ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 7, January 2015

Solution of Integro differential equation by Taylor collocation Method

Chirag R Trivedi
Department Science and Humanity (Mathematics)
Vadodara Institute of Engineering Kotambi Vadodara -391510-Gujarat

Abstract: *In this paper we use Taylor Series as an approximate solution of the differential equation. We develop an algorithm of Taylor Collocation Method based on Taylor series for solving various integro differential equation using the prescribed method the solution of this problem is reducing to a system equation. The result demonstrates the applicability and accuracy of the algebraic method.*

Keywords: **Taylor Collocation Method, Integro differential equation, Voltera Integro differential equation**.

I. INTRODUCTION

The Taylor series algorithm is one of the earliest algorithms for the approximate solution for initial value problem for ordinary differential equation. Newton used it in his calculation and Euler described in his work. The basic idea of these developments was the recursive calculation of the coefficients of Taylor series. A Taylor expansion approach to solving integral equation is represented by Knawel and Liu and the method extended by Sezer to Volteral equations and Integro differential equations.

Integro differential equations find useful in a wide range of application field such as Computer, Graphics, Image Processing, Biological problems and financial problems. Therefore their numerical solutions are very useful to analyze the problems related to various field.

- ***Algorithm of Taylor Collocation Method for Integro Differential Equation***

Consider the Integro differential equation of the form

$p(x)y'(x) + y(x) = f(x) + \int_a^b g(x)\, y(t)\, dt$ (1)

$y(a) = k_1$

Where $p(x)$, $f(x)$ and $g(x)$ are función of x. k_1 is Known constant

Let $y(x) = \sum_{n=0}^{N+1} \frac{y^n(0)}{n!} x^n$

be the approximate solution of equation (1),Now differentiating equation (1) with respect to x

$$y'(0) = \sum_{n=0}^{N+1} \frac{y^n(0)}{n!} n x^{n-1};$$

(2)

Substituting this value in equation (1) we get

$$p(x)\sum_{n=0}^{N+1}\frac{y^n(0)}{n!} n x^{n-1} + \sum_{n=0}^{N+1}\frac{y^n(0)}{n!} x^n = f(x) + \int_a^b g(x)\sum_{n=0}^{N+1}\frac{y^n(0)}{n!} t^n dt$$

By simplification we get,

$$\sum_{n=0}^{N+1}\frac{y^n(0)}{n!}\left[p(x) n x^{n-1} + x^n - g(x)\left[\frac{b^{n+1}-a^{n+1}}{n+1}\right]\right] = f(x)$$

(3)

At the Collocation points $x = x_i$ we obtain

$\sum_{n=0}^{N+1}\frac{y^n(0)}{n!}\left[p(x_i) n x_i^{n-1} + x_i^n - g(x_i)\left[\frac{b^{n+1}-a^{n+1}}{n+1}\right]\right] = f(x_i)$;

i = 0, 1,2,…N (4)

Now the condition $y(a) = k_1$ gives

$y(0) + y'(0)a + \frac{y^2(0)}{2!}a^2 + \text{--------} \frac{y^{N+1}}{(N+1)!} a^{N+1} = k_1$

(5)

The equation (4) gives the following set of equations

At $x = x_0$

$y(0)\left[1 - g(x_0)\left(\frac{b-a}{1}\right)\right] + y'(0)\left[p(x_0) + x_0 - g(x_0)\left[\frac{b^2-a^2}{2}\right]\right] +$

$\frac{y''(0)}{2!}\left[2x_0 p(x_0) + x_0^2 - g(x_0)\left[\frac{b^3-a^3}{3}\right]\right] + \ldots\ldots. +$

$\frac{y^{N+1}(0)}{(N+1)!}\left[(N+1)x_0^N p(x_0) + x_0^{N+1} - g(x_0)\left[\frac{b^{N+2}-a^{N+2}}{N+2}\right]\right] = f(x_0)$

(6)

At $x = x_1$

$y(0)\left[1 - g(x_1)\left(\frac{b-a}{1}\right)\right] + y'(0)\left[p(x_1) + x_1 - g(x_1)\left[\frac{b^2-a^2}{2}\right]\right] +$

$\frac{y''(0)}{2!}\left[2x_1 p(x_1) + x_1^2 - g(x_1)\left[\frac{b^3-a^3}{3}\right]\right] + \ldots\ldots\ldots +$

$\frac{y^{N+1}(0)}{(N+1)!}\left[(N+1)x_1^N p(x_1) + x_1^{N+1} - g(x_1)\left[\frac{b^{N+2}-a^{N+2}}{N+2}\right]\right] = f(x_1)$

