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Optimum Solution of Integer Programming Problem by New Approach

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Abstract- In this paper, new alternative simplex method for the solution of IPP is introduced. This method is easy to solve Integer programming problem. This is powerful method to get improved solution. It reduces number of iterations and save valuable time by skipping calculations of net evaluation.

Key words: **Integer programming problem, optimal solution, simplex method, alternative method.**

I. INTRODUCTION

Integer programming problem is a special class of L.P.P. where all or some variables are constrained to assume non – negative integer values. This type of problem is of particular importance in business and industry where discrete nature of the variables are involved in many decision – making situations.

Khobragade et al. [1-3, 6-14] suggested an alternative approach to solve linear programming problem.

In this paper, an attempt has been made to solve integer programming problem (IPP) by new method which is an alternative simplex method. This method is different from Khobragade et al. [1-3, 6-14] Method.

II. ALL I.P.P. ALGORITHM

The iterative procedure for the solution of an all – integer programming problem is as follow:

Step (1). Convert the minimization I.P.P. into that of maximization, if it is in the minimization form. Ignore the integerality condition.

Step (2). Introduce stack/or surplus variables, if necessary to convert the inequations into equations and obtain the optimum solution of the given I.P.P. by using new simplex algorithm [1]

Step (3). Select max $C_j \sum x_{ij}$, $x_{ij} \geq 0$, for entering vector.

Step (4). Choose greatest coefficient of decision variables.

(i) If greatest coefficient is unique, then element corresponding to this row and column becomes pivotal (leading) element.

(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step (5). Use usual simplex method for this table and go to next step.

Step (6). Ignore corresponding row and column. Proceed to step 4 for remaining elements and repeat the same procedure until an optimal solution is obtained or there is an indication for unbounded solution.

Step (7). Test the integerality of the optimum solution

(a) If the optimum solution includes all integer values, an optimum basic feasible integer solution has been obtained.

(b) If the optimum solution does not include all – integer values then proceed onto next step.

Step (8). Examine the constraint equations corresponding to the current optimum solution. Let these equations be represented by

$$\sum_{j=0}^{n'} y'_{ij}\, x_j = b'_i \qquad [i = 0 1 2 \ldots m']$$

Where n' denotes the number of variables and m' the number of equations.

Choose the largest fraction of b'_i s i.e. find $\max_i \{b'_i\}_f$.

Let it be $[b'_k]_f$ or write it simply as f_{k0} .

Step (9). Express each of the negative fractions if any, in the kth row of the optimum simplex table as the sum of a negative integer and a non – negative fraction.

Step (10). Find the Gomorian constraint $\sum_{j=0}^{n'} f_{kj} x_j \geq f_{k0}$

and append the equation

$$G_{sla}^{(1)} = -f_{k0} + \sum_{j=0}^{n'} f_{kj} x_j$$

to the current set of equation constraints.

Step (11). Starting with this new set of equation constraints, find the new optimum solution by dual simplex algorithm (so that $G_{sla}^{(1)}$ is the initial leaving basic variable).

Step (12). If this new optimum solution for the modified I.P.P. is an integer solution, it is also feasible and optimum for the given I.P.P. Otherwise return to step (4) and repeat the process until an optimum feasible integer solution has been obtained.

III. SOLVED PROBLEMS

PROBLEM- 1

Maximize $Z = 7x_1 + 9x_2$

Subject to the constraints:

$x_1 + 3x_2 \leq 6$,

$7x_1 + x_2 \leq 35$, $x_1, x_2 \geq 0$ and are integers.

SOLUTION: Introducing the slack variables $x_3 \geq 0$ and $x_4 \geq 0$ in the equations of the constraints, the

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inequations becomes equations. An initial basic feasible solution is

$x_B = [x_3,\ x_4] = [6,\ 35]$

Starting Table:

			7	9	0	0
C_B	y_B	x_B	x_1	x_2	S_1	S_2
0	S_1	6	–1	3	1	0
0	S_2	35	7	1	0	1
0	S_1	11	0	22/7	1	1/7
7	x_1	5	1	1/7	0	1/7
				↑	↓	
9	x_2	7/2	0	1	7/22	1/22
7	x_1	9/2	1	0	-1/2	3/22

Applying Gomory Constraint Technique

			7	9	0	0	0
C_B	y_B	x_B	x_1	x_2	S_1	S_2	G^1
9	x_2	7/2	0	1	7/22	1/22	0
7	x_1	9/2	1	0	-1/22	3/22	0
0	G^1	-1/2	0	0	-7/22	-1/22	1
					↑		↓
9	x_2	3	0	1	0	0	1
7	x_1	32/7	1	0	0	1/7	-1/7
0	S_1	11/7	0	0	1	1/7	-22/7

			7	9	0	0	0	0
C_B	y_B	x_B	x_1	x_2	S_1	S_2	G^1	G^2
9	x_2	3	0	1	0	0	1	0
7	x_1	32/7	1	0	0	1/7	-1/7	0
0	S_1	11/7	0	0	1	1/7	- 22/7	0
0	G^2	-4/7	0	0	0	-1/7	-6/7	1
9	x_2	3	0	1	0	0	1	0
7	x_1	4	1	0	0	0	-1	1
0	S_1	1	0	0	1	0	-4	1
0	S_2	4	0	0	0	1	6	-7

This table shows that the optimum feasible solution has been obtained in integers. Hence an integer optimum solution to the given I.P.P. is

$x_1 = 4$, $x_2 = 3$ and max $Z = 55$.

