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# Optimum Design of Trussed Dome Structures

S. Abdel Salam, A.EL-shihy, A. Eraky, and M. Salah
Department of Structural Engineering, faculty of engineering, Zagazig University, Zagazig, Egypt

***Abstract*— Genetic optimization algorithm (GAs) for weight minimization of trussed dome is discussed in this paper. Topology, sizing and shape optimization types are used to find the minimum weight of trussed dome by taking inner spacing, inner height between rings, total height of the dome and cross section of trussed dome members as a variable with constant span to find the optimum value of them which give minimum weight of the dome by using genetic algorithm technique. Two examples are presented in this paper solved by matlab, the first example is 272-Bar dome with height 4.34 m and 40m span and the second example is 444-Bar dome with height 20 m and 20m span. The results are compared with previous researches to emphasize the benefit of the used technique.**

***Index Terms*—Optimization, Genetic Algorithm, Weight minimization, Trussed dome structure.**

## I. INTRODUCTION

Optimization means making things the best. Thus, structural optimization is the subject of making an assemblage of materials sustains loads in the best way. Depending on the geometric feature, structural optimization problems divide into three classes; sizing optimization, shape optimization and Topology optimization. Abundance of different search and optimization techniques are used in optimum structural design applications, genetic algorithm one of this techniques. Genetic algorithm is a method for solving optimization problems that is based on natural selection, the process that drives biological evolution.

Christensen & Klarbring [1] presented a general expression for structural optimization. Coello [2] dictated that the sizing optimization refers to finding the optimal cross section area of each member of the structure; shape optimization means optimizing the outer shape of the structure; and topology optimization describes the search for the best inner connectivity of the members. Roman GÄatzi [3] presented a tool developed to optimize structures for a large range of objectives. The optimization method uses genetic algorithms which can be described as stochastic search algorithms, based on the mechanics of natural selection and natural genetics**.** Kang Tai [4] evaluate an effective topology/shape design optimization methodology via a simulated topology optimization problem and then applied to an actual structural problem of automatically synthesizing a path generating compliant mechanism which produces a vertical straight line output path when given a horizontal straight line input. R. Kamyab [5] focused on size optimization of scallop domes subjected to static loading. As this type of space structures includes a large number of the structural elements, optimum design of such structures results in efficient structural configurations. A. Kaveh [6] Studied frequency constraint optimization of truss structures on shape and size. O. Hasançebi [7] examined minimum weight design of pin-jointed geodesic steel domes using seven metaheuristic search techniques; namely, simulated annealing, genetic algorithms, evolution strategies, particle swarm optimizer, tabu search, ant colony optimization and harmony search methods. The optimum design problem of geodesic steel domes is formulated according to design limitations stipulated by ASD-AISC (Allowable Stress Design Code of American Institute of Steel Institution). Yavuz Sarac [8] used two modern optimization techniques (GAs and SA) in the optimum structural design applications of pin-jointed 3-D dome structures. R. Kamyab [9] discussed the sizing optimization of double-layer scallop domes subjected to static loading. As this type of space structures possesses a large number of the structural elements, optimum design of such structures results in efficient structural configurations. Optimization task is achieved using firefly algorithm (FA) by taking into account linear and nonlinear responses of the structure. In the nonlinear optimization process only the geometrical nonlinearity effects are included. The numerical results demonstrate that nonlinear optimization provides more efficient structures compared with the linear one. Henant Chickerman [10] proposed a new method for solving structural optimization problems using a local function approximation algorithm. This new algorithm called the generalized convex approximation (GCA)**.** Eduardo [11] proposed a genetic algorithm to evolve the structural configuration for weight minimization of industrial buildings, with rectangular geometry projection, made up of uniform planar structures (steel truss roofs) along the longitudinal direction which are interconnected by purlins.