(7)

At $x = x_2$

$y(0)\left[1 - g(x_2)\left(\frac{b-a}{1}\right)\right] + y'(0)\left[p(x_2) + x_2 - g(x_2)\left[\frac{b^2-a^2}{2}\right]\right] +$

$\frac{y''(0)}{2!}\left[2x_2 p(x_2) + x_2^2 - g(x_2)\left[\frac{b^3-a^3}{3}\right]\right] + \ldots\ldots\ldots\ldots$

$+\frac{y^{N+1}(0)}{(N+1)!}\left[(N+1)x_2^N p(x_2) + x_2^{N+1} - g(x_2)\left[\frac{b^{N+2}-a^{N+2}}{N+2}\right]\right] = f(x_2)$

(8)

Similarly At $x = x_N$

$y(0)\left[1 - g(x_N)\left(\frac{b-a}{1}\right)\right] + y'(0)\left[p(x_N) + x_N - g(x_N)\left[\frac{b^2-a^2}{2}\right]\right] +$

$\frac{y''(0)}{2!}\left[2x_N p(x_N) + x_N^2 - g(x_N)\left[\frac{b^3-a^3}{3}\right]\right] +$

$\ldots\ldots. + \frac{y^{N+1}(0)}{(N+1)!}\left[(N+1)x_N^N p(x_N) + x_N^{N+1} - g(x_N)\left[\frac{b^{N+2}-a^{N+2}}{N+2}\right]\right] = f(x_N)$

(9)

From the equation (6) to (9) we get the matrix equation

AX=B, **Refer Appendix-I**.

Solving this system we get the value of $y(0), y'(0), y''(0)\ldots\ldots., y^{N+2}(0)$ and hence an approximate solution is obtained from equation (1)

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 7, January 2015

❖ *Application of Taylor collocation Method*

In this section we solved a BVP and various integro-differential equations by the method of Taylor collocation

Example 1: Let us consider the Integro differential equation

$$y'(x) = 1 - \frac{3}{2}x + \int_0^1 xy(t)dt$$

$$y(0) = 1$$

Assume that the Taylor series $y(x) = \sum_{n=0}^{N+1} \frac{y^n(0)}{n!} x^n$ as an approximate solution of equation (1)

Find the value of $y'(x)$ from the above equation and putting the value of $y(x), y'(x)$ in equation (1) and by simplification we get

$$\sum_{n=0}^{N+1} \frac{y^n(0)}{n!} [nx^{n-1} - \frac{x}{n+1}] = 1 - \frac{3}{2}x$$

(10)

At the collocation point $x = x_i$ the equation (10) gives the collocation equation

$$\sum_{n=0}^{N+1} \frac{y^n(0)}{n!} \left[nx_i^{n-1} - \frac{x_i}{n+1}\right] = 1 - \frac{3}{2}x_i$$

where i = 0,1,2 …5 (11)

For N = 5 we obtain the collocation equation

$$y(0)[-x_i] + y'(0)[1 - \frac{x_i}{2}] + \frac{y''(0)}{2!}\left[2x_i - \frac{x_i}{3}\right] + \frac{y'''(0)}{3!}\left[3x_i^2 - \frac{x_i}{4}\right] + \frac{y^{IV}(0)}{4!}\left[4x_i^3 - \frac{x_i}{5}\right] + \frac{y^{V}(0)}{5!}\left[5x_i^4 - \frac{x_i}{6}\right] + \frac{y^{VI}}{6!}\left[6x_i^5 - \frac{x_i}{7}\right] \quad (12)$$

i = 0,1,..5

Now the condition $y(0) = 1$ gives

$$(1)y(0) + (0)y'(0) + (0)y''(0) + (0)y'''(0) + (0)y^{IV}(0) + (0)y^{V}(0) + (0)y^{VI}(0) = 1$$

(13) Now equation (11) and (12) for i=0,1,2,3,4,5 gives the following System of equation.

