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Thermal Stress Analysis of Semi Infinite Square Beam with Internal Heat Source

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Abstract- *This paper is concerned with transient thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a semi-infinite square beam when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.*

Key Words: **Semi-infinite square beam, transient problem, Integral transform, heat source**

I. INTRODUCTION

Khobragade et al. [2-7, 9] have investigated temperature distribution, displacement function, and stresses of a thin rectangular plate and Khobragade et al. [8] have established displacement function , temperature distribution and stresses of a semi-infinite rectangular beam.

In this paper, an attempt has been made to determine the temperature distribution, displacement function and thermal stresses of a semi-infinite square beam occupying the region D : $-a \le x \le a$; $-a \le y \le a$, $0 \le z \le \infty$ with known boundary conditions. Here Marchi-Fasulo transforms and Fourier cosine transform techniques have been used to find the solution of the problem.

II. STATEMENT OF THE PROBLEM

Consider a thin rectangular plate occupying the space D: $-a \le x \le a$; $-a \le y \le a$, $0 \le z \le \infty$. The displacement components u_x and u_y u_z in the x and y and z directions respectively as Tanigawa et al. [1] are

$$u_x = \int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-v\frac{\partial^2 U}{\partial x^2}\right)+\lambda T\right]dx \tag{1}$$

$$u_y = \int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}-v\frac{\partial^2 U}{\partial y^2}\right)+\lambda T\right]dy \tag{2}$$

$$u_z = \int_{0}^{\infty}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}-v\frac{\partial^2 U}{\partial z^2}\right)+\lambda T\right]dz \tag{3}$$

where E, ν, and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and U (x,y,z,t) is the Airy's stress functions which satisfy the differential equation as Tanigawa et al. [1] is

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z,t) = -\lambda E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) \times T(x,y,z,t) \tag{4}$$

where T(x,y,z,t) denotes the temperature of a rectangular beam satisfy the following differential equation as Tanigawa et al. [1] is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g(x,y,z,t)}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{5}$$

Where k is the thermal conductivity and α is the thermal diffusivity of the material,

Subject to initial condition

$$T(x,y,z,0) = f(x,y,z) \tag{6}$$

The boundary conditions are

$$\left[T(x,y,z,t)+k_1\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a} = f_1(y,z,t) \tag{7}$$

$$\left[T(x,y,z,t)+k_2\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=-a} = f_2(y,z,t) \tag{8}$$

$$\left[T(x,y,z,t)+k_3\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=a} = f_3(x,z,t) \tag{9}$$

$$\left[T(x,y,z,t)+k_4\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=-a} = f_4(x,z,t) \tag{10}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial z}\right]_{Z=0} = 0 \tag{11}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial z}\right]_{Z=\infty} = 0 \tag{12}$$

The stress components in terms of U(x, y, z, t) Tanigawa et al. [1] are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right] \tag{13}$$

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$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} \right] \tag{14}$$

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right] \tag{15}$$

The equations (1) to (15) constitute the mathematical formulation of the problem under consideration.

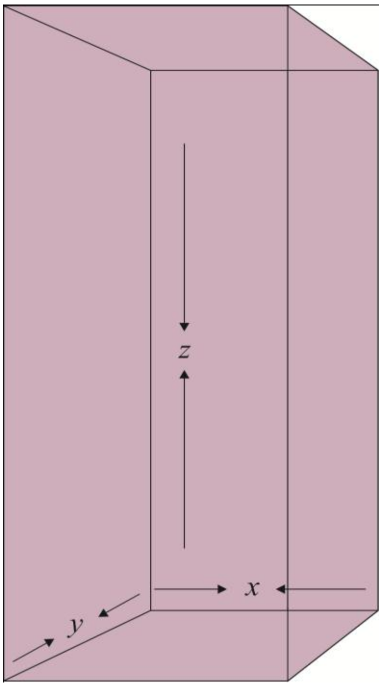

Fig 1: Geometry of the problem

III. SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo transform and finite Fourier cosine transform to the equation (5), we get

$$\frac{d\overline{\overline{T}}^{*}}{dt} + \alpha q^2 \overline{\overline{T}}^{*} = \Psi \tag{16}$$

