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Optimum Solution of Integer Programming Problem by an Alternative Simplex Method

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Abstract- In this paper, new alternative methods for the solution of IPP is introduced. This method is easy to solve Integer programming problem. This is powerful method to get improved solution. It reduces number of iterations and save valuable time by skipping calculations of net evaluation.

***Key words:* Integer programming problem, optimal solution, simplex method, alternative method.**

I. INTRODUCTION

Integer programming problem is a special class of L.P.P. where all or some variables are constrained to assume non – negative integer values. This type of problem is of particular importance in business and industry where discrete nature of the variables is involved in many decision – making situations. Khobragade et al. [2, 3, 4] suggested an alternative approach to solve linear programming problem.

In this paper, an attempt has been made to solve linear programming problem (LPP) by new method which is an alternative for simplex method. This method is different from Khobragade et al. [2-4] Method.

II. ALL I.P.P. ALGORITHM

The iterative procedure for the solution of an all – integer programming problem is as follow:

Step (1). Convert the minimization I.P.P. into that of maximization, if it is in the minimization form. Ignore the integerality condition.

Step (2). Introduce stack/or surplus variables, if necessary to convert the in equations into equations and obtain the optimum solution of the given I.P.P. by using new simplex algorithm [1]

Step (3). Select max $\sum x_{ij}$, $x_{ij} \geq 0$, for entering vector.

Step (4). Choose greatest coefficient of decision variables.

(i) If greatest coefficient is unique, then element corresponding to this row and column becomes pivotal (leading) element.

(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step (5). Use usual simplex method for this table and go to next step.

Step (6). Ignore corresponding row and column. Proceed to step 4 for remaining elements and repeat the same procedure until an optimal solution is obtained or there is an indication for unbounded solution.

Step (7). Test the integerality of the optimum solution

(a) .If the optimum solution includes all integer values; an optimum basic feasible integer solution has been obtained.

(b) .If the optimum solution does not include all – integer values then proceed onto next step.

Step (8). Examine the constraint equations corresponding to the current optimum solution. Let these equations be represented by

$$\sum_{j=0}^{n'} y'_{ij}\, x_j = b'_i \qquad [i = 0 1 2 \ldots m']$$

Where n' denotes the number of variables and m' the number of equations.

Choose the largest fraction of b'_i s i.e. find $\max_i \{b'_i\}_f$.

Let it be $[b'_k]_f$ or write it simply as f_{k0} .

Step (9). Express each of the negative fractions if any, in the kth row of the optimum simplex table as the sum of a negative integer and a non – negative fraction.

Step (10). Find the Gomorian constraint $\sum_{j=0}^{n'} f_{kj} x_j \geq f_{k0}$

and append the equation

$$G_{sla}^{(1)} = -f_{k0} + \sum_{j=0}^{n'} f_{kj} x_j$$

to the current set of equation constraints.

Step (11). Starting with this new set of equation constraints, find the new optimum solution by dual simplex algorithm (so that $G_{sla}^{(1)}$ is the initial leaving basic variable).

Step (12). If this new optimum solution for the modified I.P.P. is an integer solution, it is also feasible and optimum for the given I.P.P. Otherwise return to step (4) and repeat the process until an optimum feasible integer solution has been obtained.

III. SOLVED PROBLEMS

PROBLEM 1

Maximize $Z = x_1 + x_2$

Subject to the constraints:

$3x_1 + 2x_2 \leq 5$,

$x_2 \leq 2$, $\quad x_1, x_2 \geq 0$ and are integers.

SOLUTION: Introducing the slack variables $s_1 \geq 0$ and $s_2 \geq 0$ in the equations of the constraints, the inequations becomes equations. An initial basic feasible solution is

$x_B = [s_1,\ s_2] = [5,\ 2]$

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Starting Table :

			1	1	0	0
C_B	y_B	x_B	x_1	x_2	S_1	S_2
0	S_1	5	3	2	1	0
0	S_2	2	0	1	0	1
1	x_1	5/3	1	2/3	1/3	0
0	S_2	2	0	1	0	1
				↑	↓	
1	x_1	1/3	1	0	1/3	-2/3
1	x_2	2	0	1	0	1

Applying GC

			1	1	0	0	0	
C_B	y_B	x_B	x_1	x_2	S_1	S_2	G^1	
1	x_1	1/3	1	0	1/3	-2/3	0	
1	x_2	2	0	1	0	1	0	
0	G^1	-1/3	0	0	-1/3	-1/3	1	
					↑		↓	
1	x_1	0	1	0	0	-1	1	
1	x_2	2	0	1	0	1	0	
0	S_1	1	0	0	1	1	-3	

This table shows that the optimum feasible solution has been obtained in integers. Hence the integer optimum solution to the given I.P.P. is

$x_1 = 0$, $x_2 = 2$ and **max** $Z = 2$.

