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Optimum Solution of Linear Programming Problem by New Method

Putta Baburao; Supriya N. Khobragade and N.W.Khobragade
Department of Mathematics, RTM Nagpur University, Nagpur -440033.

Abstract- *In this paper, new alternative methods for simplex method, Big M method and dual simplex method are introduced. These methods are easy to solve linear programming problem. These are powerful methods. It reduces number of iterations and save valuable time by skipping calculations of net evaluation.*

Key words: **Linear programming problem, optimal solution, simplex method, alternative method.**

I. INTRODUCTION

Khobragade et al. [1-3, 6-14] suggested an alternative approach to solve linear programming problem.

In this paper, an attempt has been made to solve linear programming problem (LPP) by new method which is an alternative for simplex method. This method is different from Khobragade et al. [1-3, 6-14] Method.

II. AN ALTERNATIVE ALGORITHM FOR SIMPLEX METHOD

To find optimal solution of any LPP by an alternative method for simplex method, algorithm is given as follows:

Step (1). Check objective function of LPP is of maximization or minimization type. If it is to be minimization type then convert it into a maximization type by using the result:

$$\text{Min. } Z = -\text{ Max.}(-Z).$$

Step (2). Check whether all b_i (RHS) are non-negative. If any b_i is negative then multiply the corresponding equation of the constraints by(-1).

Step (3). Express the given LPP in standard form then obtain initial basic feasible solution.

Step (4). Select max $C_j \sum x_{ij}$, $x_{ij} \geq 0$, for entering vector.

Step (5). Choose greatest coefficient of decision variables.
(i) If greatest coefficient is unique, then element corresponding to this row and column becomes pivotal (leading) element.

(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step (6). Use usual simplex method for this table and go to next step.

Step (7). Ignore corresponding row and column. Proceed to step 5 for remaining elements and repeat the same procedure until an optimal solution is obtained or there is an indication for unbounded solution.

Step (8). If all rows and columns are ignored, then current solution is an optimal solution.

III. SOLVED PROBLEMS

PROBLEM -1

Max. $Z = 5x_1 + 3x_2$

Subject to the constraints:

$x_1 + x_2 \leq 2$, $5x_1 + 2x_2 \leq 10$, $3x_1 + 8x_2 \leq 12$, $x_1, x_2 \geq 0$.

SOLUTION: We have the constraints

$$x_1 + x_2 + s_1 = 2,$$
$$5x_1 + 2x_2 + s_2 = 10$$
$$3x_1 + 8x_2 + s_3 = 12$$

where S_1, S_2, S_3 are slack variables.

New Simplex Table.

C_B	basis	x_B	x_1	x_2	s_1	s_2	s_3
0	s_1	2	1	1	1	0	0
0	s_2	10	5	2	0	1	0
0	s_3	12	3	8	0	0	1
0	s_1	0	0	3/5	1	-1/5	0
0	x_1	2	1	2/5	0	1/5	0
0	s_3	6	0	34/5	0	-3/5	1
0	s_1	-9/17	0	0	1	-5/34	-3/34
5	x_1	28/17	1	0	0	4/7	-1/17
3	x_2	15/17	0	1	0	–3/34	5/34

Since all rows and columns are ignored, hence an optimum basic feasible solution has been reached.

$\therefore$ Optimum solution is $x_1 = 28/17$, $x_2 = 15/17$ and max. $Z = 185/17$.

PROBLEM -2

Minimum $Z = 3x_1 - 7x_2 + 5x_3$

Subject to the constraints:

$5x_1 - x_2 + 4x_3 \leq 15$, $-3x_1 + 4x_2 \leq 8$, $4x_1 + 3x_2 - 8x_3 \leq 31$, $x_1, x_2, x_3 \geq 0$.

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SOLUTION. We have the constraints

$$5x_1 - x_2 + 4x_3 + s_1 = 15$$
$$-3x_1 + 4x_2 + s_2 = 8$$
$$4x_1 + 3x_2 - 8x_3 + s_3 = 31$$

Where s_1, s_2, s_3 are slack variables.

New Simplex table.

