The problem of player B is to determine q_1, q_2, q_3 so as to Maximize $q_0 = \frac{1}{v} = q_1' + q_2' + q_3'$ subject to the constraints :

$$3q_1' - q_2' - 3q_3' \leq 1$$
$$-3q_1' + 3q_2' - q_3' \leq 1$$
$$-4q_1' - 3q_2' + 3q_3' \leq 1,\ q_1', q_2', q_3' \geq 0.$$

where $q_j' = \frac{q_j}{v}$; $v =$ maximum expected loss of B.

Let us solve B's problem by simplex method. Introducing the slack variable q_4', q_5', q_6' respectively in the constraints of the problem, one obtains the following simplex tables :

Initial Simplex Table: - for player B

C_B	Basis	x_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	1	3	-1	-3	1	0	0
0	S_2	1	-3	3	-1	0	1	0
0	S_3	1	-4	-3	3	0	0	1
1	x_1	1/3	1	-1/3	-1	1/3	0	0
0	S_2	2	0	2	-4	1	1	0
0	S_3	7/3	0	-13/3	-1	4/3	0	1
1	x_1	2/3	1	0	-5/3	1/2	1/6	0
1	x_2	1	0	1	-2	1/2	1/2	0
0	S_3	20/3	0	0	-29/3	7/2	13/6	1

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Since all rows and columns are ignored, the optimum solution has been attained. Thus, for the original problem, the expected value of the game is given by

$$v^* = \frac{1}{q_0}$$
$$= 3/5$$

and the optimum mixed strategy for B is given by

$$q_1^* = \frac{q_1'}{q_0} = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5},$$

$$q_2^* = \frac{q_2'}{q_0} = 1 \times \frac{3}{5} = 3/5,$$

$$q_3^* = \frac{q_3'}{q_0} = 0.$$

The optimum strategies for A are obtained from the dual solution to the above problem.

The optimum values for p_1', p_2' and p_3', where $p_i' = \frac{p_i}{u}$ $(i = 1, 2 \ldots 3)$ are read off from the net evaluation row of the above optimum simplex table under y_4 y_5 and y_6, because A's problem is the dual of B's problem.

Thus $p_1' = 1$, $p_2' = 2/3$, $p_3' = 0$, $p_0 = q_0 = 7/5$.

Hence the optimum mixed strategy for A is given by

$$p_1^* = \frac{p_1'}{p_0} = 1 \times \frac{3}{5} = 3/5,$$

$$p_2^* = \frac{p_2'}{p_0} = \left(\frac{2}{3}\right)\left(\frac{3}{5}\right) = \frac{2}{5},$$

$$p_3^* = \frac{p_3'}{p_0} = 0$$

Hence the optimum solution to the original game problem is

$$S_A = \begin{bmatrix} A_1 & A_2 & A_3 \\ 3/5 & 2/5 & 0 \end{bmatrix},$$

$$S_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ 2/5 & 3/5 & 0 \end{bmatrix}, \quad v^* = 3/5.$$

PROBLEM- 2

Solve the following game by linear programming

	Player B		
Player A	1	–1	–2
	–1	1	1
	2	–1	0

SOLUTION: Exercise for students.

Max $q_0 = q_1' + q_2' + q_3'$ s.t.

$$q_1' - q_2' - 2q_3' \le 1,$$
$$-q_1' + q_2' + q_3' \le 1$$
$$2q_1' - q_2' + 0q_5' \le 1, \quad q_1', q_2', q_3' \ge 0]$$

Simplex table:

C_B	Basis	x_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	1	1	-1	-2	1	0	0
0	S_2	1	-1	1	1	0	1	0
0	S_3	1	2	-1	0	1	0	1
0	S_1	1/2	0	-1/2	-2	1	0	-1/2
0	S_2	3/2	0	1/2	1	0	1	1/2
1	x_1	1/2	1	-1/2	0	0	0	1/2
0	S_1	7/2	0	1/2	0	1	2	½
1	x_3	3/2	0	1/2	1	0	1	½
1	x_1	1/2	1	-1/2	0	0	0	1/2
1	x_2	7	0	1	0	2	4	1
1	x_3	-2	0	0	1	-1	-1	0
1	x_1	4	1	0	0	1	2	1
1	x_2	3	0	1	2	0	2	1
0	S_1	2	0	0	-1	1	1	0
1	x_1	2	1	0	1	0	1	1

Since all rows and columns are ignored, the optimum solution has been attained. Thus, for the original problem, the expected value of the game is given by

$$v^* = \frac{1}{q_0}$$
$$= 1/5$$

and the optimum mixed strategy for B is given by

$$q_1^* = \frac{q_1'}{q} = (2)\left(\frac{1}{5}\right) = \left(\frac{2}{5}\right),$$

$$q_2^* = \frac{q_2'}{q_0} = (3)\left(\frac{1}{5}\right) = 3/5,$$

$$q_3^* = \frac{q_3'}{q_0} = 0.$$

The optimum strategies for A are obtained from the dual solution to the above problem.