The use of modern optimization technique (GA) in the optimum structural design applications of trussed dome structure is investigated in this paper .The work is aimed mainly to minimize the total weight of trussed dome structures in civil engineering applications. In this paper, two numerical examples are studied using genetic algorithm technique and matlab**.**

## II. THEORY AND MODELING

Using the stiffness matrix method to analyze the trussed dome structures, shown the stiffness matrix equation:

$$F = K.D \qquad (1)$$

where F, K and D are the load vector, stiffness matrix and displacement vector respectively.

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Volume 4, Issue 8, February 2015

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_n \end{bmatrix} \qquad (2)$$

$$K = \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ K_{n1} & \cdots & \cdots & K_{nn} \end{bmatrix} \qquad (3)$$

$$D = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ \vdots \\ D_n \end{bmatrix} \qquad (4)$$

Where:

n: DOF which equal (no. of free point $\times$ 3).

By solving the stiffness matrix equation and find the displacement value at points in trussed dome, the internal forces in the trussed dome members are calculated by equation (5)

$$N = \frac{EA}{L}.\Delta \qquad (5)$$

where:
N: the internal force in member
E: modules of elasticity
A: initial cross section area
Δ: total change in length member, which given in equation (6)

$$\Delta = (D_{bx} - D_{ax})\cos\theta_x + (D_{by} - D_{ay})\cos\theta_y + (D_{bz} - D_{az})\cos\theta_z \qquad (6)$$

, as shown in figure (1).

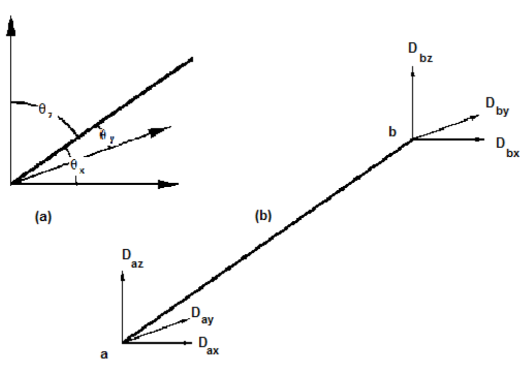


**Fig 1: (a) Axis direction and axis angle, (b) member displacements.**

The Egyptian steel code is used in the design of compression members as follow:
The allowable stress in axial compression:

$$F_{act} = \frac{N}{A} \le F_{pb} * 1.2 \qquad (7)$$

where:
$F_{act}$: the allowable stress
N: internal force at member
A: cross section area
$F_{pb}$: permissible buckling stress, given in equation (8), (9).

a) If $\lambda < 100$

$$F_{pb} = 1.4 - 0.000065\lambda^2 \qquad (8)$$

b) If $\lambda > 100$ $F_{pb} = 7500/\lambda^2$

(9)

Where:
λ: slenderness ratio = $K.L/r$
K: buckling length factor
L: unsupported length for member
r: radius of gyration

For tension member:

$$F_{act} = \frac{N}{A} \le F_{pt} \qquad (10)$$

where:
$F_{act}$: the allowable stress
N: internal force at member
A: cross section area
$F_{pt}$: permissible tension stress

In this study the trussed domes is design only due to vertical loads, which have the largest effect in determination the cross section area of members, and the vertical displacement of dome. This vertical loads come from the dome own weight, covering and the live load. These loads can be calculated as follow:

$$W = W_{\text{own weight}} + W_{\text{covering}} + W_{L.L} \qquad (11)$$

Where:

$$W_{\text{own weight}} = \sum_{1}^{n} 7.85 * L * A \qquad (12)$$

$W_{\text{covering}}$ = 5-20 kg/ m$^2$

$$W_{L.L} = 60 - (200/3) * \tan\delta \ \text{kg/ m}^2 \qquad (13)$$

where:
A: member cross section area.
n: number of total members.
L: member length.
δ: inclination of inaccessible roof

The vertical load acting on every joint can be calculated by multiply the load intensity by the area surrounded by this joint as shown in Figure (2).