C_B	basis	x_B	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	15	5	–1	4	1	0	0
0	s_2	8	–3	4	0	0	1	0
0	s_3	31	4	3	-8	0	0	1
0	s_1	17	17/4	0	4	1	1/4	0
7	x_2	2	-3/4	1	0	0	1/4	0
0	s_3	25	25/4	0	-8	0	-3/4	1
-5	x_3	17/4	17/16	0	1	1/4	1/16	0
7	x_2	2	-3/4	1	0	0	1/4	0
0	s_3	59	59/4	0	0	2	-1/4	1
-5	x_3	0	0	0	1	25/24	19/24	-17/24
7	x_2	5	0	1	0	6/59	14/59	3/59
-3	x_1	4	1	0	0	8/59	-1/59	4/59

Since all rows and columns are ignored, hence an optimum basic feasible solution has been reached.

$\therefore$ Optimum solution is $x_1 = 4$, $x_2 = 5$, $x_3 = 0$ and max. $Z = 23$.

$\therefore$ Min $Z = -(\max Z) = -23$

PROBLEM -3

Maximize $Z = 2x_1 + 3x_2$

Subject to the constraint:

$x_1 + x_2 \le 4$, $-x_1 + x_2 \le 1$, $x_1 + 2x_2 \le 5$, $x_1, x_2 \ge 0$.

SOLUTION: We have the constraints

$$x_1 + x_2 + s_1 = 4,$$
$$-x_1 + x_2 + s_2 = 1,$$
$$x_1 + 2x_2 + s_3 = 5$$

New Simplex Table.

Since all rows and columns are ignored, hence an opt. basic feasible solution has been reached.

C_B	basis	x_B	x_1	x_2	s_1	s_2	s_3
0	s_1	4	1	1	1	0	0
0	s_2	1	-1	1	0	1	0
0	s_3	5	1	2	0	0	1
0	s_1	3/2	1/2	0	1	0	-1/2
0	s_2	-3/2	-3/2	0	0	1	-1/2
3	x_2	5/2	1/2	1	0	0	1/2
2	x_1	3	1	0	2	0	-1
0	s_2	3	0	0	3	1	-2
3	x_2	1	0	1	-1	0	1

$\therefore$ Optimum solution is $x_1 = 3$ $x_2 = 1$ and max. $Z = 9$.

PROBLEM -4

Minimum $Z = x_1 - 3x_2 + 2x_3$

Subject to the constraints:

$3x_1 - x_2 + 2x_3 \le 7$, $-2x_1 + 4x_2 \le 12$, $-4x_1 + 3x_2 + 8x_3 \le 10$, $x_1, x_2, x_3 \ge 0$.

SOLUTION. We have the constraints

$$3x_1 - x_2 + 2x_3 + s_1 = 7,$$
$$-2x_1 + 4x_2 + s_2 = 12,$$
$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10,$$

where s_1, s_2, s_3 are slack variables.

New simplex table.

C_B	basis	x_B	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	7	3	-1	2	1	0	0
0	s_2	12	–2	4	0	0	1	0
0	s_3	10	-4	3	8	0	0	1
0	s_1	10	5/2	0	2	1	1/4	0
3	x_3	3	-1/2	1	0	0	1/4	0
0	s_3	1	-5/2	0	8	0	-3/4	1

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-1	x_1	4	1	0	4/5	2/5	1/10	0
3	x_2	5	0	1	2/5	1	3/10	0
0	s_3	11	0	0	10	1	-1/2	1
-1	x_1	103/25	1	0	0	8/25	7/50	-2/25
3	x_2	114/25	0	1	0	4/25	8/25	-1/25
-2	x_3	11/10	0	0	1	1/10	-1/20	1/10

Since all rows and columns are ignored, an optimum basic feasible solution has been reached.
Hence solution is

$x_1 = 103/25$, $x_2 = 114/25$, $x_3 = 11/10$ and Min $Z = -(\max Z) = -184/25$

IV. ALTERNATIVE ALGORITHM FOR BIG-M METHOD

To find optimal solution of any LPP by an alternative method for Big-M method, algorithm is given as follows:
Step (1). Check objective function of LPP is of maximization or minimization type. If it is to be minimization type then convert it into a maximization type by using the result:

$$\text{Min. } Z = -\text{ Max.}(-Z).$$

Step (2). Check whether all b_i (RHS) are non-negative. If any b_i is negative then multiply the corresponding equation of the constraints by -1.

Step (3). Express the given LPP in standard form then obtain initial basic feasible solution.

If basic solution is non-feasible due to the constraints of the type $\geq$ and $=$ then we add artificial variable to the corresponding constraint in standard form. Assign very large value $+M$ for maximization and $-M$ for minimization in objective function.