This is a first order differential equation whose solution is given by

$$\overline{\overline{T}}^{*}(m,n,s,t) = \left(\overline{\overline{f}}^{*} + \int_0^t \Psi e^{-\alpha q^2 (t-t')} dt' \right) \tag{17}$$

where, m, n, s are parameters of Marchi-Fasulo transform and cosine transform respectively.

$$\Psi = \alpha \left[\frac{P_l(a)}{k_1} \overline{f}_1^{*} - \frac{P_l(-a)}{k_2} \overline{f}_2^{*} + \frac{P_m(a)}{k_3} \overline{f}_3^{*} - \frac{P_m(-a)}{k_4} \overline{f}_4^{*} + \frac{\overline{\overline{g}}^{*}}{k} \right]$$

$$q^2 = \mu_m{}^2 + \zeta_n{}^2 + s^2 \pi^2 \tag{18}$$

Now, applying inversion of Fourier Cosine transform, and finite Marchi-Fasulo transform to the equation (17), one obtains the expression for temperature distribution and unknown temperature gradient as

$$T(x,y,z,t) = \frac{2}{\pi} \sum_{m,n=1}^{\infty} \frac{P_m(x) P_n(y)}{\mu_m \ \zeta_n} \int_0^{\infty} \cos sz \, ds \times \left(\overline{\overline{f}}^{*} + \int_0^t \Psi e^{-\alpha q^2 (t-t')} dt' \right) \tag{19}$$

Equation (19) is the required solution.

IV. AIRY'S STRESS FUNCTIONS

Substituting the value of temperature distribution T(x,y,z,t) from (20) in equation (4) one obtains

$$U(x,y,z,t) = \frac{-2\lambda E}{\pi} \sum_{m,n=1}^{\infty} \frac{P_m(x) P_n(y)}{\mu_m \ \zeta_n} \int_0^{\infty} \cos sz \, ds \times \left(\overline{\overline{f}}^{*} + \int_0^t \Psi e^{-\alpha q^2 (t-t')} dt' \right) \tag{20}$$

V. DISPLACEMENT COMPONENTS

Substituting the values of Airy's stress function from equation (22) in the equation (1) to (3), one obtains

$$u_x = \frac{-2\lambda}{\pi} \int_{-a}^{a} \int_0^{\infty} \sum_{m,n=1}^{\infty} \left[\frac{1}{\mu_m \ \zeta_n} \left(P_m(x) P_n''(y) - \nu P_m''(x) P_n(y) - P_m(x) P_n(y) \right) \overline{\overline{f}}^{*} + \begin{pmatrix} P_m(x) P_n''(y) - (s^2+1) P_m(x) P_n(y) \\ - \nu P_m''(x) P_n(y) \end{pmatrix} \cos(sz) B(t) \right] ds dx$$

(21)

$$u_y = \frac{-2\lambda}{\pi} \int_{-a}^{a} \int_0^{\infty} \sum_{m,n=1}^{\infty} \left[\frac{1}{\mu_m \ \zeta_n} \left(P_m''(x) P_n(y) - \nu P_m(x) P_n''(y) - P_m(x) P_n(y) \right) \overline{\overline{f}}^{*} + \begin{pmatrix} P_m''(x) P_n(y) - (s^2+1) P_m(x) P_n(y) \\ - \nu P_m(x) P_n''(y) \end{pmatrix} \cos(sz) B(t) \right] ds dy \tag{22}$$

$$u_z = \frac{-2\lambda}{\pi} \int_0^{\infty} \int_0^{\infty} \sum_{m,n=1}^{\infty} \left[\frac{1}{\mu_m \ \zeta_n} \left(P_m''(x) P_n(y) - P_m(x) P_n''(y) - P_m(x) P_n(y) \right) \overline{\overline{f}}^{*} + \begin{pmatrix} P_m''(x) P_n(y) - (\nu s^2 - 1) P_m(x) P_n(y) \\ + P_m(x) P_n''(y) \end{pmatrix} \cos(sz) B(t) \right] ds dz \tag{23}$$