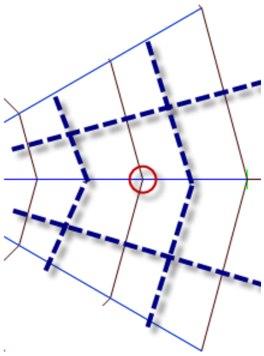

**Fig 2: The Area Surrounded of One Joint of the Dome**

The strut material (st. 37) characteristics are assumed according to Egyptian steel code [12] as shown in table (1).

**Table (1) the member material characteristics.**

| | |
|---|---|
| Mass Density | 7.85 t/m$^3$ |
| Modulus of Elasticity | 2100 t/ cm$^2$ |
| Yield strength | 2.4 t/cm$^2$ |
| Ultimate strength | 3.6 t/cm$^2$ |

The general expression of structure optimization has been given by Christensen & Klarbring [1]:

$$(SO)\begin{cases} \text{minimize } f(x,y) \text{ with respect to } x \text{ and } y \\ \text{subject to} \begin{cases} \text{behavioral constrains on } y \\ \text{design constrains on } x \\ \text{stability constraint} \end{cases} \end{cases}$$

Where:
$f$: is the objective function.
x: is a function or vector representing the design variables.
y: is a function or vector representing the state variables.

**Objective Function:**
To generate a vector of cross-sectional areas **A** for $N_m$ members of the dome [7]:

$$A^T = [A_1, A_2, \ldots\ldots A_{N_m}] \quad (14)$$

Such that **A** minimizes the objective function

$$W = \sum_{m=1}^{N_m} \rho_m L_m A_m \quad (15)$$

where $W$ refers to the weight of the dome,
$A_m, L_m$ *and* $\rho_m$ are cross-sectional area, length and unit weight of the dome member, respectively.

**Design Constraints**:
The structural behavioral and performance limitations of trussed dome structures can be formulated as follows [7]:

$$g_m = \frac{\sigma_m}{(\sigma_m)_{all}} - 1 \le 0, m = 1, \ldots, N_m \quad (16)$$

$$s_m = \frac{\lambda_m}{(\lambda_m)_{all}} - 1 \le 0, m = 1, \ldots, N_m \quad (17)$$

$$\delta_{j,K} = \frac{d_{j,K}}{(d_{j,K})_{all}} - 1 \le 0, m = 1, \ldots, N_m \quad (18)$$

where, the functions $g_m$ , $s_m$ and $\delta_{j,K}$ are referred to as constraints being bounds on stresses, slenderness ratios and displacements, respectively ; $\sigma_m$ and $(\sigma_m)_{all}$ are the computed and allowable axial stresses for the member, respectively; $\lambda_m$ and $(\lambda_m)_{all}$ are the slenderness ratio and its upper limit for the member, respectively; $N_m$ is the total number of joints; and finally $d_{j,K}$ and $(d_{j,K})_{all}$ , are the displacements computed in the k-th direction of the j-th joint and its permissible value, respectively. In the present study, these limitations are implemented according to Egyptian steel code [12].

## III. RESULTS

In this section, two numerical examples are studied using genetic algorithm technique and matlab.

**Dome (272-Bar dome):**

It is circular in plan view. It has a diameter of 40 m. The building consists of a pin-connected type steel dome with 272 members and 81 joints. The height of the steel dome is only 4.34 m. The 3-dimensional view of the steel dome is presented in Figure (4) and the side and top views of the building are shown in Figures (5) and (6).