Step (4). Select max $C_j \sum x_{ij}$, $x_{ij} \geq 0$ for entering vector.

Step (5). Choose greatest coefficient of decision variables. (i) If greatest coefficient is unique, then variable corresponding to this column becomes incoming variable.

(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step (6). Compute the ratio with X_B. Choose minimum ratio, then variable corresponding to this row is outgoing variable. The element corresponding to incoming variable and outgoing variable becomes pivotal (leading) element.

Step (7). Use usual simplex method for this table and go to next step.

Step (8). Ignore corresponding row and column. Proceed to step 5 for remaining elements and repeat the same procedure until an optimal solution is obtain or there is an indication for unbounded solution.

Step (9). If all rows and columns are ignored, then current solution is an optimal solution.

PROBLEM -5

Max $Z = 6x_1 + 4x_2$
Subject to:
$2x_1 + 3x_2 \leq 30$, $3x_1 + x_2 \leq 24$, $x_1 + x_2 \geq 3$, $x_1, x_2 \geq 0$.

SOLUTION: We have the constraints

$2x_1 + 3x_2 + s_1 = 30$,
$3x_1 + x_2 + s_2 = 24$,
$x_1 + x_2 - s_3 + A_1 = 3$,
$x_1, x_2 \geq 0$.
$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0.$

Where s_1, s_2, s_3 are slack variables and A_1 is artificial variable.

Simplex table:

C_B	basis	x_B	x_1	x_2	s_1	s_2	s_3	A_1
0	s_1	30	2	3	1	0	0	0
0	s_2	24	3	1	0	1	0	0
-M	A_1	3	1	1	0	0	-1	1
0	s_1	14	0	7/3	1	-2/3	0	0
6	x_1	8	1	1/3	0	1/3	0	0
-M	A_1	5	0	2/3	0	-1/3	-1	1
4	x_2	6	0	1	3/7	-2/7	0	0
6	x_1	6	1	0	-1/7	3/7	0	0
-M	A_1	-9	0	0	-2/7	-1/7	-1	1

Since all rows and column are ignored, hence an optimum solution has been reached. Therefore optimum solution is:

$$x_1 = 6,\ x_2 = 6\ ;\ \text{Max}Z = 60$$

PROBLEM- 6

Min $Z = 2x_1 + x_2$
Subject to:
$3x_1 + x_2 = 3$, $4x_1 + 3x_2 \geq 6$, $x_1 + x_2 \leq 3$, $x_1, x_2 \geq 0$.

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SOLUTION: We have the constraints

$3x_1 + x_2 + A_1 = 3$,

$4x_1 + 3x_2 - s_1 + A_2 = 6$,

$x_1 + x_2 + s_2 = 3$,

$x_1, x_2, s_1, s_2 \geq 0$.

where s_1, s_2 are surplus and slack variables respectively and A_1, A_2 are artificial variables.

Simplex table:

C_B	basis	x_B	x_1	x_2	s_1	s_2	A_1	A_2
-M	A_1	3	3	1	0	0	1	0
-M	A_2	6	4	3	-1	0	0	1
0	s_2	3	1	1	0	1	0	0
-M	A_1	1	5/3	0	1/4	0	1	-1/3
-1	x_2	2	4/3	1	-1/3	0	0	1/3
0	s_2	1	-1/3	0	1/3	1	0	-1/3
-2	x_1	3/5	1	0	1/5	0	3/5	-1/5
-1	x_2	6/5	0	1	-3/5	0	-4/5	3/5
0	s_2	6/5	0	0	2/5	1	1/5	-4/15

Since all rows and column are ignored, hence an optimum solution has been reached. Therefore optimum solution is:

$x_1 = 3/5,\ x_2 = 6/5$; Min Z $= 12/5$

V. ALTERNATIVE ALGORITHM FOR DUAL SIMPLEX METHOD

To find optimal solution of any LPP by an alternative method for dual simplex method, algorithm is given as follows:

Step (1). The objective function of the LPP must be maximize. If it is minimize then convert it into maximize by using the result:

$$\text{Min. } Z = -\text{ Max.}(-Z).$$

Step (2). Convert all $\geq$ constraints into $\leq$ by multiplying the corresponding equation of the constraints by -1.

Step (3). Convert inequality constraints into equality by addition of slack variables and obtain an initial basic solution. Express the above information in the form of a table known as dual simplex table.

