ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 8, February 2015

Effective Utilization of Linear Programming Technique for Time Optimized Task Management System

P H Patel*, K S Patel
Science and Humanity Department, BITS edu campus, Varnmama, Vadodara-391240

***Abstract*—This paper signifies the application of linear programming in the area of human resources administration in minimizing the time for perform task, is given to teaching staff to particular department. The method gives an integer optimum solution to all the models formulated. Data collected may not yield a feasible solution, when this occurs the model needs to be reformed to give an optimum solution. However, this study recommends to the management of the Babaria Institute of Technology, BITS edu campus, affiliated to Gujarat Technology University (GTU), Gujarat, India.**

***Index Terms*— Linear Programming, Constraint, Objective function, Operations research, Math lab.**

I. INTRODUCTION

Operations research, or operational research in British usage, is a discipline that deals with the application of advanced analytical methods to help make better decisions.[1] It is often considered to be a sub-field of mathematics.[2] The terms management and decision science are sometimes used as synonyms.[3]. Employing techniques from other mathematical sciences, such as mathematical modeling, statistical analysis, and mathematical, operations research arrives at optimal or near-optimal solutions to complex decision-making problems. Because of its emphasis on human-technology interaction and its focus on practical applications. Operations research has overlapped with other disciplines, notably industrial engineering, operations management, draws on psychology and organization science. Operations research is often concerned with determining the maximum, performance (of profit, or yield) or minimum loss (of loss, or cost) of some real-world objective. Originating in military efforts before World War II, its techniques have grown to concern problems in a variety of industries [4]. As a formal discipline, operational research originated in the efforts of military planners during World War II. In the decades after the war, the techniques were more widely applied to the problems in business, industry and society. Since that time, operational research has expanded into a field widely used in industries ranging from petrochemicals to airlines, finance, logistics, and government, moving to a focus on the development of mathematical models that can be used to analyses and optimize complex systems, and has been an area of active academic and industrial research [4].

Early work in operational research was carried out by individuals such as Charles Babbage. His research into the cost of transportation and sorting of mail led to England's universal "Penny Post" in 1840, and studies into the dynamical behavior of railway vehicles in defense of the GWR's broad gauge [5]. Percy Bridgman brought operational research to bear on problems in physics in the 1920s and would later attempt to extend these to the social sciences [6]. Modern operational research originated at the Bawdsey Research Station in the UK in 1937 [7] and it was the result of an initiative of the station's superintendent, A. P. Rowe. Rowe conceived the idea as a means to analyses and improves the working of the UK's early warning radar system, Chain Home (CH) initially, he analyzed the operating the radar equipment and its communication networks, expanding later to include the operating personnel's behavior. This revealed unappreciated limitations of the CH network and allowed remedial action to be taken [8].

The assignment problem is one of the fundamental combinatorial optimization problems in the branch of optimization or operations in mathematics. It consists of finding a maximum weight matching in a weighted bipartite graph. And also this is a variant of transportation model which provides the basis for optimal allocation of tasks to facilities at minimum cost or time.

II. FORMAL MATHEMATICAL DEFINITION

Formal definition of the assignment problem (or linear assignment problem) is given two sets, A and T, of equal size, together with a weight function C: A×T→R. Find a bijection f:A→T such that the cost function:

$\sum_{a\in A} C(a, f(a))$ Is minimized [9].

Usually the weight function is viewed as a square real-valued matrix C, so that the cost function is written down as:

$\sum_{a\in A} C_{a,f(a)}$

The problem is "linear" because the cost function to be optimized as well as all the constraints contains only linear terms.

The problem can be expressed as a standard linear program with the objective function

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$\sum_{i\in A}\sum_{j\in T} C(i,j)x_{ij}$ Subject to the constraints $\sum_{j\in T} x_{ij} = 1 \quad for\ i \in A$,

$\sum_{i\in A} x_{ij} = 1 \quad for\ j \in T$, $x_{ij} \geq 0\ for\ i, j \in A, T.$

The variable x_{ij} represents the assignment of agent i to task j , taking value 1 if the assignment is done and 0 otherwise. This formulation allows also fractional variable values, but there is always an optimal solution where the variables take integer values. This is because the constraint matrix is totally unimodular. The first constraint requires that every agent is assigned to exactly one task, and the second constraint requires that every task is assigned exactly one agent. An assignment problem can be solved using different methods viz. Complete Enumeration Method, Transportation Method, simplex method and Hungarian Assignment Method.