VI. DETERMINATION OF STRESS FUNCTION

Substituting the value of Airy's stress function U(x,y,z,t) from equation (20) in the equation (14) to (16)

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one obtain the stress functions as,

$$\sigma_{xx} = \frac{-2\lambda E}{\pi} \int_0^\infty \sum_{m,n=1}^\infty \frac{P_m(x)}{\mu_m \ \zeta_n} \begin{bmatrix} P_n''(y)\overline{f^*} + (P_n''(y) - P_n(y)s^2) \\ \cos(sz)B(t)ds \end{bmatrix} \tag{24}$$

$$\sigma_{yy} = \frac{-2\lambda E}{\pi} \int_0^\infty \sum_{m,n=1}^\infty \frac{P_n(y)}{\mu_m \ \zeta_n} \begin{bmatrix} P_m''(x)\overline{f^*} + (P_m''(x) - P_m(x)s^2) \\ \cos(sz)B(t)ds \end{bmatrix} \tag{25}$$

$$\sigma_{zz} = \frac{-2\lambda E}{\pi} \int_0^\infty \sum_{m,n=1}^\infty \frac{P_n(y)}{\mu_m \ \zeta_n} \begin{bmatrix} (P_m''(x)P_n(y) + P_m(x)P_n''(y)) \\ (\overline{f^*} + \cos(sz)B(t))ds \end{bmatrix}$$

Where $B(t) = \left(\int_0^t \Psi e^{-\alpha q^2 (t-t')} dt' \right)$ (26)

VII. SPECIAL CASE AND NUMERICAL RESULTS

Set

$$f(x,y,z,t) = (x-a)^2 (x+a)^2 (z+e^{-z}) \times (y-a)^2 (y+a)^2 (e^{-t}) \tag{27}$$

$$\therefore \overline{\overline{f}}^*(l,m,z,s) = (z+e^{-z}) \times \left[\frac{a_m \cos^2(a_m a) - \cos(a_m a)\sin(a_m a)}{a_m^{\ 2}} \right]^2 \tag{28}$$

$a = 2$, $\alpha = 0.86$, $t = 1$ sec in the equations (20) to obtain

$$T(x,y,z,t) = \frac{2}{\pi} \sum_{m,n=1}^\infty \frac{P_m(x)P_n(y)}{\mu_m \ \zeta_n} \int_0^\infty \cos sz \ ds \times \left(\left[\frac{a_m \cos^2(2a_m) - \cos(2a_m)\sin(2a_m)}{a_m^{\ 2}} \right]^2 + \int_0^1 \Psi e^{-(0.86)\alpha q^2 (1-t')} dt' \right) \tag{29}$$

VIII. MATERIAL PROPERTIES

The numerical calculations has been carried out for an Aluminum (pure) rectangular beam with the material properties as,

Density ρ =169 lb/ft ^{3}

Specific heat = 0.208 Btu/lbOF

Thermal conductivity K = 117 Btu/(hr. ftOF)

Thermal diffusivity α = 3.33 ft ^{2}/hr.

Poisson ratio ν = 0.35

Coefficient of linear thermal expansion

α_t = 12.84 x 10^{-6}1/F

Lame constant μ = 26.67

Young's modulus of elasticity E = 70G Pa

IX. DIMENSIONS

The constants associated with the numerical calculation are taken as

Length of rectangular beam x = 4ft

Breath of rectangular beam y = 3 ft

Height of rectangular beam z = 10^3ft

X. CONCLUSION

In this article, the temperature distribution, displacement function and thermal stresses of a semi-infinite square beam have been obtained; when the boundary conditions are known with the aid of finite Marchi-Fasulo transform and Fourier cosine transform techniques. The results are obtain in the form of infinite series in terms of Bessel's function.

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