Step (4). Choose most negative X_B, then variable corresponding to this row becomes outgoing variable. Select the most negative $C_j \sum x_{ij}$, $x_{ij} \leq 0$ of, then variable corresponding to this column becomes incoming variable. The element corresponding to incoming variable and outgoing variable is pivotal (leading) element.

Step (5). Use usual simplex method for this table and go to next step.

Step (6). Ignore corresponding row and column. Proceed to step 4 for remaining elements and repeat the same procedure until an optimal solution is obtained in finite number steps or there is an indication of the non-existence of a feasible solution.

Step (7): If all rows and columns are ignored, then current solution is an optimal solution.

PROBLEM -7

Minimize Z= $x_1 + 2x_2 + 3x_3$

Subject to

$x_1 - x_2 + x_3 \geq 4$, $x_1 + x_2 + 2x_3 \leq 8$, $x_2 - x_3 \geq 2$, $x_1, x_2, x_3 \geq 0$

SOLUTION: We have the constraints

$-x_1 + x_2 - x_3 + s_1 = 4$

$x_1 + x_2 + 2x_3 + s_2 = 8$

$0x_1 + x_2 + x_3 + s_1 = 2$

$x_1, x_2, x_3 \geq 0$

Initial simplex table

C_B	basis	x_B	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	-4	-1	1	-1	1	0	0
0	s_2	8	1	1	2	0	1	0
0	s_3	-2	0	-1	1	0	0	1
0	s_1	4	0	2	1	1	1	0
-1	x_1	8	1	1	2	0	1	0
0	s_3	-2	0	-1	1	0	0	1
-2	x_2	2	0	1	1/2	1/2	1/2	0
-1	x_1	6	1	0	3/2	-1/2	1/2	0
0	s_3	0	0	0	3/2	1/2	1/2	1

Since all X_B are positive, current solution is an optimal solution.

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$x_1 = 6 \;\; x_2 = 2, x_3 = 0$; Max $Z^* = -10$;

Min Z = 10

PROBLEM -8

Maximize $Z = x_1 + 2x_2$

Subject to: $3x_1 + x_2 \geq 3$,

$4x_1 + 3x_2 \geq 6, x_1 + 2x_2 \geq 3, \quad x_1, x_2 \geq 0$

SOLUTION: We have the constraints

$$-3x_1 - x_2 + s_1 = -3$$
$$-4x_1 - 3x_2 + s_2 = -6$$
$$-x_1 - 2x_2 + s_3 = -3$$
$$x_1, x_2, x_3 \geq 0.$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0.$$

Simplex table:

C_B	basis	x_B	x_1	x_2	s_1	s_2	s_3
0	s_1	-3	-3	-1	1	0	0
0	s_2	-6	-4	-3	0	1	0
0	s_3	-3	-1	-2	0	0	1
0	s_1	3/2	0	5/4	1	-3/4	0
1	x_1	3/2	1	3/4	0	-1/4	0
0	s_3	-3/2	0	-5/4	0	-1/4	1
0	s_1	0	0	0	1	-1	1
1	x_1	3/5	1	0	0	-2/5	3/5
2	x_2	6/5	0	1	0	1/5	-4/5

Since all X_B are positive, current solution is an optimal solution.

$x_1 = 3/5$, $x_2 = 6/5$ MAX Z = 3

IX. CONCLUSION

Alternative methods for simplex method , Big M method and dual simplex method have been derived to obtain the solution of linear programming problem. The proposed algorithms have simplicity and ease of understanding. These reduces number of iterations and improves the optimum solutions in most of the cases. These methods save valuable time as there is no need to calculate the net evaluation Zj-Cj.

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AUTHOR BIOGRAPHY

Dr. N.W. Khobragade for being M.Sc in statistics and Maths, he attained Ph.D in both subjects. He has been teaching since 1986 for 28 years at PGTD of Maths,

RTM Nagpur University, Nagpur and successfully handled different capacities.

At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems (Thermoelasticity in particular) and Operations Research. Published more than 180 research papers in reputed journals. Fourteen students awarded Ph.D Degree and six students submitted their thesis in University for award of Ph.D Degree under their guidance.

Supriya Khobragade Student of M.E final in Computer Science, R.A.I.T College, Nerul, Navi Mumbai.

Putta Baburao for being M.Sc, Mphil in Maths, he has been teaching since 2001 for 14 years at P B Siddhartha College of Arts and Sci, Vijaywada (A.P).