An efficient method for solving an assignment problem is Hungarian Assignment Method (also known as reduced matrix method), which is based on the concept of opportunity cost. Opportunity costs show the relative penalties associated with assignment cost matrix to the extent of having at least one zero in each row and each column, the it will be possible to make optimal assignments (opportunity costs are all zero). The Hungarian method is a combinatorial optimization algorithm that solves the assignment in polynomial time and which anticipated later primal-dual methods. It was developed and published by Harold Kuhn in 1955, who gave the name "Hungarian method" because the algorithm was largely based on the earlier works of two Hungarian mathematicians: Denes Konig and Jeno Egervary [10].

James Munkres reviewed the algorithm in 1957 and observed that it is (strongly) polynomial [11]. Since then the algorithm has been known also as the Kuhn–Munkres algorithm or Munkres assignment algorithm. The time complexity of the original algorithm was $O(n^4)$, however Edmonds and Karp, and independently Tomizawa noticed that it can be modified to achieve an $O(n^3)$ running time. Ford and Fulkerson extended the method to general transportation problems. In 2006, it was discovered that Carl Gustav Jacobi had solved the assignment problem in the 19th century, and the solution had been published posthumously in 1890 in Latin.

III. RATIONAL OF THE STUDY

A departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate, of the time each man would take to perform each task, is given in the matrix below.

Problem: How should the tasks be allocated, one to a man, so as time minimize the total man-hours?

Tasks: A: Result Analysis of B.E. 1st year Examination; B: Work shop on their fields; C: Designing Lesson Panning; D: Exam oriented Assignments or Tutorials

Discipline: E: Mathematics; F: Physics; G: Communication Skills; H: Management

Table 1: describes information on various tasks completed by different disciplines in mentioned hours

Tasks	Discipline			
	E	F	G	H
A	13	14	14	15
B	18	17	17	19
C	19	19	18	18
D	22	26	24	25

IV. MATLAB PROGRAMMING FOR HUNGARIAN METHOD

```
function [assignment,cost] = hungarian_assign(costMat)

assignment = zeros(1,size(costMat,1));
cost = 0;

validMat = costMat == costMat & costMat < Inf;
bigM = 10^(ceil(log10(sum(costMat(validMat))))+1);
costMat(~validMat) = bigM;

costMat(costMat~=costMat)=Inf;
validMat = costMat<Inf;
validCol = any(validMat,1);
validRow = any(validMat,2);

nRows = sum(validRow);
nCols = sum(validCol);
n = max(nRows,nCols);
if ~n
  return
end

maxv=10*max(costMat(validMat));

dMat = zeros(n) + maxv;
dMat(1:nRows,1:nCols) = costMat(validRow,validCol);

hungarian_assign' Assignment Algorithm starts here
  STEP 1: Subtract the row minimum from each row.
minR = min(dMat,[],2);
minC = min(bsxfun(@minus, dMat, minR));
STEP 2: Find a zero of dMat. If there are no starred zeros in
its column or row start the zero. Repeat for each zero
zP = dMat == bsxfun(@plus, minC, minR);

starZ = zeros(n,1);
while any(zP(:))
  [r,c]=find(zP,1);
  starZ(r)=c;
  zP(r,:)=false;
  zP(:,c)=false;
end
while 1
STEP 3: Cover each column with a starred zero. If all the
columns arecovered then the matching is maximum
  if all(starZ>0)
    break
  end
```

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ISO 9001:2008 Certified
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```
    coverColumn = false(1,n);
    coverColumn(starZ(starZ>0))=true;
    coverRow = false(n,1);
    primeZ = zeros(n,1);
    [rIdx, cIdx] =
find(dMat(~coverRow,~coverColumn)==bsxfun(@plus,minR(~coverRow),minC(~coverColumn)));
    while 1
```

STEP 4: Find a noncovered zero and prime it. If there is no starred zero in the row containing this primed zero, Go to Step 5. Otherwise, cover this row and uncover the column containing the starred zero. Continue in this manner until there are no uncovered zeros left. Save the smallest uncovered value and Go to Step 6.