*For these **Refer Appendix-II***

Solve the System of equation we get
$y(0) = 1, y'(0) = 1, y''(0) = 1.88738E - 15,$
$y'''(0) = 3.55271E - 14, y^{IV}(0) = 9.9476E - 14$
$y^{V}(0) = -6.82121E - 13, y^{VI}(0) = 1.81899E - 12$

Substituting all these values in equation (1) we get the approximate solution in the form of Taylor series as follow.

$$y(x) = 1 + x + (1.88738E - 15)x^2 + (3.55271E - 14)x^3 + (9.947E - 14)x^4 + (-6.8212E - 13)x^5 + (1.81899E - 12)$$

Comparison of solution with exact solutions is shown in below Table

Value of x	Solution by Taylor Collocation Method	Exact Solution X+1
0	1	1
0.2	1.2	1.2
0.4	1.4	1.4
0.6	1.6	1.6
0.8	1.8	1.8
1	2	2

Example-2: Let us consider the Integro differential equation

$$y'(x) = 1 - \frac{1}{3}x + \int_0^1 xty(t)dt \quad y(0) = 0 \quad (14)$$

Solution: Assume that the Taylor series

$y(x) = \sum_{n=0}^{N+1} \frac{y^n(0)}{n!} x^n$ as an approximate solution of equation (14)

Find the value of $y'(x)$ from the above equation and putting the value of $y(x), y'(x)$ in equation (14) and by simplification we get

$$\sum_{n=0}^{N+1} \frac{y^n(0)}{n!} [nx^{n-1} - \frac{x}{n+2}] = 1 - \frac{x}{3}$$

(15)

At the collocation point $x = x_i$ the equation (2) gives the collocation equation

$$\sum_{n=0}^{N+1} \frac{y^n(0)}{n!} \left[nx_i^{n-1} - \frac{x_i}{n+2}\right] = 1 - \frac{x_i}{3}$$

(16)

Where i = 0,1,2 …5

For N = 5 we obtain the collocation equation

$$y(0)[-x_i] + y'(0)[1 - \frac{x_i}{2}] + \frac{y''(0)}{2!}\left[2x_i - \frac{x_i}{3}\right] + \frac{y'''(0)}{3!}\left[3x_i^2 - \frac{x_i}{4}\right] + \frac{y^{IV}(0)}{4!}\left[4x_i^3 - \frac{x_i}{5}\right] + \frac{y^{V}(0)}{5!}\left[5x_i^4 - \frac{x_i}{6}\right] + \frac{y^{VI}}{6!}\left[6x_i^5 - \frac{x_i}{7}\right]$$

i = 0,1,..5

Now the condition $y(0) = 0$ gives

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 7, January 2015

$(1)y(0) + (0)y'(0) + (0)y''(0) + (0)y'''(0) + (0)y^{IV}(0) + (0)y^{V}(0) + (0)y^{VI}(0) = 0$

(17)

Now equation (16) and (17) for i=0, 1, 2, 3, 4,5

Gives the following System of equation

For these Refer **Appendix-III**

Solve the System of equation we get

$y(0) = 0, y'(0) = 1, y''(0) = 3.16352E-05,$
$y'''(0) = -0.000601605\, y^{IV}(0) = 0.004725681$
$y^{V}(0) = -0.013619701, y^{VI}(0) = -0.012812147$

Substituting all these values in equation (1) we get the approximate solution in the form of Taylor series as follow.

$$y(x) = x + (3.16352E-05)x^2 + (-0.000601605)x^3 + (0.004725681)x^4 + (-0.013619701)x^5 + (-0.012812147)x^6$$

Comparison of solution with exact solutions is shown in Table

Value of X	Solution by Taylor Collocation Method	Exact Solution X
0	0	0
0.2	0.2	0.2
0.4	0.3999	0.4
0.6	0.5999	0.6
0.8	0.7999	0.8
1	0.9999	1

II. CONCLUSION

- For the first order IDE, the coefficient matrix is of order (N+1) × (N+1).
- In this method the unknown Taylor coefficients are calculated numerically and the solution is obtained in the form of a function of independent variable, and hence this method can be regarded as numerical analytical method.
- Computation of this method, using computers, is very easy, as the function evaluation can be performed using simple matrix –vector operations.
- The function evaluation at any point in the interval is possible.
- The numerical solutions obtained by the present method are in agreement with the exact solutions when the step size is large enough.