PROBLEM- 2

Maximize $Z = x_1 + 2x_2$.

$x_1 + x_2 \le 7$,

$2x_1 \le 11$, $2x_2 \le 7$

$x_1, x_2 \ge 0$ and are integer.

SOLUTION

Introduce the slack variables $s_1 \ge 0$, $s_2 \ge 0$ and $s_3 \ge 0$ in the constraints of the given problem, an initial basic feasible solution is $x_B = [s_1, s_2.s_3] = [7, 11,7]$

Starting Table:

			1	4	0	0	0
C_B	y_B	x_B	x_1	x_2	S_1	S_2	S_3
0	S_1	7	1	1	1	0	0
0	S_2	11	2	0	0	1	0
0	S_3	7	0	2	0	0	1
0	S_1	7/2	1	0	1	0	-1/2
0	S_2	11	2	0	0	1	0
2	x_2	7/2	0	1	0	0	1/2

Second Iteration: [Modified Table]

C_B	y_B	x_B	x_1	x_2	S_1	S_2	S_3	G^1
0	S_1	-2	0	0	1	-1/2	-1/2	
1	x_1	11/2	1	0	0	1/2	0	
2	x_2	7/2	0	1	0	0	1/2	
						↑	↓	
0	S_2	4	0	0	-2	1	1	
1	x_1	7/2	1	0	1	0	-1/2	
2	x_2	7/2	0	1	0	0	1/2	
0	S_2	4	0	0	-2	1	1	0
1	x_1	7/2	1	0	1	0	-1/2	0
2	x_2	7/2	0	1	0	0	1/2	0
0	G^1	-1/2	0	0	0	0	-1/2	1
0	S_2	3	0	0	-2	1	0	2
1	x_1	4	1	0	1	0	0	-1
2	x_2	3	0	1	0	0	0	1
0	S_3	1	0	0	0	0	1	-2

This gives us an optimum integer solution is: $x_1 = 4$, $x_2 = 3$ and **max.** $Z = 10$.

PROBLEM- 3

Maximize $Z = x_1 + x_2$.

$2x_1 + 5x_2 \le 16$,

$6x_1 + 5x_2 \le 30$ and $x_1, x_2 \ge 0$.

Solution: Introduce the slack variable $s_1 \ge 0$ and $s_2 \ge 0$ in the constraints of the given problem, an initial basic feasible solution is $x_B = [s_1, s_2] = [16, 30]$.

Starting Table:

C_B	y_B	x_B	x_1	x_2	S_1	S_2
0	S_1	16	2	5	1	0
0	S_2	30	6	5	0	1
1	x_2	16/5	2/5	1	1/5	0
0	S_2	14	4	0	-1	1
1	x_2	9/5	0	1	3/10	-1/10

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1	x_1	7/2	1	0	-1/4	1/4

Third Iteration: [Modified Table]

C_B	y_B	x_B	x_1	x_2	S_1	S_2	G^1
1	x_2	9/5	0	1	3/10	-1/10	0
1	x_1	7/2	1	0	-1/4	1/4	0
0	G^1	-4/5	0	0	-3/10	-9/10	1
1	x_2	1	0	1	0	-1	1
1	x_1	25/6	1	0	0	1	-5/6
0	S_1	8/3	0	0	1	3	-10/3

Apply G C T

C_B	y_B	x_B	x_1	x_2	S_1	S_2	G^1	G^2
1	x_2	1	0	1	0	-1	1	0
1	x_1	25/6	1	0	0	1	-5/6	0
0	S_1	8/3	0	0	1	3	-10/3	0
0	G^2	-1/6	0	0	0	0	-1/6	1
1	x_2	0	0	1	0	-1	0	6
1	x_1	5	1	0	0	1	0	-5
0	S_1	6	0	0	1	3	0	-20
0	G^1	1	0	0	0	0	1	-6

This gives the optimum integer solution is

$x_1 = 5,\ x_2 = 0$ and **max.** $Z = 5.$

Thus the manufacturer should produce 3 dolls of type X, 2 dolls of type Y in order to get the maximum profit of Rs. 5.

PROBLEM- 4

Max $z = 2x_1 + 20x_2 - 10x_3$

$2x_1 + 20x_2 + 4x_3 \le 15$, ,

$6x_1 + 20x_2 + 4x_3 = 20$ and $x_1, x_2, x_3 \ge 0$

SOLUTION

			2	20	-10	0	-M
. C_B	y_B	x_B	x_1	x_2	x_3	S_1	A_1
0	S_1	15	2	20	4	1	0
-M	A_1	20	6	20	4	0	1
20	x_2	3/4	1/10	1	1/5	1/20	0
-M	A_1	5	4	0	0	-1	1
20	x_2	5/8	0	1	1/5	3/40	-1/40
2	x_1	5/4	1	0	0	-1/4	1/4