PROBLEM- 2

The owner of a ready – made garments store makes two types of shirts known as zee shirt and Button – Down shirts. He makes a profit of Re. 1 and Rs. 4 per shirt on Zee Shirts and Button – Down shirts respectively. He has two tailors [Tailor A and Tailor B] at his disposal to stitch these shirts. Tailor A and Tailor B can devote at the most 7 hours and 15 hours per day respectively. Both these shirts are to be stitched by both the Tailors. Tailor A and Tailor B spend two hours and five hours respectively in stitching a Zee Shirt and four hours and three hours respectively in stitching of a Button – Down Shirt. How many shirts of both the types should be stitched in order to maximize daily profits ?

SOLUTION: [Formulation of the problem]

Suppose the owner of ready made garments decide to make x_1 Zee Shirts and x_2 Button – Down shirts. Then the availability of time to Tailors has the following restrictions:

$2x_1 + 4x_2 \le 7$, $5x_1 + 3x_2 \le 15$, $x_1, x_2 \ge 0$ and x_1 is an integer.

The problem of the owner is to find values of x_1 and x_2 to maximize the profit $Z = x_1 + 4x_2$.

Introduce the slack variables $x_3 \ge 0$ and $x_4 \ge 0$ in the constraints of the given problem, an initial basic feasible solution is $x_B = [x_3, x_4] = [7,\ 15]$

Starting Table:

			1	4	0	0
C_B	y_B	x_B	x_1	x_2	S_1	S_2
0	S_1	7	2	4	1	0
0	S_2	15	5	3	0	1
			↑			↓
0	S_1	1	0	14/5	1	-2/5
1	x_1	3	1	3/5	0	1/5
4	x_2	5/14	0	1	5/14	-1/7
1	x_1	39/14	1	0	-3/14	2/7

Second Iteration: [Modified Table]

C_B	y_B	x_B	x_1	x_2	S_1	S_2	G^1
4	x_2	5/14	0	1	5/14	-1/7	0
1	x_1	39/14	1	0	-3/14	2/7	0
0	G^1	- 11/14	0	0	- 11/14	-2/7	1
						↑	↓

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4	x_2	3/4	0	1	0	0	-1/2
1	x_1	2	1	0	0	0	1
0	S_2	11/4	0	0	11/4	1	

This gives us an optimum integer solution is: $x_1 = 2$, $x_2 = 3/4$ and max. $Z = 5$.

PROBLEM- 3

A manufacturer of baby – dolls makes two types of dolls : Doll X and Doll Y. Processing of these two dolls is done on two machines, A and B. Doll X requires two hours on machine A and six hours, on machine B. Doll Y requires five hours on machine A and also five hours on machine B. There are sixteen hours of time per day available on machine A and thirty hours on machine B. The profit gained on both the dolls is same i.e. one rupee per doll. What should be the daily production of each of the two dolls?

SOLUTION. [Formulation of the Problem] **:**
Suppose the manufacturer decides to produce
x_1 Dolls of type X and x_2 dolls of type Y.
Then availability of time on two machines has the following restrictions.

$2x_1 + 5x_2 \le 16$, $6x_1 + 5x_2 \le 30$ and $x_1, x_2 \ge 0$.

The manufacturer wishes to determine the value of x_1 and x_2 so as to maximize the profit $Z = x_1 + x_2$.

Introduce the slack variable $x_3 \ge 0$ and $x_4 \ge 0$ in the Constraints of the given problem, an initial basic feasible solution is $x_B = [x_3, x_4] = [16, 30]$.

Starting Table:

Apply G C T

C_B	y_B	x_B	x_1	x_2	S1	S_2
0	S_1	16	2	5	1	0
0	S_2	30	6	5	0	1
1	x_2	16/5	2/5	1	1/5	0
0	S_2	14	4	0	-1	1
1	x_2	9/5	0	1	3/10	-1/10
1	x_1	7/2	1	0	-1/4	1/4

C_B	y_B	x_B	x_1	x_2	S_1	S_2	G^1
1	x_2	9/5	0	1	3/10	-1/10	0
1	x_1	7/2	1	0	-1/4	1/4	0
0	G^1	-4/5	0	0	-3/10	-9/10	1
1	x_2	1	0	1	0	-1	1

1	x_1	25/6	1	0	0	1	-5/6
0	S_1	8/3	0	0	1	3	-10/3

Apply G C T

C_B	y_B	x_B	x_1	x_2	S_1	S_2	G^1	G^2
1	x_2	1	0	1	0	-1	1	0
1	x_1	25/6	1	0	0	1	-5/6	0
0	S_1	8/3	0	0	1	3	-10/3	0
0	G^2	-1/6	0	0	0	0	-1/6	1
1	x_2	0	0	1	0	-1	0	6
1	x_1	5	1	0	0	1	0	-5
0	S_1	6	0	0	1	3	0	-20
0	G^1	1	0	0	0	0	1	-6

This gives the optimum integer solution is
$x_1 = 5$, $x_2 = 0$ and max. $Z = 5$.
Thus the manufacturer should produce 3 dolls of type X, 2 dolls of type Y in order to get the maximum profit of Rs. 5.