```
        cR = find(~coverRow);
        cC = find(~coverColumn);
        rIdx = cR(rIdx);
        cIdx = cC(cIdx);
        Step = 6;
        while ~isempty(cIdx)
            uZr = rIdx(1);
            uZc = cIdx(1);
            primeZ(uZr) = uZc;
            stz = starZ(uZr);
            if ~stz
                Step = 5;
                break;
            end
            coverRow(uZr) = true;
            coverColumn(stz) = false;
            z = rIdx==uZr;
            rIdx(z) = [];
            cIdx(z) = [];
            cR = find(~coverRow);
            z = dMat(~coverRow,stz) == minR(~coverRow) +
minC(stz);
            rIdx = [rIdx(:);cR(z)];
            cIdx = [cIdx(:);stz(ones(sum(z),1))];
        end
        if Step == 6
```

STEP 6: Add the minimum uncovered value to every element of each covered row, and subtract it from every element of each uncovered column.Return to Step 4 without altering any stars, primes, or covered lines.

```
[minval,rIdx,cIdx]=outerplus(dMat(~coverRow,~coverColumn),minR(~coverRow),minC(~coverColumn));
            minC(~coverColumn) = minC(~coverColumn) +
minval;
            minR(coverRow) = minR(coverRow) - minval;
        else
            break
        end
    end
```

STEP 5:

Construct a series of alternating primed and starred zeros as follows:
Let Z0 represent the uncovered primed zero found in Step 4.
Let Z1 denote the starred zero in the column of Z0 (if any).
Let Z2 denote the primed zero in the row of Z1 (there will always be one). Continue until the series terminates at a primed zero that has no starred zero in its column. Unstar each starred zero of the series, star each primed zero of the series, erase all primes and uncover every line in the matrix. Return to Step 3.

```
rowZ1 = find(starZ==uZc);
    starZ(uZr)=uZc;
    while rowZ1>0
        starZ(rowZ1)=0;
        uZc = primeZ(rowZ1);
        uZr = rowZ1;
        rowZ1 = find(starZ==uZc);
        starZ(uZr)=uZc;
    end
end
```

Cost of assignment

```
rowIdx = find(validRow);
colIdx = find(validCol);
starZ = starZ(1:nRows);
vIdx = starZ <= nCols;
assignment(rowIdx(vIdx)) = colIdx(starZ(vIdx));
pass = assignment(assignment>0);
pass(~diag(validMat(assignment>0,pass))) = 0;
assignment(assignment>0) = pass;
cost =
trace(costMat(assignment>0,assignment(assignment>0)));

function [minval,rIdx,cIdx]=outerplus(M,x,y)
ny=size(M,2);
minval=inf;
for c=1:ny
    M(:,c)=M(:,c)-(x+y(c));
    minval = min(minval,min(M(:,c)));
end
[rIdx,cIdx]=find(M==minval);
```

V. DISCUSSIONS

As mentioned in table 1 if by randomization Task A is assigned to E discipline, Task B is assigned to F discipline, Task C is assigned to G discipline and Task D is assigned to H discipline then the total work hours to complete this task was taking 73 hours. By optimizing the same task using Hungarian Method with the help of MAT LAB, the task will be completed within minimum period of time 71 hours. By assigning the task A to H, B to E, C to F and D to G.

VI. CONCLUSION

In this present study, we have gone with four tasks to complete in minimum period of the time by distributing among four different disciplines. But, we can optimize number of task distributed among same number of disciplines.

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ISO 9001:2008 Certified
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Through this study, we get the maximum performance in the minimum period of the time.

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AUTHOR BIOGRAPHY

Prakashkumar H. Patel, presently working as Head of Science and Humanity Department, BITS edu campus. His areas of interest are Operation Research, Wavelet Analysis and Linear Algebra.

Kaushal S. Patel, presently working as an Assistant Professor in Science and Humanity Department, BITS edu campus. His areas of interest are Operation Research, Wavelet Analysis and Porous media.