Appendix-I

$$\begin{bmatrix} 1 & a & & \frac{a^{N+1}}{(N+1)!} \\ 1 - g(x_0)(a-b) & x_0 + p(x_0) - g(x_0)\left(\frac{a^2-b^2}{2}\right) & \text{------} & x_0^{N+1} + (N+1)x_0^{n}\, p(x_0) - g(x_0)\left(\frac{a^{N+2}-b^{N+2}}{N+2}\right) \\ 1 - g(x_1)(a-b) & x_1 + p(x_1) - g(x_1)\left(\frac{a^2-b^2}{2}\right) & \text{------} & x_1^{N+1} + (N+1)x_1^{N}\, P(x_1) - g(x_1)\left(\frac{a^{N+2}-b^{N+2}}{N+2}\right) \\ \ldots & \ldots. & & \ldots\ldots\ldots \\ 1 - g(x_N)(a-b) & x_N + p(x_N) - g(x_N)\left(\frac{a^2-b^2}{2}\right) & \text{------} & x_N^{N+1} + (N+1)x_N^{N}\, p(x_n) - g(x_N)\left(\frac{a^{N+1}-b^{N+2}}{N+2}\right) \end{bmatrix}$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 7, January 2015

$$X = \begin{bmatrix} y(0) \\ y'(0) \\ y''(0) \\ y'''(0) \\ \cdots\cdots\cdots \\ y^{n}(0) \end{bmatrix} \qquad B = \begin{bmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \\ f(x_3) \\ \cdots\cdots\cdots \\ f(x_n) \end{bmatrix}$$

Appendix-II

1	0	0	0	0	0	0
0	1	0	0	0	0	0
-0.2	0.9	0.1666	0.011666	-0.0003333	-0.00021111	-3.70159E-05
-0.4	0.8	0.3333	0.06333	0.0073333	0.00051141	5.968259E-06
-0.6	0.7	0.5	0.155	0.031	0.0045666	5.28952E-04
-0.8	0.6	0.666	0.28666	0.078666	0.0159555	0.00271965
-1	0.5	0.8333	0.458333	0.158333	0.040277	0.00813492

$$\begin{bmatrix} y(0) \\ y'(0) \\ y''(0) \\ y'''(0) \\ y^{IV}(0) \\ y^{V}(0) \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 0.7 \\ 0.4 \\ 0.1 \\ -0.2 \end{bmatrix}$$

Appendix-III

A =

1	0	0	0	0	0	0
0	1	0	0	0	0	0
-0.1	0.9333	0.175	0.013333	-0.0000555	-0.000171429	-3.20555E-05
-0.2	0.8666	0.35	0.06666	0.0078888	0.0005904	1.588888E-05
-0.3	0.8	0.525	0.16	0.0318333	0.004685714	5.43833E-04
-0.4	0.7333	0.7	0.29333	0.079777	0.016114285	0.002591778

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 7, January 2015

-0.5	0.6666	0.875	0.46667	0.1597222	0.04047619	0.000815972

$$\begin{bmatrix} y(0) \\ y'(0) \\ y''(0) \\ y'''(0) \\ y^{IV}(0) \\ y^{V}(0) \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \\ 0.9333 \\ 0.8666 \\ 0.8 \\ 0.7333 \end{bmatrix}$$

REFERENCES

[1] Sastry.s.s Introductory Methods of Numerical Analysis PHI Learning 4thEd. (2009).

[2] Jain.M.K. Numerical solution of Differential equations, 2nd Ed., New Age Int.Pt.Ltd, 2002.

[3] Karamete A and sezer M. A Taylor collocation method for the solution of linear integro differential equations. intern. Computer Math., 2002.Vol 79(9).pp-987-1000.

[4] M.I. Berenguer, M.A. Fortes, A.I. Gerralda- Guillem, M.RuizGalán, linear Volterraintegro – differential equation and schauder bases, Appl. math. Com – put. 159(2) (2004) 495-507.

[5] M.I. Berenguer, A.I. Gerralda - Guillem, M.RuizGalán , Biorthogonal systems approximating the solution of the nonlinear Volterraintegro – differential equation, Fixed point theory Appl Article ID 970/49, 9 p., doi: 1o. 1155/2010/470149, 2010.

[6] M.I. Berenguer , M.V. Fernando 2 Munoz , A.I. Gerralda – Guillem , M.RuizGalán , A sequential approach for solving the Fredholmintegro – differential equation , Applied Numerical mathematics 6297-304.

[7] P.Darania, Ali, Ebadian, A method for the numerical solution of the integro – differential equations, Appl. Math. Comput. 188 (2007) 657-668.

[8] E.Yusufoğlu (Agadjanov), Improved homotopy perturbation method for solving Fredholm type integro – differential EQUATIONS, chaos solutions fractals 41(2009) 28-37.

AUTHOR BIOGRAPHY

Chirag kumar Rajendrabhai Trivedi

Qualification: Msc (Applied Mathematics)

Year of Passing: 2008

University: The M. S. University Baroda

Mphil (Applied Mathematics)

November-2011

VNSGU,Surat

At Present I am working as Lecturer in Vadodara Institute of Engineering since July 2011.