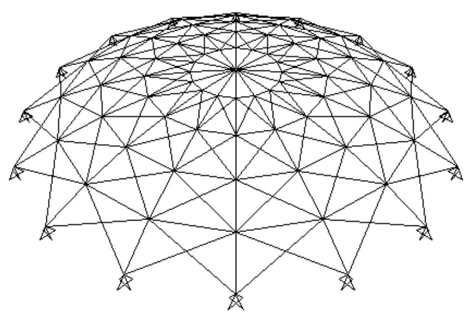

**Fig 4: 3-D View of the Dome.**

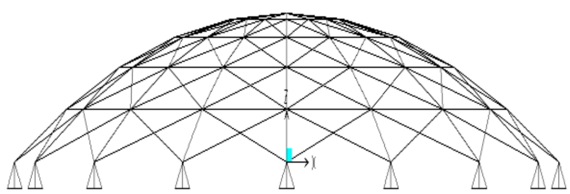

**Fig 5: Side View of the Dome.**

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Volume 4, Issue 8, February 2015

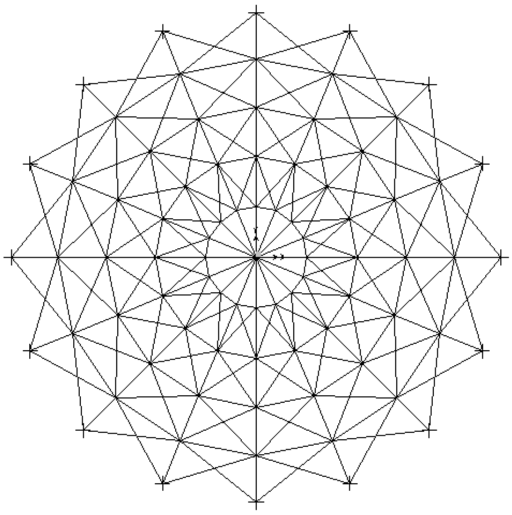

**Fig 6: Top View of the Dome**

Circular hollow pipes are used in the design. Table (2) shows cross sections area of trussed dome members and total weight, figure (7) shows the numbering of this member.

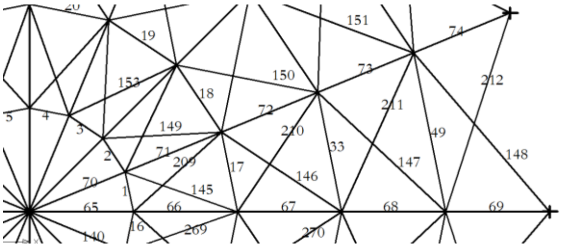

**Fig 7: Numbering of Dome members**

**Table (2) the cross section areas of trussed dome member.**

| Number of Members | Cross Sectional Area ($cm^2$) |
|---|---|
| 1:16 | 19.7645376070930 |
| 17:32 | 18.0414380425521 |
| 33:48 | 26.6462470080779 |
| 49:64 | 16.8822771716842 |
| 65:144 | 33.3458493646358 |
| 145:272 | 31.1442368705109 |
| Total weight (ton) | 25.399 |

Using Genetic algorithm to make optimization for this example (minimization of weight) by take inner diameter and inner height as a variable with constant span (40 m) and constant height (4.34 m), figure(8) shows the minimum weight and the relationship between weight and number of iterations.