PROBLEM- 4

Max $z = 3x_1 + x_2 + 3x_3$

Subject to the constraints:

$-x_1 + 2x_2 + x_3 \le 4, 4x_2 - 3x_3 \le 2,$

$x_1 - 3x_2 + 2x_3 \le 3$ and $x_1, x_2, x_3 \ge 0$

SOLUTION. Starting table

C_B	y_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	4	-1	2	1	1	0	0
0	S_2	2	0	4	-3	0	1	0
0	S_3	3	1	-3	2	0	0	1
0	S_1	3	-1	0	5/2	1	-1/2	0
1	x_2	1/2	0	1	-3/4	0	1/4	0
0	S_3	9/2	1	0	-1/4	0	3/4	1
3	x_3	6/5	-2/5	0	1	2/5	-1/5	0
1	x_2	7/5	-3/10	1	0	3/10	2/3	0
0	S_3	24/5	9/10	0	0	1/10	7/10	1
3	x_3	10/3	0	0	1	4/9	1/9	4/9
1	x_2	3	0	1	0	1/3	1/3	1/3
3	x_1	16/3	1	0	0	1/9	7/9	10/9

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Apply G C T

C_B	y_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3	G^1
3	x_3	10/3	0	0	1	4/9	1/9	4/9	0
1	x_2	3	0	1	0	1/3	1/3	1/3	0
3	x_1	16/3	1	0	0	1/9	7/9	10/9	0
0	G^1	-1/3	0	0	0	-1/9	-7/9	-10/9	1
3	x_3	2	0	0	1	0	-3	-4	4
1	x_2	2	0	1	0	0	-2	-3	3
3	x_1	5	1	0	0	0	0	0	1
0	S_2	3	0	0	0	1	7	10	-9

Solution is, $x_1=5$, $x_2=2$, $x_3=2$, Max z= 23

PROBLEM- 5

Max $z = 4x_1 + 6x_2 + 2x_3$

Subject to the constraints:

$4x_1 - 4x_2 \leq 5, -x_1 + 6x_2 \leq 5,$

$-x_1 + x_2 + x_3 \leq 5$ and $x_1, x_2, x_3 \geq 0$

SOLUTION. Stating table

C_B	y_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	5	4	-4	0	1	0	0
0	S_2	5	-1	6	0	0	1	0
0	S_3	5	-1	1	1	0	0	1
0	S_1	25/3	10/3	0	0	1	2/3	0
6	x_2	5/6	-1/6	1	0	0	1/6	0
0	S_3	25/6	-5/6	0	1	0	-1/6	1
4	x_1	5/2	1	0	0	3/10	1/5	0
6	x_2	5/4	0	1	0	1/20	1/5	0
0	S_3	25/4	0	0	1	1/4	0	1

Apply G C T

C_B	y_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3	G^1
4	x_1	5/2	1	0	0	3/10	1/5	0	0
6	x_2	5/4	0	1	0	1/20	1/5	0	0
0	S_3	25/4	0	0	1	1/4	0	1	0
0	G^1	-1/2	0	0	0	-3/10	-1/5	0	1
4	x_1	2	1	0	0	0	0	0	1
6	x_2	7/6	0	1	0	0	1/6	0	-1/6
0	S_3	35/6	0	0	1	0	-1/6	1	5/6
0	S_1	5/3	0	0	0	1	2/3	0	-10/3

Again apply GCT :

C_B	y_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3	G^1	G^2
4	x_1	2	1	0	0	0	0	0	1	0
6	x_2	7/6	0	1	0	0	1/6	0	1/6	0
2	x_3	35/6	0	0	1	0	-1/6	1	5/6	0
0	S_1	5/3	0	0	0	1	2/3	0	-10/3	0
0	G^2	-5/6	0	0	0	0	-5/6	0	-5/6	1
4	x_1	2	1	0	0	0	0	0	1	0
6	x_2	1	0	1	0	0	0	0	1/6	1/5
2	x_3	6	0	0	1	0	0	1	5/6	-1/5
0	S_1	1	0	0	0	1	0	0	-10/3	4/5
0	S_2	1	0	0	0	0	1	0	1	-6/5

Solution is , $x_1=2$, $x_2=1$, $x_3=6$ and Max Z = 26

IV. CONCLUSION

An alternative simplex method have been derived to obtain the solution of Integer programming problem. The proposed algorithms have simplicity and ease of understanding. From the above examples, authors observed that this method reduces number of iterations and improves the optimum solutions in most of the cases. This method save valuable time as there is no need to calculate the net evaluation Zj-Cj.

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