Apply G C T

C_B	y_B	x_B	x_1	x_2	x_3	S_1	A_1	G^1
20	x_2	5/8	0	1	1/5	3/40	-1/40	0
2	x_1	5/4	1	0	0	-1/4	1/4	0
30	G^1	-5/8	0	0	-1/5	-3/40	1/40	1
20	x_2	0	0	1	0	0	0	1
2	x_1	10/3	1	0	2/3	0	1/6	-10/3
0	S_1	25/3	0	0	8/3	1	-1/3	-40/3

Apply GCT

C_B	y_B	x_B	x_1	x_2	x_3	S_1	A_1	G^1	G^2
20	x_2	0	0	1	0	0	0	1	0
2	x_1	10/3	1	0	2/3	0	1/6	-10/3	0
0	S_1	25/3	0	0	8/3	1	-1/3	-40/3	0
0	G^2	-1/3	0	0	-2/3	0	-1/6	10/3	1
20	x_2	0	0	1	0	0	0	1	0
2	x_1	3	1	0	0	0	0	0	1
0	S_1	7	0	0	0	1	-1	0	4
-10	x_3	1/2	0	0	1	0	1/4	-5	-3/2

Apply GCT

C_B	y_B	x_B	x_1	x_2	x_3	S_1	A_1	G^1	G^2	G_3
20	x_2	0	0	1	0	0	0	1	0	0
2	x_1	3	1	0	0	0	0	0	1	0
0	S_1	7	0	0	0	1	-1	0	4	0
-10	x_3	1/2	0	0	1	0	1/4	-5	-3/2	0
0	G_3	-1/2	0	0	0	0	-1/4	5	-1/20	1
20	x_2	0	0	1	0	0	0	1	0	0
2	x_1	2	1	0	0	0	-1/2	10	0	2
0	S_1	3	0	0	0	1	-3	40	0	8
-10	x_3	2	0	0	1	0	1	-20	0	-3
0	G^2	1	0	0	0	0	1/2	-10	1	-2

Solution is, x_1 =2, x_2 =0, x_3 =2, Max z= -16

PROBLEM- 5

Max $z = 200x_1 + 400x_2 + 300x_3$

$30x_1 + 40x_2 + 20x_3 \le 600$,

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$20x_1 + 10x_2 + 20x_3 \le 400$,
$10x_1 + 30x_2 + 20x_3 \le 800$ and $x_1, x_2, x_3 \ge 0$ and are integers.

SOLUTION

C_B	y_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3
0	**S_1**	**60**	**3**	**4**	**2**	**1**	**0**	**0**
0	**S_2**	**40**	**2**	**1**	**2**	**0**	**1**	**0**
0	**S_3**	**80**	**1**	**3**	**2**	**0**	**0**	**1**
400	x_2	**15**	**3/4**	**1**	**1/2**	**1/4**	**0**	**0**
0	**S_2**	**25**	**5/4**	**0**	**3/2**	**-1/4**	**1**	**0**
0	**S_3**	**35**	**-5/4**	**0**	**1/2**	**-3/4**	**0**	**1**
400	x_2	**20/3**	**1/3**	**1**	**0**	**1/3**	**-1/3**	**0**
300	x_3	**50/3**	**5/6**	**0**	**1**	**-1/6**	**2/3**	**0**
0	**S_3**	**80/3**	**-5/3**	**0**	**0**	**-2/3**	**-1/3**	**1**

Apply G C T

C_B	y_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3	G^1
400	x_2	**20/3**	**1/3**	**1**	**0**	**1/3**	**-1/3**	**0**	**0**
300	x_3	**50/3**	**5/6**	**0**	**1**	**-1/6**	**2/3**	**0**	**0**
0	**S_3**	**80/3**	**-5/3**	**0**	**0**	**-2/3**	**-1/3**	**1**	**0**
0	**G^1**	**-2/3**	**-5/6**	**0**	**0**	**-5/6**	**-2/3**	**0**	**1**
400	x_2	**7**	**3/4**	**1**	**0**	**3/4**	**0**	**0**	**-1/2**
300	x_3	**16**	**0**	**0**	**1**	**-1**	**0**	**0**	**1**
0	**S_3**	**27**	**-5/12**	**0**	**0**	**-1/4**	**0**	**1**	**-1/2**
0	**S_2**	**1**	**5/4**	**0**	**0**	**5/4**	**1**	**0**	**-3/2**

Solution is , x_1 =0 , x_2 =7 , x_3 =16 and Max Z = 7600.

IV. CONCLUSION

An alternative methods for simplex method have been derived to obtain the solution of Integer programming problem. The proposed algorithms have simplicity and ease of understanding. This reduces number of iterations and improves the optimum solutions in most of the cases. These methods save valuable time as there is no need to calculate the net evaluation Zj-Cj.

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