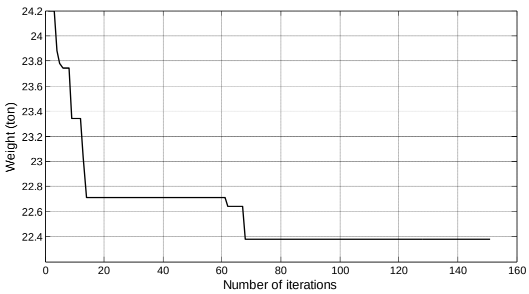


**Fig 8: minimum weight of the trussed dome.**

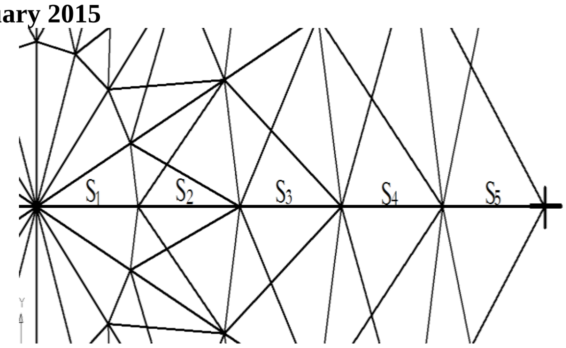


**Fig 9: spacing variables.**

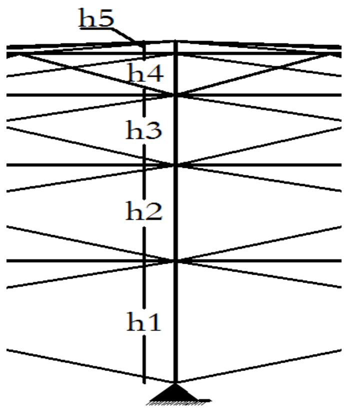


**Fig 10: Height variables**

**Table (3) the min. values of the spacing.**

| Variable | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ |
|---|---|---|---|---|---|
| Value(m) | 2.13 | 4.0 | 3.73 | 3.8667 | 6.2667 |

**Table (4) the min. values height variables.**

| Variable | $H_5$ | $H_4$ | $H_3$ | $H_2$ | $H_1$ |
|---|---|---|---|---|---|
| Value(m) | 0.34 | 0. 6 | 0.9 | 0.6 | 1.99 |

Then change the total height of trussed dome to find the optimum height for previous example which gives minimum weight, figure (11) shows the minimum weight and the relationship between weight and total height if total height is taken as a variable (4-10) m with constant span (40 m).

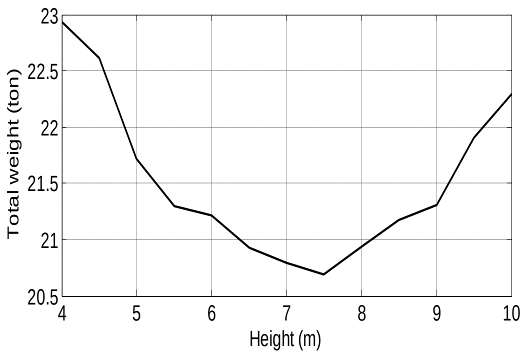


**Fig 11: minimum weight of the trussed dome with total height**

Figure (11) shows the different values of minimum weight of the trussed dome which fined at variable heights, the minimum weight act when total height equal 7.5 m, figure (12) show the minimum total weight for the trussed dome at height equal 7.5m.

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International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 8, February 2015

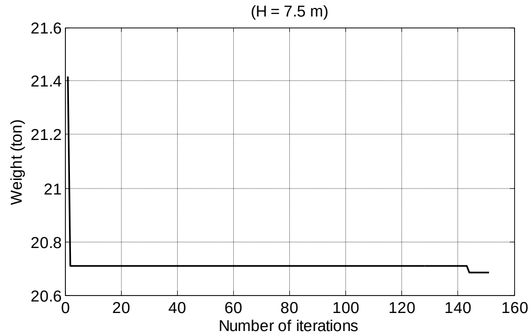


**Fig 12: minimum weight of the trussed dome at (H =7.5 m).**

If inner diameter, inner height and total height are taken as a variable with constant span (40 m) to make optimization for previous example using genetic algorithm, the minimum weight is founded 20.7370 ton as shown in figure (13).

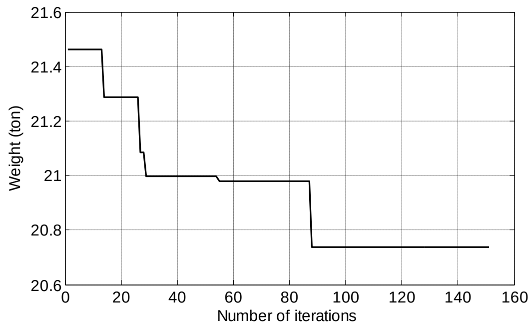


**Fig 13: minimum weight of the trussed dome.**

**Table (5) the min. values of the spacing.**

| Variable | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ |
|---|---|---|---|---|---|
| value(m) | 3.066 | 3.733 | 2.133 | 4 | 7.067 |