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The optimum values for p_1', p_2' and p_3', where $p_i' = \frac{p_i}{u}$ $(i = 1, 2 \dots 3)$ are read off from the net evaluation row of the above optimum simplex table under y_4 y_5 and y_6, because A's problem is the dual of B's problem.

Thus $p_1' = 0$, $p_2' = 3$, $p_3' = 2$, $p_0 = q_0 = 5$.

Hence the optimum mixed strategy for A is given by

$$p_1^* = \frac{p_1'}{p_0} = 0,$$

$$p_2^* = \frac{p_2'}{p_0} = 3\left(\frac{1}{5}\right) = \frac{3}{5},$$

$$p_3^* = \frac{p_3'}{p_0} = 2\left(\frac{1}{5}\right) = \frac{2}{5}$$

Hence the optimum solution to the original game problem is

Ans. $S_A = \begin{bmatrix} A_1 & A_2 & A_3 \\ 0 & 3/5 & 2/5 \end{bmatrix}$,

$S_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ 2/5 & 3/5 & 0 \end{bmatrix}$, $v = \frac{1}{5}$.

PROBLEM- 3

Solve the following game by linear programming

	Player B		
Player A	–1	–2	8
	7	5	–1
	6	0	12

SOLUTION:

Max $q_0 = q_1' + q_2' + q_3'$ s.t.

$$-q_1' - 2q_2' + 8q_3' \le 1,$$

$$7q_1' + 5q_2' - q_3' \le 1$$

$$6q_1' + 12q_5' \le 1, \quad q_1', q_2', q_3' \ge 0\,]$$

Simplex table:

C_B	Basis	x_B	x_1	x_2	x_3	S$_1$	S$_2$	S$_3$
0	S$_1$	1	-1	-2	8	1	0	0
0	S$_2$	1	7	5	-1	0	1	0
0	S$_3$	1	6	0	12	0	0	1
0	S$_1$	1/3	-5	-2	0	1	0	-2/3
0	S$_2$	13/12	15/2	5	0	0	1	1/12
1	x_3	1/12	1/2	0	1	0	0	1/12
0	S$_1$	19/18	0	4/3	0	1	2/3	-23/57
1	x_1	13/90	1	2/3	0	0	2/15	1/19
1	x_3	1/90	0	-1/3	1	0	-1/15	13/23
1	x_2	19/24	0	1	0	3/4	1/2	-23/76
1	x_1	-23/60	1	0	0	-1/2	-1/5	29/11
1	x_3	11/40	0	0	1	1/4	1/10	-5/11
1	x_2	13/60	3/2	1	0	0	1/5	3/38
0	S$_1$	23/30	-2	0	0	1	2/5	-29/57
1	x_3	1/12	1/2	0	1	0	0	1/12

Since all rows and columns are ignored, the optimum solution has been attained. Thus, for the original problem, the expected value of the game is given by

$$v^* = \frac{1}{q_0}$$
$$= 10/3$$

and the optimum mixed strategy for B is given by

$$q_1^* = \frac{q_1'}{q_0} = 0,$$

$$q_2^* = \frac{q_2'}{q_0} = \frac{13}{60} \times \frac{10}{3} = 13/18,$$

$$q_3^* = \frac{q_3'}{q_0} = \frac{1}{1} \times \frac{10}{3} = 5/18.$$

The optimum strategies for A are obtained from the dual solution to the above problem.

The optimum values for p_1', p_2' and p_3', where $p_i' = \frac{p_i}{u}$ $(i = 1, 2 \dots 3)$ are read off from the net evaluation row of the above optimum simplex table under y_4 y_5 and y_6, because A's problem is the dual of B's problem.

Thus $p_1' = 0$, $p_2' = 3/5$, $p_3' = 4/5$, $p_0 = q_0 = 7/5$.