**Table (6) the min. values height variables.**

| Variable | $H_5$ | $H_4$ | $H_3$ | $H_2$ | $H_t$ |
|---|---|---|---|---|---|
| value(m) | 0.3133 | 0.9667 | 0.7 | 0.7 | 6.42 |

Using genetic algorithm to minimize the total weight of trussed dome by change the cross section area of trussed dome members (sizing optimization), the trussed dome consist of six group of cross section member , four groups of them is angular members , group for radial members and group for bracing members as shown in figure (14).

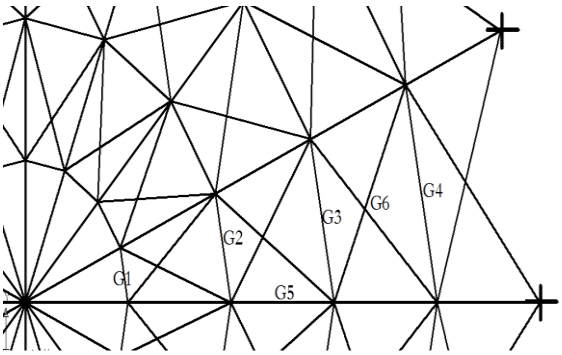


**Fig 14: Members groups numbering.**

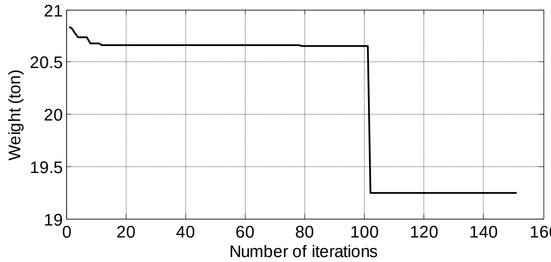


**Fig 15: minimum weight of the trussed dome.**

Figure (15) shows the value of minimum weight of the trussed dome when cross sections are variable, the minimum weight equal 19.2510 ton. So the weight percentage minimize is between (20 % - 30%).

***Dome (444-Bar dome)***

The dome structure which has 444 elements and 121 nodes is presented below. This example is taken from literature. It was solved by Sarac [8]. The top and side views of the structure are presented in Fig. 16 and Fig.17. The height of the structure is 20 m. and the radius at the ground level (z=0) is 10 m.

The download vertical loads are assumed to apply to the structure; 100,000 lbf (45,359.24 kgf) at node 1 and 10,000 lbf (4535.924 kgf) at each other free node. The structure is restrained by 12 joints in the bottom layer. The continuous cross sectional areas are used in this design. The lower bound of the cross-sectional areas is 0.1 in². The allowable tensile stress is 10,000 psi (7.031 kgf/mm²), the stress limit in compression is Euler buckling load. The displacements of the free nodes must be less than 0.25 in. (0.635 cm).

The same design conditions stated in the related article are used in order to make a comparison between results. As stated above, real load conditions and code-based design are not used. Besides, predetermined point loads and stress check for tensile and compression members are made use of.

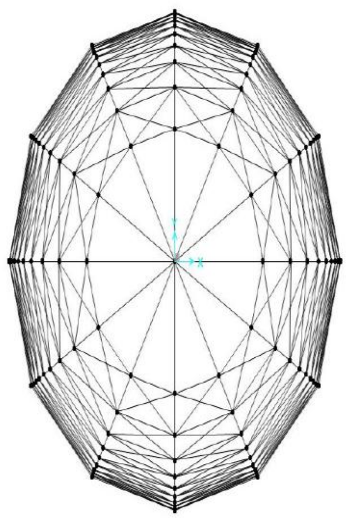

**Fig 16: Top View of 444-Bar Dome.**

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International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 8, February 2015

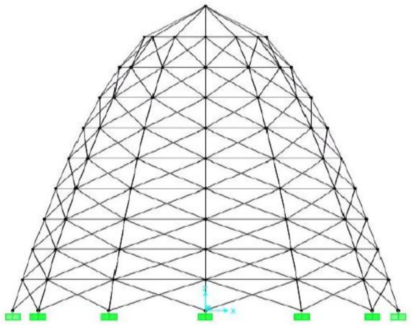

**Fig 17: Side View of 444-Bar Dome.**

**Table (7) the trails of genetic algorithm optimization.**

| Trial No. | Best design (ton) |
|---|---|
| 1 | 9.8526 |
| 2 | 9.7773 |
| 3 | 9.9146 |
| 4 | 9.9051 |

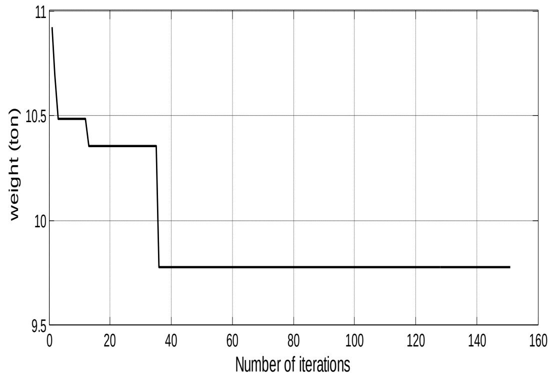


**Fig 18: minimum weight of the trussed dome.**

Then change the total height of trussed dome to find the optimum ratio of height to span for previous example which gives minimum weight by taking this ratio within range from 0.25 to 1.0 with constant span, figure (19) shows the minimum weight and the relationship between weight and total height.

**Table (8) the values of the min. weight For Different height-to-Span Ratios.**

| (Height/span) ratio | Weight (ton) |
|---|---|
| 0.1 | 18.3045 |
| 0.15 | 14.4563 |
| 0.20 | 11.6038 |
| 0.25 | 9.6375 |
| 0.30 | 9.6574 |
| 0.35 | 9.2929 |
| 0.40 | 9.5168 |
| 0.5 | 9.7773 |
| 0.75 | 12.6175 |
| 1.0 | 15.4324 |

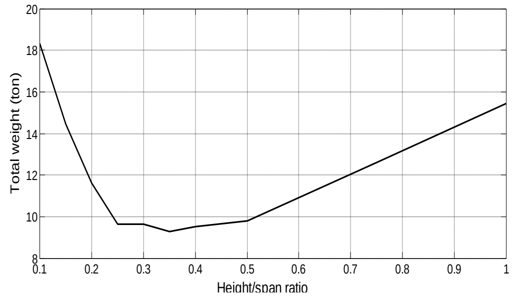


**Fig 19: (height /Span) Ratio with total Weight.**

Figure (19) shows the minimum weight act when (height/span) ratio equal 0.35, figure (20) shows the minimum total weight for the trussed dome at height equal 7m, with constant span.

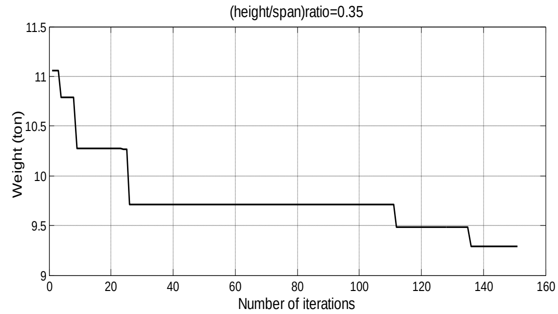


**Fig 20: minimum weight of the trussed dome at height = (7m).**

The comparison of the results of the current work and the works from literature is shown in Table (9).