Hence the optimum mixed strategy for A is given by

$$p_1^* = \frac{p_1'}{p_0} = 0,$$

$$p_2^* = \frac{p_2'}{p_0} = \left(\frac{1}{5}\right)\left(\frac{10}{3}\right) = \frac{2}{3},$$

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$$p_3^* = \frac{p_3'}{p_0} = \left(\frac{1}{10}\right)\left(\frac{10}{3}\right) = \frac{1}{3}$$

Hence the optimum solution to the original game problem is

$$\text{Ans. } S_A = \begin{bmatrix} A_1 & A_2 & A_3 \\ 0 & 2/3 & 1/3 \end{bmatrix},$$

$$S_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ 0 & 13/18 & 5/18 \end{bmatrix}, \ v = 10/3.$$

PROBLEM- 4

Solve the following game by linear programming

		Player B	
	–1	2	1
Player A	1	-2	2
	3	4	-3

SOLUTION:

Max $q_0 = q_1' + q_2' + q_3'$ s.t.

$-q_1' + 2q_2' + q_3' \le 1,$

$q_1' - 2q_2' + 2q_3' \le 1$

$3q_1' + 45q_2' - 3q'_3 \le 1, \quad q_1', q_2', q_3' \ge 0]$

Simplex table:

C_B	Basis	x_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	1	-1	2	1	1	0	0
0	S_2	1	1	-2	2	0	1	0
0	S_3	1	3	4	-3	0	0	1
0	S_1	1/2	-5/2	0	5/2	1	0	-1/2
0	S_2	3/2	5/2	0	1/2	0	1	½
1	x_2	1/4	3/4	1	-3/4	0	0	¼
0	S_1	2	0	**0**	3	1	1	0
1	x_1	3/5	1	0	1/5	0	2/5	1/5
1	x_2	-1/5	0	1	- 9/10	0	- 3/10	1/10
1	x_3	2/3	0	0	1	1/3	1/3	0
1	x_1	7/15	1	0	0	- 1/15	1/3	1/5
1	x_2	2/5	0	1	0	3/10	0	1/10

Since all rows and columns are ignored, the optimum solution has been attained. Thus, for the original problem, the expected value of the game is given by

$$v^* = \frac{1}{q_0}$$

$$= 15/23$$

and the optimum mixed strategy for B is given by

$$q_1^* = \frac{q_1'}{q_0} = \frac{7}{15} \times \frac{15}{23} = \frac{7}{23},$$

$$q_2^* = \frac{q_2'}{q_0} = \frac{2}{5} \times \frac{15}{23} = 6/23,$$

$$q_3^* = \frac{q_3'}{q_0} = \frac{2}{3} \times \frac{15}{23} = 10/23.$$

The optimum strategies for A are obtained from the dual solution to the above problem.

The optimum values for p_1', p_2' and p_3', where $p_i' = \frac{p_i}{u}$ $(i = 1, 2 \ldots 3)$ are read off from the net evaluation row of the above optimum simplex table under y_4 y_5 and y_6, because A's problem is the dual of B's problem.

Thus $p_1' = 17/30$, $p_2' = 2/3$, $p_3' = 3/10$, $p_0 = q_0 = 23/15$.

Hence the optimum mixed strategy for A is given by

$$p_1^* = \frac{p_1'}{p_0} = \frac{17}{30} \times \frac{15}{23} = \frac{17}{46},$$

$$p_2^* = \frac{p_2'}{p_0} = \left(\frac{2}{3}\right)\left(\frac{15}{23}\right) = \frac{10}{23},$$

$$p_3^* = \frac{p_3'}{p_0} = \left(\frac{3}{10}\right)\left(\frac{15}{23}\right) = \frac{9}{46}$$

Hence the optimum solution to the original game problem is

$$\text{Ans.:- } S_A = \begin{bmatrix} A_1 & A_2 & A_3 \\ 17/46 & 10/23 & 9/46 \end{bmatrix},$$

$$S_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ 7/23 & 6/23 & 10/23 \end{bmatrix}, \ v^* = \frac{15}{23}.$$

V. CONCLUSION

An alternative simplex method have been derived to obtain the solution of Integer programming problem. The proposed algorithm have simplicity and ease of understanding. From the above examples, authors observed that this method reduces number of iterations and improves the optimum solutions in most of the cases. This method save valuable time as there is no need to calculate the net evaluation Zj-Cj.

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