**Table (9) Comparison of the Results.**

| Technique Used | Related Article | Weight (kg) |
|---|---|---|
| Sequential Linear Programming (LESLP) | Lamberti and Pappalettere | 9202.308 |
| Genetic Algorithms (SSTOGA) | YAVUZ SARAÇ | 9801.85 |
| Simulated Annealing (SSTOSA) | YAVUZ SARAÇ | 9242.09 |
| Genetic Algorithms | Current Work | 9292.9 |

As seen from Table (9), in this problem the value of total weight by using Genetic Algorithms is very near the values founded by Lamberti and Pappalettere, and Yavuz Sarac [8] and less than values founded by using genetic algorithms (SSTOGA) in Yavuz Sarac work.

## IV. CONCLUSIONS

Genetic algorithm was used in this paper to solve the configuration and sizing design for weight minimization of trussed dome structures, changing the node coordinates (continuous design variables) and cross-sections area (discrete design variables) to find the optimum variable

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International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 8, February 2015

which result the minimum total weight of structure . First example show the steps to minimize the total weight of trussed dome by change the inner spacing , inner height , total height and the cross sections of structure members, second example was used to make a comparison with previous researches results. Points can be concluded from this research:

1- The best (height/span) ratio in trussed dome structure is between 0.25 and 0.40.
2- Using genetic algorithm technique in optimization of trussed dome can save 20 % -30% from the cost.
3- Generic algorithm technique gave results close to previous researches which used other techniques.

## REFERENCES

[1] Christensen, Peter W. Klarbring, Anders. (2008),"An Introduction to Structural Optimization" Springer Netherlands.

[2] Coello Coello, Carlos A. Christiansen, Alan D. (2000), "Multiobjective optimization of trusses using genetic algorithms. Computers and Structures" Volume 75, issue 6. Pages 647-660.

[3] Roman GÄatzi, Marion Uebersax, Oliver KÄonig. (2000), "Structural Optimization Tool using Genetic Algorithms and Ansys".

[4] Kang Tai, Shengyin Wang, Shamim Akhtar and Jitendra Prasad. (2002), "Structural Topology Optimization Using a Genetic Algorithm and a Morphological Representation of Geometry".

[5] R. Kamyab, E. Salajegheh. (2012), "Size optimization of nonlinear scallop domes by an enhanced particle swarm algorithm".

[6] A. Kavehand, A. Zolghadr. (2011), "Shape and size optimization of truss structures with frequency constraints using enhanced charged system search algorithm" Asian journal of civil engineering (building and housing) vol. 12, no. 4 pages 487-509.

[7] O. Hasançebi, F. Erdal, M. P. Saka. (2010), "Optimum design of geodesic steel domes under code provisions using metaheuristic techniques" International Journal of Engineering and Applied Sciences (IJEAS) Vol.2, Issue 2(2010)88-103.

[8] Yavuz Sarac. (2005), "Optimum design of pin-jointed 3-D dome structures using global optimization techniques "thesis submitted to the graduate school of natural and applied sciences of middle east technical university.

[9] R. Kamyab Moghadas, A. Garakani, and Kalantarzadeh Hindawi. (2013), "Optimum Design of Geometrically Nonlinear Double-Layer Domes by Firefly Metaheuristic Algorithm" Mechanical Engineering Volume 2013.

[10] Hemant Chickerman and Hea Changgea. (1996), "Structural optimization using anew local approximation method" International journal for numerical methods in engineering, vol.39, 829-846.

[11] Eduardo S. Croc & Elaine G. Ferreira & Afonso C. C. Lemonge "A genetic algorithm for structural optimization of steel truss roofs".

[12] Egyptian code for practice for steel construction and bridges. Code no ecp 205-2001 edition 2008.

## AUTHOR'S PROFILE

S. Abdel-salam "Prof. of Structural Eng., Faculty of Engineering, Zagazig University".

A.M.EL-shihy "Prof. of Structural Eng., Faculty of Engineering, Zagazig University".

Atef Eraky "Prof. of Structural Eng., Faculty of Engineering, Zagazig University".

Mohammed Salah "Demonstrator, Department of Structural Eng., Faculty of Engineering, Zagazig University